

DISPERSION OF MINERALS

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The new methods of studying powdered minerals lead naturally, not only to a knowledge of their indices of refraction in ordinary light, but also and simultaneously to a knowledge of the variation in their indices with variation in the wave-length (or color) of light; that is, the new methods lead to a knowledge of the dispersion of minerals. Of course, the dispersion of a mineral is as definite a physical property as its refringence or birefringence and, therefore, may be used to identify it. At present, data regarding the dispersion of minerals are remarkably scanty and in some cases not very accurate. All available data on dispersion (of reasonable accuracy) are assembled in the following table. In order to make these data useful for determinative purposes, the minerals are classified first in groups of increasing dispersion, and then arranged in each group in the order of increasing refringence.

The dispersion given is always the difference between the index in light of 4861 Å wave-length (= the Fraunhofer line F or the β line of hydrogen) and the index in light of 6563 Å wave-length (= the Fraunhofer line C or the α line of hydrogen). In the tables this dispersion is meant by the expression F-C.

In anisotropic crystals the dispersion of the substance is, in general, not the same for light vibrating in directions which are crystallographically unlike. Therefore the dispersion for N_o is not the same as for N_p , and the dispersion for N_m is not equal to that for N_o nor to that for N_p . However, these differences are usually small, and the only dispersion given in the table is that for N of isotropic substances, N_o of uniaxial substances and N_m of biaxial substances (with rare exceptions, in cases where data are not available for N_m).

In the literature the dispersion is rarely given directly and the indices measured for different wave-lengths do not always include the wave-lengths of the F and C lines, which are the standards used in computing the dispersion. Therefore, it is necessary to obtain this dispersion by graphic (or mathematical) solution in many cases; when this involves considerable extrapolation the value of the dispersion is followed by the \pm sign. In order to permit checking of this extrapolation and also to permit the preparation of dispersion curves for all minerals, the original data

are given for yellow (the sodium D line unless otherwise noted), blue (or green—the exact wave-length given whenever known) and red (the wave-length given, if known). If a chemical analysis of the mineral sample whose dispersion was measured is available, the name of the mineral is followed by an asterisk. Finally, in order to permit verification of the data, the name of the observer and the place of publication are given in all cases. If a later publication of the data is probably more accessible than the first, that is commonly given preference.

The dispersion of minerals increases, in general, with their refringence; nevertheless, minerals of the same refringence do not necessarily have the same dispersion. An interesting and possibly important illustration of this fact is furnished by the data regarding sillimanite and mullite. These two minerals are so much alike that Eitel¹ and Rinne² consider that they are not essentially different. They are both orthorhombic and have crystal forms which are very similar. The common prism faces are nearly at right angles to each other and both minerals have perfect cleavage parallel to the brachypinacoid. In fact, Wyckoff³ was at first unable to distinguish between them by means of X-ray patterns, though he later found⁴ "very slight but probably real differences." In optical properties, also, they are almost identical. According to Bowen, Greig and Zies,⁵ they both have the optic plane parallel with the perfect cleavage and in both the slow ray vibrates parallel with the vertical axis, giving positive elongation. They have the same optic sign, nearly the same refringence, nearly the same birefringence and nearly the same optic angle; in fact, when mullite contains four to five percent of ($\text{TiO}_2 + \text{Fe}_2\text{O}_3$), it is indistinguishable from sillimanite even by means of accurate measurements of these optic properties. According to Bowen, Grieg and Zies, such mullite can be distinguished from sillimanite by chemical analysis or by the fact that it is colored and pleochroic while sillimanite is colorless. Since differences between mullite and sillimanite are so difficult to find, it seems important to note that the dispersion of mullite (.026) is more than double that of

¹ *Zeit. Kryst.*, **LXIV**, 1927, p. 535.

² *Zeit. Kryst.*, **LXI**, 1925, p. 113.

³ *Jour. Am. Cer. Soc.*, **VII**, 1924, p. 253.

⁴ *Am. Jour. Sci.*, **CCXI**, 1926, p. 459.

⁵ *Jour. Wash. Acad. Sci.*, **XIV**, 1924, p. 183.

sillimanite (.01 to .012) according to data now available. This difference, like the color, is doubtless due in part to the small tenor of ($\text{TiO}_2 + \text{Fe}_2\text{O}_3$) in the mullite. It seems improbable to the writer that it is due wholly or even chiefly to the titanitic acid and ferric iron, and, if mullite containing none of these substances has a dispersion notably higher than that of sillimanite, it would be difficult to explain that condition on the basis of submicroscopic inclusions of corundum in sillimanite, since the dispersion of corundum is no greater than that of sillimanite. Of course, it is also difficult to understand why admixed corundum (with $N=1.76$) should lower the indices of sillimanite from about 1.67 to about 1.65.

It is clear that minerals differ in their dispersion, and in some cases this difference will be very useful in distinguishing between them. The scanty data now available are summarized in the following table. Such data will doubtless become much more abundant as the new dispersion methods of measuring indices of refraction come into wide usage.

DISPERSION OF MINERALS

F-C	N, N ₆ or N _m			Mineral	Authority	Reference
	D	Blue	Red			
Section 1. N_F - N_C = .000 to .0075						
.0062	1.309	1.3129 (492)	1.3069 Li	Ice	A. Ehringhaus:-	<i>N. Jahrb. Min. Bl. Bd. XLI, 1917, p. 369.</i>
.0039	1.3258	1.3285 F	1.3246 C	Williamite	K. Spangenberg:-	<i>Zeit. Kryst., LVII, 1923, p. 494.</i>
.004 ±	1.3395	1.3408 TI	1.3382 Li	Cryolithionite*	N. V. Ussing:-	Dana: <i>System Min., App. II, 1909, p. 33.</i>
.004	1.4339	1.4369 F	1.4326 Li	Fluorite	S. Kozu:-	<i>Mineral. Mag., XVII, 1916, p. 256.</i>
.0049	1.4425	1.4462 F	1.4413 C	Yttrifluorite*	F. Zambonini:-	<i>Zeit. Kryst., LVI, 1921, p. 219.</i>
.0071	1.4629	1.4678 F	1.4607 C	Picromerite (art.)	A. E. H. Tutton	<i>Trans. Chem. Soc. London, LXXXVII, 1905, p. 1173.</i>
.006 ±	1.502	1.506 bl	1.500 red	Prosopite	A. Des Cloizeaux:-	Dana: <i>System Min. 1892, p. 178.</i>
.006	1.5084	1.5131 (475)	1.5073 (633)	Noceite*	F. Zambonini:-	<i>Zeit. Kryst., LVI, 1921, p. 219.</i>
.0065	1.5329	1.5343 TI	1.5281 Li	Langbeinite	O. Lueddecke:-	<i>Zeit. Kryst., XXIX, 1897, p. 255.</i>
.005 ±	1.5457	1.5480 TI	1.5448 red	Eudidymite*	W. C. Broegger:-	<i>Zeit Kryst., XVI, 1890, p. 586.</i>

	1.5812	1.5849 TI	1.5802 Li	Scapolite	A. Laitakari:- G. Flink:-	<i>Bull. Com. Geol. Finland,</i> LIV, 1921, p. 76. Dana:- <i>System. Min.</i> , 1892, p. 388.
.007 ±		1.620 TI	1.616 Li	Tremolite		
.006 ±						
Section 2. N_F - N_C = .0075 to .0085						
.0079	1.4554	1.4611 F	1.4532 C	Epsomite	Borel:-Groth:-	<i>Chem. Kryst.</i> , II, 1908, p. 430.
.0077	1.4560	1.4614 F	1.4537 C	Potassalumite	A. Mulheims:-	<i>Zeit. Kryst.</i> , XIV, 1888, p. 223.
.0081	1.4694	1.4750 F	1.4669 C	Borax (art.)	Dufet:-Groth:-	<i>Chem. Kryst.</i> , II, 1900, p. 132.
.0081	1.4730	1.4786 F	1.4705 C	Boussingaultite (art.)	A. E. H. Tutton:-	<i>Trans. Chem. Soc. London,</i> LXXXVII, 1905, p. 1123.
.0084	1.4801	1.4860 F	1.4776 C	Goslarite (art.)	Topsoë and Christen- son:-Groth:-	<i>Ann. Phys.</i> , I, 1874, p. 63; <i>Chem. Kryst.</i> , II, p. 437;
.008 ±	1.4888	1.4949 F	?	Morenosite (art.)	Topsoë and Chr:- Groth:-	<i>Ann. Phys.</i> , I, 1874, p. 63; <i>Chem. Kryst.</i> , II, p. 437.
.008	1.5080	1.5107 TI	1.5050 Li	Leucite	F. Zambonini:-	<i>Zeit. Kryst.</i> , LV, 1915, p. 299.
.008	1.5095	1.5115 TI	1.5050 Li	Pirssonite*	J. H. Pratt:-	<i>Am. Jour. Sci.</i> , II, 1896, p. 126.
.008 ±	1.5124	1.5148 TI	1.5093 Li	Flagstaffite (art.)	Arzrumi:-Groth:-	<i>Chem. Kryst.</i> , III, 1910, p. 658.

* Analysis given.

DISPERSION OF MINERALS

F-C	N ₁ , N ₀ or N _m			MINERAL	AUTHORITY	REFERENCE
	D	Blue	Red			
.008	1.5156 (570)	1.5207 (475)	1.5128 (633)	Cancrinite*	F. Zambonini:-	<i>Zeit. Kryst.</i> , LV, 1915, p. 305.
.008	1.5225	1.5282 F	1.5195 Li	Adularia	S. Kozu:-	<i>Mineral. Mag.</i> , XVII, 1916, p. 253.
.0079	1.5229	1.5283 F	1.5204 C	Gypsum	Lang:-	<i>Sitz. Akad. Wiss. Wien</i> , LX XVI, 1877, p. 793.
.0083	1.5251	1.5312 F	1.5225 Li	Sanidine	S. Kozu:-	<i>Mineral. Mag.</i> , XVII, 1916, p. 238.
.008	1.533	1.539 F	1.531 C	Albite	S. Tsuboi:-	<i>Mineral. Mag.</i> , XX, 1923, p. 108.
.0084	1.5376	1.5474 (453)	1.5343 (667)	Nephelite*	F. Zambonini:-	<i>Zeit. Kryst.</i> , LV, 1915, p. 302.
.0084	1.5372	1.5431 F	1.5347 C	Cordierite*	L. Oppenheimer:-	<i>Zeit. Kryst.</i> , LVII, 1922, p. 312.
.0083	1.543	1.548 F	1.540 C	Oligoclase	S. Tsuboi:-	<i>Mineral. Mag.</i> , XX, 1923, p. 108.
.008 ±	1.5441	1.5466 green	1.5417 red	Epididymite	G. Flink:-	<i>Med. Gröb.</i> , XXIV, 1901, p. 70; <i>Zeit. Kryst.</i> , XXIV, 1901, p. 653.
.0078	1.5442	1.5497 F	1.5419 C	Quartz	A. Ehringhaus:-	<i>N. Jahrb. Min., Bl. Bd.</i> , XLIII, 1920, p. 602.

.0084	1.5492	1.5522 Tl	1.5466 Li	Edingtonite	O. Nordenskiöld:-	<i>Geol. För. Förh.</i> , XVII, 1895, p. 597.
.008	1.554 (570)	1.559 (475)	1.550 (667)	Grothite	F. Zambonini:-	<i>Zeit. Kryst.</i> , LVII, 1922, p. 107.
.008	1.5579	1.5604 Tl	1.5550 Li	Beryllonite*	E. S. Dana:-	<i>Am. Jour. Sci.</i> , XXXVII, 1889, p. 23.
.0078	1.5752	1.5808 F	1.5730 C	Anhydrite	A. Mühlheims:-	<i>Zeit. Kryst.</i> , XIV, 1888, p. 228.
.008	N _p =1.587 N _m =1.593	1.593 F	1.590 Tl	Celsian (art.)	P. Eskola:-	<i>Am. Jour. Sci.</i> , CCIV, 1922, p. 331.
.008	1.5990	1.6016 Tl	1.5953 Li	Muscovite*	R. C. Sabot:-	<i>Zeit. Kryst.</i> , LVII, 1922, p. 417.
.0078	1.6181	1.6234 F	1.6154 C	Topaz	A. Mühlheims:-	<i>Zeit. Kryst.</i> , XIV, 1888, p. 202.
.0084	1.6237	1.6296 F	1.6212 C	Celestite	A. Arzruni:-	<i>Zeit. Kryst.</i> , I, 1877, p.188.
.008±	1.6339	?	1.6312	Melilite	Henniges:-	<i>Mikr. Phys.</i> , 1905, p. 69.
.008±	1.637±	1.643 blue	1.635 red	Anthophyllite*	Rosenbusch:-	<i>Nov. Rech.</i> , 1867, p. 541;
.008±	N _p =1.6691 N _p =1.6420	1.6706 Tl 1.6438 Tl	1.6643 Li 1.6386 Li	Eosphorite*	A. Des Cloitzaux:-	<i>Hintze-Hdb. Mineralck.</i> , II, 1897, p. 1180.
					J. Drugman:	<i>Mineral. Mag.</i> , XVII, 1914, p. 199.

Section 3. N_F-N_C= .0085 to .0095

.009	1.470	1.478 blue	1.469 red	Melanterite	Des Cloitzaux:-	Dana:- <i>System Min.</i> , 1892, p. 942.
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* Analysis given.

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F-C	N, N ₀ or N _m			MINERAL	AUTHORITY	REFERENCE
	D	Blue	Red			
.009 ±	1.4790	1.4817 Tl	1.4763 Li	Natrolite*	W. A. Broegger:-	<i>Zeit. Kryst.</i> , III, p. 480; XVI, p. 615.
.009 ±	1.4827	1.4855 Tl	1.4796 Li	Sodalite*	K. Feussner:-	<i>Zeit. Kryst.</i> , V, p. 581.
.0091	1.5246	1.5309 F	1.5218 C	Cancrinite	M. Albis:-	<i>Rend. Accad. Lincei</i> , XXX, 1920, p. 472.
.009 ±	1.5247	1.5273 Tl	1.5215 Li	Pollucite*	S. L. Penfield:-	<i>Am. Jour. Sci.</i> , XLI, 1891, p. 213.
.009 ±	1.5281	?	1.5251	Anorthoclase*	F. Fouqué:-	<i>Bull. Soc. Fr. Min.</i> , XVII, 1894, p. 283.
.0089	1.5344	1.5408 F	1.5319 C	Apophyllite	A. Ehringhaus:-	<i>N. Jahrb. Min., Bl. Bd.</i> , XII, 1917, p. 374; and XLIII, 1920, p. 602.
.0087	1.5376	1.5474 (453)	1.5343 (667)	Nephelite*	F. Zambonini:-	<i>Zeit. Kryst.</i> , LV, 1915, p. 302.
.009 ±	1.5385	1.5409 Tl	1.5348 Li	Gismondite	F. Rinne:-	<i>Sitz. Akad. Wiss. Berlin</i> , 1890, p. 1023; <i>Zt. Kr.</i> , XX, 1892, p. 302.
.009	1.546	1.5523 F	1.5433 C	Cordierite*	L. Oppenheimer:-	<i>Zeit. Kryst.</i> , LVII, 1922, p. 312.
.0086	1.552 ±	1.558 F	1.549 C	Andesine	S. Tsuboi:-	<i>Mineral. Mag.</i> , XX, 1923, p. 108.

.0087	1.5579	1.5604 TI	1.5550 Li	Beryllonite*	E. S. Dana:—	<i>Am. Jour. Sci.</i> , XXXVII, 1889, p. 23.
.009	1.563 ±	1.569	1.560	Labradorite	S. Tsuboi:—	<i>Mineral. Mag.</i> , XX, 1923, p. 108.
.0093	1.573	1.579 F	1.570 C	Bytownite	S. Tsuboi:—	<i>Mineral. Mag.</i> , XX, 1923, p. 108.
.009	1.574 ±	1.580 (480)	1.571 Li	Beryl	Offret:—Hintze:—	<i>Hdb., Min.</i> , II, 1897, p. 1273.
.0095	1.583	1.585 F	1.580 C	Anorthite	S. Tsuboi:—	<i>Mineral. Mag.</i> , XX, 1923, p. 108.
.009	1.5873	1.5901 TI	1.5838 Li	Scapolite*	R. Brauns:—	<i>Zeit. Kryst.</i> , LVII, 1922, p. 214.
.009	1.5893 (578)	1.5944 (492)	1.5851 (691)	Hamburgite	A. Ehringhaus:—	<i>N. Jahrb. Min., Bl. Bd.</i> , XII, 1917, p. 381.
.009	1.5920	1.5981 F	1.5892 C	Colemanite	A. Mühlheims:—	<i>Zeit. Kryst.</i> , XIV, 1888, p. 230.
.0094	1.6022	1.6085 F	1.5991 C	Zunyite	M. Albis:—	<i>Rend. Accad. Lincei</i> , XXX, 1921, p. 472.
.009	1.6371	1.6436 F	1.6346 C	Barite	A. Arzruni:—	<i>Zeit. Kryst.</i> , I, 1877, p. 102.
.009	1.6455 ±	1.6527 F	1.6426 C	Tourmaline (Yellow)	K. Schlossmacher:—	<i>Cent. Min.</i> , 1919, p. 106.
.009	1.654	1.661 (480)	1.651 Li	Phenacite	Offret:—Groth:—	<i>Chem. Kryst.</i> , II, 1908, p. 247.
.009	1.6553	1.6606 (502)	1.6521 (688)	Euclaseite	L. Weber:—	<i>Zeit. Kryst.</i> , LVII, 1923, p. 407.
.009 ±	1.6596	1.6625 TI	1.6557 Li	Sillimanite	L. J. Spencer:—	<i>Mineral. Mag.</i> , XIX, 1920, p. 107.

* Analysis given.

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				N, N ₀ or N _m		MINERAL	AUTHORITY	REFERENCE
F-C	D	Blue	Red					
Section 4. N_F-N_C= .0095 to .0105								
.010	1.4839	1.4869 Tl	1.4808 Li	Sodalite	K. Feussner:-	<i>Zeit. Kryst.</i> , V, 1881, p. 581.		
.010	1.5096	1.5180 bl.	1.5078 red	Petalite	A. Des Cloizeaux:-	<i>Man. Mineral.</i> , II, 1874, p. 36.		
.01 ±	1.5144	1.518 Tl	1.5117 Li	Northupite*	J. H. Pratt:-	<i>Am. Jour. Sci.</i> , II, 1896, p. 123.		
.010 ±	1.5734	1.5771 E	1.5703 B	Beryl	A. Schrauf:-	<i>Sitz. Akad. Wiss. Wien</i> , XLII, 1860, p. 116.		
.0095	1.583	1.5895 F	1.580 C	Anorthite	S. Tsuboi:-	<i>Mineral. Mag.</i> , XX, 1922, p. 108.		
.01 ±	1.5870	1.5902 Tl	1.5831 Li	Muscovite	R. C. Sabot:-	<i>Zeit. Kryst.</i> , LVII, 1922, p. 420.		
.010	1.5921	1.5974 (502)	1.5883 (668)	Colemanite	L. Weber:-	<i>Zeit. Kryst.</i> , LX, 1924, p. 164.		
.010 ±	1.6170	1.6202 green	1.6142 red	Hemimorphite	Lang:-	<i>Sitz. Akad. Wiss. Wien.</i> , XXXVII, 1859, p. 379.		
.01 ±	1.6342	1.6383 Tl	1.6283	Danburite	C. Hintze:-	<i>Zeit. Kryst.</i> , VII, 1883, p. 302.		
.01 ±	1.6348	1.6418 F	1.6318 C	Tourmaline*	K. Becht:-	<i>Zeit. Kryst.</i> , LVII, 1922, p. 336.		

.010 ±	1.6382 (578)	1.6452 (492)	1.6345 (691)	Apatite	A. Ehringhaus:-	<i>N. Jahrb. Min., Bl. Bd.,</i> XLI, 1917, p. 360.
.01 ±	1.6577	1.6611 Tl	1.6542 Li	Sillimanite	G. Melczer:-	<i>Zeit. Kryst.,</i> XXIII, 1900, p. 253.
.010 ±	1.6653	1.6681 Tl	1.6617 Li	Spodumenite*	L. Duparc <i>et al.</i> :-	<i>Zeit. Kryst.,</i> LII, 1913, p. 299.
.010	1.6717	1.6785 (475)	1.6679 (667)	Parisite	E. Quercigh:-	<i>Zeit. Kryst.,</i> IV, 1915, p. 273.
.01 ±	1.7201	1.7272 blue	1.7171 red	Spinel	M. Bauer:-	<i>N. Jahrb. Min.,</i> 1895, I, p. 282.
.01 ±	1.722	1.729 blue	1.719 red	Diasporite	A. Des Cloizeaux:-	<i>Dana.-System Min.,</i> 1892, p. 246.
.01 ±	1.7287	?	1.7250 Li	Ganophyllite	A. Hamberg:-	<i>Geol. För. Förh.,</i> XII, 1890, p. 586.

Section 5. N_F-N_C=.0105 to .0115

.011	1.4904	1.4983 F	1.4872 C	Sylvite	K. Spangenberg:-	<i>Zeit. Kryst.,</i> LVII, 1922, p. 494.
.011	1.5220	1.5252 Tl	1.5189 Li	Microcline*	L. Duparc <i>et al.</i> :-	<i>Zeit. Kryst.,</i> LII, 1913, p. 294.
.0115	1.5662	1.5764 (471)	1.5617 (706)	Brucite	A. Ehringhaus:-	<i>N. Jahrb. Min., Bl. Bd.,</i> XLI, 1917, p. 371.
.011 ±	1.5702	1.5737 E	1.5662 B	Beryl	A. Schrauf:-	<i>Sitz. Akad. Wiss. Wien.,</i> XLI, 1860, p. 116.

* Analysis given.

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F-C	N _v , N _o or N _m			MINERAL	AUTHORITY	REFERENCE
	D	Blue	Red			
.0113	1.5903	1.5937 Tl	1.5869 Li	Scapolite*	R. Brauns:-	<i>Zeit. Kryst.</i> , LVII, 1922, p. 214.
.011	1.6455	1.6532 F	1.6421 C	Tourmaline*	K. Becht:-	<i>Zeit. Kryst.</i> , LVII, 1922, p. 336.
.0106	1.6776	1.6850 F	1.6744 C	Diopside	H. Dufet:-	<i>Bull. Soc. Fr. Min.</i> , X, 1887, p. 214.
.0113	1.7153	1.724 F	1.7107 C	Clinozoisite*	F. Zambonini:-	<i>Zeit. Kryst.</i> , LVII, 1922, p. 317.
.0114	1.7219	1.7303 F	1.7189 C	Cyanite	E. Taubert:-	<i>Zeit. Kryst.</i> , XLIV, 1908, p. 314.
.011 ±	1.7470	1.7503 Tl	1.7431 Li	Chrysoberyl	G. Melczer:-	<i>Zeit. Kryst.</i> , XXXIII, 1900, p. 240.
.0106	1.7684	1.7759 F	1.7652 C	Corundum	G. Melczer:-	<i>Zeit. Kryst.</i> , XXXV, 1902, p. 561.
.011	1.8142	1.8172 Tl	1.8088 Li	Spessartite*	R. C. Sabot:-	<i>Zeit. Kryst.</i> , LVII, 1922, p. 224.

Section 6. $N_F - N_C = .0115$ to $.0125$

.012 ±	1.526	1.536 bl.	1.526 red	Wavellite	A. Des Cloizeaux:-	<i>Mon. Mineral.</i> , II, 1893, p. 455.
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.012 ±	1. 539	1. 5345 B	1. 5435 E	Mellite	A. Schrauf:--	Dana:-- <i>System Min.</i> , 1892, p. 994.
.0115	1. 5662	1. 5764 (471)	1. 5617 (706)	Brucite	A. Ehringhaus:--	<i>N. Jahrb. Min., Bl. Bd.</i> , XII, 1917, p. 371.
.012 ±	1. 5704	1. 5748 E	1. 5664 B	Beryl	A. Schrauf:--	<i>Sitz. Akad. Wiss. Wien.</i> , XLII, 1860, p. 116.
.012 ±	1. 5948	?	1. 5909 red	Leucophanite	W. C. Broegger:--	<i>Zeit. Kryst.</i> , XVI, 1890, p. 246.
.012 ±	1. 5956	1. 5992 Tl	1. 5922 Li	Clinochlorite	C. Pulfrich:--	<i>Wied. Ann.</i> , XXX, 1887, p. 501.
.012 ±	1. 6126	1. 6161 Tl	1. 6097 Li	Meliphanite*	W. C. Broegger:--	<i>Zeit. Kryst.</i> , XVI, 1890, p. 279.
.012 ±	1. 6515	1. 6549 Tl	1. 6485 Li	Tourmaline*	R. C. Sabot:--	<i>Zeit. Kryst.</i> , LVII, 1922, p. 339.
.0117	1. 6600	1. 6671 F	1. 6554 C	Sillimanite	E. Taubert:--	<i>Zeit. Kryst.</i> , XLIV, 1908, p. 314.
.012 ±	1. 6616	1. 6653 Tl	1. 6594 Li	Monticellite*	S. L. Penfield:--	<i>Am. Jour. Sci.</i> , I, 1896, p. 134.
.012 ±	1. 667	1. 675 bl.	1. 663 red	Boracite	A. Des Cloizeaux:--	<i>Mon. Mineral.</i> , II, 1874, p. 3.
.0117	1. 6697	1. 6780 F	1. 6663 C	Olivine*	E. Ernst:--	<i>N. Jahrb. Min., Bl. Bd.</i> , LI, 1925, p. 133.
.0123	1. 6810	1. 6912 (471)	1. 6748 (706)	Aragonite	B. Marbach:--	<i>Zeit. Kryst.</i> , LVI, 1921, p. 233.
.012 ±	1. 682	1. 686 gr.	1. 678 red	Pyrosmalite	G. Flink:--	<i>Zeit. Kryst.</i> , LVII, 1922, p. 425.

* Analysis given.

DISPERSION OF MINERALS

F-C	N, N ₀ or N _m			MINERAL	AUTHORITY	REFERENCE
	D	Blue	Red			
.012 ±	1.688	1.692 Tl	1.684 Li	Triphylite*	S. L. Penfield:-	<i>Am. Jour. Sci.</i> , CL , 1895, p. 387.
.012	1.6935	1.6965 Tl	1.6895	Rhodizite*	Duparc, <i>et al.</i> :-	<i>Bull. Soc. Fr. Min.</i> , XXIV , 1911, p. 136.
.012 ±	1.705	1.709 Tl	1.701 Li	Pyrope (pure)	H. Philipsborn:-	<i>Abh. Sachs. Akad. Wiss.</i> , XL , 1928, No. III.
.0124	1.7125	1.721 F	1.7086 C	Olivine *	F. S. Starrabba:-	<i>Zeit. Kryst.</i> , LVI , 1921, p. 436.
.012	1.7188	1.7227 Tl	1.7153 Li	Spinel	G. Melcher:-	<i>Zeit. Kryst.</i> , XXXII , 1900, p. 260.
.0125	1.7450	1.7538 F	1.7413 C	Stauriolite*	F. Hörner:-	<i>Zeit. Kryst.</i> , LVII , 1922, p. 311.
.012 ±	1.7634	1.7725 (477)	1.7599 C	Epidote*	M. Goldschlag:-	<i>Tsch. Min. Pet. Mh.</i> , XXXIV , 1917, p. 47.
.012 ±	1.6616	1.6653 Tl	1.6594 Li	Monticellite*	S. L. Penfield	<i>Am. J. Sc.</i> , I , 1896, p. 134.
.012 ±	1.6515	1.6549 Tl	1.6485 Li	Tourmaline*	R. C. Sabot	<i>Zeit. Kr.</i> , LVIII , 1922, p. 339.

Section 7. N_F-N_C= .0125 to .0135

.013 ±	1.474	1.483 bl.	1.470 red	Thenardite	A. Des Cloizeaux:-	Dana:- <i>System Min.</i> , 1892, p. 895.
.0127	1.5442	1.5532 F	1.5405 C	Halite	S. P. Langley:-	<i>Am. Jour. Sci.</i> , XXX , 1885, p. 477.

.013 ±	1.5698	1.5749 E	1.5665 C	Scapolite*	A. Lacroix:-	C. R., CLXIX, 1919, p.261.
.0127	1.5818	1.5863 E	1.5777 B	Beryl	A. Schrauf:-	Sitz. Wiss. Wien., XLII, 1860, p. 116.
.013 ±	1.59 ±	1.593 bl.	1.580 red	Clinochlorite	G. Tschermak:-	Sitz. Akad. Wiss. Wien., XCIX, 1890, p. 174; C, 1891, p. 29.
.013	1.6573	1.6701 F	1.6553 (633)	Diopside*	E. Quercigh:-	N. Jahrb. Min., 1916, I, p. 159.
.0134	1.6584	1.6678 F	1.6544 C	Calcite	A. Ehringhaus:-	N. Jahrb. Min., Bl. Bd., XLIII, 1920, p. 602.
.013 ±	1.659	1.670 bl.	1.657 red	Forsterite	A. Des Cloizeaux:-	Hintze-Hdb. Min., II, 1897, p. 4.
.0132	1.6726	1.6819 F	1.6687 C	Hardystonite*	K. H. Degen:-	Sitz. Akad. Wiss. Heidel- berg, 1919, A14.
.0135	1.6799	1.6893 F	1.6758 C	Dolomite*	P. Koller:-	N. Jahrb. Min., Bl. Bd., XLII, 1918, p. 457.
.0128	1.6810	1.6900 F	1.6772 C	Aragonite	A. Mühlheims:-	Zeit. Kryst., XIV, 1888, p. 202.
.013 ±	1.685	1.691 bl.	1.678 red	Axinite	A. Des Cloizeaux	Mon. Mineral., I, 1862, p. 515.
.0132	1.7008	1.7102 F	1.6970 C	Olivine*	E. Ernst:-	N. Jahrb. Min., Bl. Bd., III, 1925, p. 133.
.0132	1.7039	1.7138 F	1.7006 C	Augite*	E. A. Wülfing:-	Tsch. Min. Pet. Mit., XV, 1896, p. 29.
.0127	1.7170	1.7255 F	1.7128 C	Clinzoisite	T. Silprandi:-	Zeit. Kryst., LVII, 1922, p. 325.

* Analysis given.

DISPERSION OF MINERALS

F-C		N, N ₀ or N _m			MINERAL	AUTHORITY	REFERENCE
		D	Blue	Red			
.013	1.7378	1.7494 (471)	1.7313 (706)	Periclasite	O. Westphal:-	Cent. Min., 1913, p. 516.	
	1.7438	1.7480 Tl	1.7394 Li				H. Rosenbusch:- <i>Mikr. Phys.</i> , p. 18.
.013 ±	1.7444	1.7476 Tl	1.7389 Li	Pyrope	B. Jezek:-	Min. Abst., II, 1923, p. 139.	
	1.7496	1.7585 F	1.7458 C				Zeit. Kryst., LVII, 1922, p. 312.
.013 ±	1.7926	1.7980 green	1.7886 red	Thortveitite	J. Schetelig:-	Norsk. Geol. Tids., VI, 1922, p. 233; Vidensk. Skr., 1922, p. 51.	
							1.8038
.0138	1.5554	1.5599 Tl	1.5513 Li	Whewellite	B. Jezek:-	Zeit. Kryst., LIV, 1915, p. 192.	
							.014 ±
.014 ±	1.6536	1.6576 Tl	1.6518 Li	Tourmaline	R. C. Sabot:-	Zeit. Kryst., LVII, 1922, p. 390.	
							.0136

Section 8. N_F-N_C=.0135 to .0155

.014 ±	1.6682	1.6727 TI	1.6627 Li	Rinkite	Osann :-	Rosenbusch:- <i>Mikr. Phys.</i> , 1905, p. 292.
.0137	1.6698	1.6795 F	1.6658 C	Justite*	K. H. Degen:-	<i>Zeit. Kryst.</i> , LVII, 1922, p. 104.
.0142	1.6712	1.6816 F	1.6674 C	Hardystonite*	K. H. Degen:-	<i>Zeit. Kryst.</i> , LVII, 1922, p. 104.
.0139	1.685 ±	1.6918 bl.	1.6779 red	Axinite	A. Des Cloizeaux :	<i>Min. Mineral.</i> , I, 1862, p. 515.
.014	1.6920	1.6965 TI	1.6872 Li	Dolomite*	M. Born:-	<i>N. Jahrb. Min., Bl. Bd.</i> , V, 1887, p. 4.
.0136	1.7172	1.7277 (477)	1.7132 C	Clinozoisite	M. Goldschlag:-	<i>Tsch. Min. Pet. Mit.</i> , XXXIV, 1917, p. 23.
.015 ±	1.7207	1.7244 TI	1.7166 Li	Xenotimite	E. H. Kraus:-	<i>Zeit. Kryst.</i> , XXXIV, 1901, p. 268.
.014 ±	1.726	1.7305 TI	1.721 C	Pyroxene* (Hess)	P. Eskola:-	<i>Jour. Geol.</i> , XXX, 1920, p. 276.
.014	1.7375	1.7475 F	1.7335 C	Periclasite (art.)	H. E. Merwin:-	<i>Am. Jour. Sci.</i> , CXCVIII, 1919, p. 82.
.014 ±	1.7439	1.7479 TI	1.7396 Li	Pyrope	E. A. Wülfing:-	Rosenbusch:- <i>Mikr. Phys.</i> , 1905, p. 18.
.014	1.7441	1.7482 TI	1.7399 Li	Grossularite*	H. Philipsborn:-	<i>Abh. Sachs. Akad. Wiss.</i> , XL, 1928, No. III.
.015 ±	1.7964	1.8013 TI	1.7910 Li	Almandite*	H. Philipsborn:-	<i>Abh. Sachs. Akad. Wiss.</i> , XL, 1928, No. III.
.015 ±	1.8145	1.8195 TI	1.8090 Li	Spessartite	R. C. Sabot:-	<i>Zeit. Kryst.</i> , LVII, 1922, p. 224.

* Analysis given.

DISPERSION OF MINERALS

F-C	N, N ₀ or N _m			MINERAL	AUTHORITY	REFERENCE
	D	Blue	Red			
		Section 9. N _F -N _C = .0155 to .0195				
.0187	1.5056	1.5191 F	1.5004 C	Niter	A. Ehringhaus:-	<i>N. Jahrb. Min., Bl. Bd., XLIII</i> , 1920, p. 602.
.018	1.546	1.559 F	1.541 C	Copiapite (art.)	H. E. Merwin:-	<i>Jour. Am. Chem. Soc., XLIV</i> , 1922, p. 1965.
.019 ±	1.550	1.564 F	1.545 C	Rhombochlorite (art.)	H. E. Merwin:-	<i>Jour. Am. Chem. Soc., XLIV</i> , 1922, p. 1965.
.019	1.5874	1.5954 E	1.5793 B	Nitratite	A. Schrauf:-	Groth:- <i>Chem. Kryst., II</i> , 1908, p. 72.
.016 ±	1.5886	1.5930 TI	1.5836 Li	Rinneite	H. E. Boeke:	<i>N. Jahrb. Min., 1909, II</i> , p. 19.
.0167	1.6426	1.6529 F	1.6380 C	Salammonite	Graillich:-	Hintze:- <i>Hdb. Min., I</i> , 1915, p. 2255.
.017	1.6494	1.6545 TI	1.6460 Li	Datolite*	O. Lueddecke:-	Hintze:- <i>Hdb. Min., II</i> , 1898, p. 168.
.016	1.6893	1.7012 (481)	1.6847 C	Willemite	A. Ehringhaus:-	<i>Zeit. Kryst., LVIII</i> , 1925, p. 460.
.017 ±	1.7202	1.7254 TI	1.7173 Li	Trimerite*	W. C. Broegger:-	<i>Zeit. Kryst., XVIII</i> , 1891, p. 373.
.016 ±	1.7366	1.7411 TI	1.7340 Li	Hedenbergite	E. A. Wülfing:-	<i>Beit. Ken. Pyroxen</i> , 1891.

.0169	1.7422	1.7551 (477)	1.7372 C	Epidote*	M. Goldschlag:-	<i>Tsch. Min. Pet. Mit.</i> , XXXIV, 1917, p. 23.
.018 ±	1.7456	1.7508 Tl	1.7396 Li	Pyrope	B. Jezek:-	<i>Min. Abst. II</i> , p. 139.
.016 ±	1.746	1.755 blue	1.739 red	Libethenite	A. Des Cloizeaux:-	<i>Dana-System Min.</i> , 1892, p. 78.
.017 ±	1.7626	1.7676 Tl	1.7575 Li	Grossularite	E. A. Wülfing:-	Rosenbusch:- <i>Mikr. Phys.</i> , 1905, p. 18.
.0165	1.7912 (578)	1.8093 (436)	1.7877 (620)	Monazite	K. Busz:-	<i>N. Jahrb. Min., Bl. Bd.</i> , XXXIX, 1914, p. 482.
.019 ±	1.8105	1.8158 Tl	1.8050 Li	Spessartite	E. A. Wülfing:-	Rosenbusch:- <i>Mikr. Phys.</i> , 1905, p. 18.
.017	1.8488	1.8623 F	1.8436 C	Smithsonite*	E. D. Mountain:-	<i>Mineral. Mag.</i> , XXI, 1926, p. 51.
.018 ±	1.85 ±	1.864 blue	1.846 red	Caledonite	A. Des Cloizeaux:-	<i>Dana-System Min.</i> , 1892, p. 924.

Section 10. $N_F - N_C = .0195$ to $.0245$

.021	1.624	1.638 F	1.617 C	Metatorbernite	N. L. Bowen:-	<i>Am. Jour. Sci.</i> , CXCVIII, 1919, p. 195.
.0196	1.6818	1.6951 F	1.7655 C	Thorite	M. Albis:-	<i>Rend. Accad. Lincei Rom.</i> , XXX, 1920, p. 472.
.022 ±	1.7418	1.7525 Tl	1.738 Li	Garnet	A. Laiakari:-	<i>Zeit. Kryst., LVII</i> , 1922, p. 229.

* Analysis given.

DISPERSION OF MINERALS

N _v , N _o or N _m					MINERAL	AUTHORITY	REFERENCE
F-C	D	Blue	Red				
.02 ±	1.755	?	1.748 Li	Arsenolite	A. Des Cloizeaux:-	Dana-System Min., 1892, p. 198.	
.0204	1.7788	1.7937 F	1.7732 C	Allactite	G. Aminoff:	Geol. För. Förh., XLIII, 1921, p. 40.	
.021 ±	1.786	1.794 Tl	1.780 Li	Tephroïte*	N. H. Magnusson:-	Geol. För. Förh., XL, 1918, p. 611.	
.02 ±	1.807	1.8196 F	?	Gahnite*	P. Eskola:-	Geol. För. Förh., XXXVI, 1914, p. 25.	
.021	1.8384	1.8449 Tl	1.8318 Li	Uvarovite	E. A. Wülfing:-	Rosenbusch:-Mikr. Phys., 1895, p. 18.	
.024 ±	1.8724	1.8799 Tl	1.8693 Li	Siderite*	A. Hutchinsonson:-	Mineral Mag., XIII, 1903, p. 209.	
.022 ±	1.9091	1.9158 Tl	1.9018 Li	Titanite*	K. Busz:-	N. Jahrb. Min., Bl. Bd., V, 1887, p. 334.	
.021	1.9200	1.9344 (475)	1.9124 C	Scheelite (art.)	A. Zambonini:-	Zeit. Kryst., LIX, p. 476.	
.023	1.9251	1.9416 E	1.9184 C	Zircon	W. F. Eppler:-	N. Jahrb. Min., Bl. Bd., LV, 1927, p. 415.	
Section 11. N _F -N _C = .0245 to .0295							
.026	1.652	1.670 F	1.644 C	Mullite (art.)	W. Eitel:-	Zeit. Anorg. Chem., LXXXVIII, p. 173.	

.027	1.664	1.672 Tl	1.657 Li	Strontianite	L. Buchrucker:--	Groth:-- <i>Chem. Kryst.</i> , II, 1908, p. 209.
.0262	1.7569	1.7666 (511)	1.7432 C	Epidote*	M. Goldschlag:--	<i>Tsch. Min. Pet. Mit.</i> , XXIV, 1917, p. 43.
.027	1.7714	1.7796 Tl	1.7445 Li	Grossularite	E. A. Wülfing:--	Rosenbusch:-- <i>Mikr. Phys.</i> , 1905, p. 18.
.027	1.812	1.831 F	1.803 (700)	Beckelite	S. Kreutz:--	<i>Zeit. Kryst.</i> , LVI, 1921, p. 634.
.026	1.8376	1.8554 (492)	1.8310 C	Roméite	H. Rose:--	<i>Cent. Min.</i> , 1919, p. 268.
.026	1.8827	1.9002 (492)	1.8739 (672)	Anglesite	A. Ehringhaus and H. Rose:--	<i>Zeit. Kryst.</i> , LVIII, 1925, p. 460.
.0251	2.4175	2.4354 F	2.4103 C	Diamond	E. A. Wülfing:--	<i>Tsch. Min. Pet. Mit.</i> , XV, 1895, p. 65.

Section 12. $N_F - N_G \equiv .0295$ to $.0495$

.03 ±	$N_g = 1.7876$ $N_p = 1.7431$	1.8025 green 1.7580 green	1.7785 red 1.7320 red	Lorenzenite	G. Flink:--	<i>Med. Grön.</i> , XIV, 1898, p. 250; XXIV, 1901, p.130.
.035 ±	1.7990	1.8096 Tl	1.7929 Li	Acmite	E. A. Wülfing:--	<i>Beit. Ken. Pyroxen</i> , 1891, p. 65.
.04 ±	1.895 ±	1.907 Tl	1.883 Li	Andradite (pure)	H. Philipsborn:--	<i>Abh. Sachs. Akad. Wiss.</i> , XI, 1928, No. III.
.033 ±	1.9048	1.9162 Tl	1.8958 Li	Titanite*	K. Busz:--	<i>N. Jahrb. Min., Bl. Bd.</i> , V, 1887, p. 338.
.04 ±	1.974 (570)	1.983 (533)	1.959 (667)	Powellite (art.)	F. Zambonini:--	<i>Bull. Soc. Fr. Min.</i> , XXXVIII, 1915, p. 206.

* Analysis given.

DISPERSION OF MINERALS

F-C	N, N ₀ or N _m			MINERAL	AUTHORITY	REFERENCE
	D	Blue	Red			
.0373	1.9986	2.0247 F	1.9874 C	Cassiterite	H. Baumhauer:--	<i>Zeit. Kryst.</i> , XLVII, 1909, p. 1.
.046 ±	2.061	2.096 F	2.050 C	Pyromorphite*	H. L. Bowman:--	<i>Mineral. Mag.</i> , XIII, 1903, p. 324.
.049 ±	2.063	2.097 F	2.048 C	Chlorargyrite	W. Wernicke:--	<i>Hintze-Hdb. Min.</i> , I, 2, 1915, p. 2279.
.035 ±	2.0763	2.1549 H	2.0595 B	Cerussite	A. Schrauf:--	<i>Sitz. Akad. Wiss. Wien.</i> , XLII, 1860, p. 120.
.04 ±	2.087	?	2.073 red	Senarmonite	A. Des Cloizeaux:--	Dana:-- <i>System Min.</i> , 1892, p. 198.
.045 ±	2.18	2.19 green	2.16 red	Manganosite	W. E. Ford:--	<i>Am. Jour. Sci.</i> , XXXVIII, 1914, p. 502.
Section 13. N _F -N _C =-.0495 to .0995						
.066	1.816	1.865 F	1.799 C	Carphosiderite (art.)	H. E. Merwin:--	<i>Jour. Am. Chem. Soc.</i> , XLIV, 1922, p. 1965.
.057 ±	1.9733	1.9909 Tl	1.9556 C	Calomel	H. Dufet:--	<i>Bull. Soc. Fr. Min.</i> , XXI, 1898, p. 90.
.0497	1.9995	2.0363 F	1.9866 C	Pyrochlorite	S. Kreuz:--	<i>Zeit. Kryst.</i> , LXI, 1925, p. 347.

.07	2.013	2.039 (530)	1.994 C	Zincite*	H. Berman:-	<i>Am. Mineral.</i> , XII, 1927, p. 168.
.07±	2.0377	2.0586 Tl	2.0171 Li	Sulphur	A. Schrauf:-	<i>Zeit. Kryst.</i> , XVIII, 1891, p. 157.
.05±	2.059	2.094 F	2.044 C	Chlorargyrite	W. Wernicke:-	<i>Hintze-Hdb. Min.</i> , I, 2, 1915, p. 2279.
.08±	2.254	2.316 F	2.234 C	Bromyrite	W. Wernicke:-	<i>Hintze-Hdb. Min.</i> , I, 2, 1915, p. 2299.
.091	2.2949	2.3634 F	2.2725 C	Finnemanite*	G. Aminoff:-	<i>Geol. För. Förh.</i> , XLV, 1923, p. 160.
.08±	2.356	?	2.330 Li	Wurtzite (art.)	H. E. Merwin:-	<i>Am. Jour. Sci.</i> , XXIV, 1912, p. 383.
.09	2.368±	2.398 Tl	?	Sphalerite (pure)	H. E. Merwin:-	<i>Am. Jour. Sci.</i> , XXIV, 1912, p. 383.
.09±	2.4039	2.4342 Tl	2.375 Li	Stibiotantalite*	Penfield and Ford:-	<i>Am. Jour. Sci.</i> , XXII, 1906, p. 61.
.064	2.654	2.700 F	2.633 Li	Moissanite (art.)	H. E. Merwin:-	<i>Jour. Wash. Acad. Sci.</i> , VII, 1917, p. 445.

Section 14. N_F-N_C= .0995 to .195

.126±	2.182	2.279 F	2.153 C	Iodyrite	W. Wernicke:-	<i>Hintze-Hdb. Min.</i> , I, 2, p. 2308.
.12±	2.30	?	2.26 Li	Brannerite*	Hess and Wells:-	<i>Jour. Frank. Inst.</i> , CLXXXIX, 1920, p. 225.
.13±	2.346	2.385 Tl	2.313 Li	Marshite	L. J. Spencer:-	<i>Mineral. Mag.</i> , XIII, 1901, p. 38.

* Analysis given.

DISPERSION OF MINERALS

F-C	N, N ₀ or N _m			MINERAL	AUTHORITY	REFERENCE
	D	Blue	Red			
		For Ng:- Na-Li=.06				
.18 ±	2.39	2.4611 (518)	2.3724 C	Goethite	H. E. Merwin:-	<i>Am. Jour. Sci.</i> , XLVII, 1919, p. 311.
.134	2.4055	?	2.38 Li	Wulfenite	A. Ehringhaus:-	<i>N. Jahrb. Min., Bl. Bd.</i> , XLIII, 1920, p. 568.
.15 ±	2.43	?	2.4408 Li	Sphalerite* (with 17% FeS)	H. E. Merwin:-	<i>Am. Jour. Sci.</i> , XXXIV, 1912, p. 383.
.12 ±	2.4804	?	2.5183 Li	Pyrophanite*	A. Hamberg:-	<i>Geol. För. Förl.</i> , XII, 1890, p. 598.
.142	2.5618	2.6586 (492)	2.5418 Li	Octahedrite	A. Ehringhaus:-	<i>N. Jahrb. Min., Bl. Bd.</i> , XLI, 1917, p. 376.
.13 ±	2.5856	?	2.5671 Li	Brookite	E. A. Wülfing:-	Rosenbusch:- <i>Mikr. Phys.</i> , 1905, p. 123.
.158 ±	2.6158	2.6725 TI		Rutile	C. Bärwald:-	<i>Zeit. Kryst.</i> , VII, 1883, p. 168.

Section 15. N_F-N_C > .195

.21 ±	2.23	2.39 blue	2.18 red	Bunsenite	A. Kundt:-	<i>Sitz. Akad. Wiss. Berlin</i> , 1888, pp. 255, 1387.
.23 ±	1.47 ±	?	2.395 Li	Sphalerite* (with 28% FeS)	H. E. Merwin:-	<i>Am. Jour. Sci.</i> , XXXIV, 1912, p. 383.

.23 ±	2.59 ±	2.688 green	2.552 red	Greenockite	Miller:-	Groth:- <i>Chem. Kryst.</i> , I, 1906, p. 150.
.55 ±	2.84	3.18 blue	2.63 red	Tenorite	A. Kundt:-	<i>Sitzb. Akad. Wiss. Berlin</i> , 1888, pp. 255, 1387.
.4 ±	2.9051 (599)	?	2.8143 Li	Cinnabarite	H. Rose:-	<i>Cent. Min.</i> , 1912, p. 527.
.33 ±	3.088	?	2.979 Li	Proustite	Fizeau:-	Groth:- <i>Chem. Kryst.</i> , II, 1908, p. 767.
.34 ±	3.176	?	3.063 C	Hutchinsonite*	G. F. H. Smith:-	<i>Mineral. Mag.</i> , XIV, 1907, p. 283.
.5 ±	3.22	?	3.042 C	Hematite	E. A. Wülfing:-	<i>Tsch. Min. Pet. Mit.</i> , XV, p. 71.

* Analysis given.