MORPHOLOGY OF MECHANICAL TWINNING IN CRYSTALS

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ABSTRACT

The methods and results of morphological studies on mechanically twinned crystals are reviewed. The derivation of the formulae relating the indices of any face before and after mechanical twinning is presented. A new and direct graphic solution for the twinning elements in a crystal which has been subjected to a twinning deformation is developed.

INTRODUCTION

As a result of the increasing interest in the interpretation of preferred orientations of the minerals in deformed rocks, the mechanism of the plastic deformation of minerals has become important to the structural geologist and the petrologist. There are two main types of plastic deformation in crystals. The first is known as translation-gliding. This is a mechanism whereby the crystal will deform by means of slip along certain planes and in certain directions. This mechanism does not produce a reorientation of the crystal structure. The second is known as twin-gliding which also takes place by means of slip along certain planes and in certain directions. This mechanism reorients the crystal structure to develop twinning. Both of these mechanisms are thought to play important roles in the development of preferred orientations in deformed rocks.

In 1930, M. J. Buerger (3)* undertook a study of translation-gliding. He compiled and evaluated all the work which had been done on this phenomenon in minerals. This served as a basis for new experiments designed to clarify and explain translation-gliding in the light of our present knowledge of crystal structure. The purpose of this paper is to make a similar study of *twin-gliding*. Most of the literature on the subject is in German and good bibliographies are not available. The writer feels that a paper dealing with the underlying principles and demonstrating the logical development of the theories of twin-gliding, which started over fifty years ago as a result of morphological studies and are still in use today, would serve as an adequate foundation for further work on the subject.

The application of the theory of simple shear to explain the phenomenon of twin-gliding, and the development of the transformation formulae (which are to be explained here on the basis of surface morphology), have been explained and derived on the basis of the Bravais crystal lattice. Since morphological studies are still the most important

* Numbers refer to corresponding references in the bibliography.

tool used in the determination of twin-gliding in mineral crystals, the writer feels that this approach to the subject is the most logical. It will also serve as a good foundation for discussions of the data which the writer is now compiling, as well as for future discussions of twin-gliding and its explanation on the basis of crystal lattices and crystal structures.

History of the Study of Twin-Gliding

In 1826, Brewster (2) discovered that the lamellae which had often been observed in cleavage rhombohedrons of calcite were really due to twinning. Over thirty years later, in 1859, Pfaff (10) found that he could press a cleavage rhombohedron of calcite with a force perpendicular to two opposite edges and thereby change the orientation of the interference figure in parts of the crystal. However, it was not until 1867 that Reusch (11) recognized that the twinning described by Brewster and the changed interference due to pressure reported by Pfaff, were one and the same thing. This marks the beginning of the study of mechanical twinning (twin-gliding).

Following Reusch's recognition that twinning could be produced mechanically, mineralogists made extensive studies of the phenomenon during the late 1800's and the early 1900's. Since that time interest seems to have been lost in this phenomenon, or else it has been considered unimportant, because it is either omitted or given very incomplete treatment in most mineralogical textbooks and reference books. This is especially true of those books published in the English language.

During the last twenty years the metallurgists have found the behavior of metal crystals during deformation to be of increasing practical importance. As a result, extensive researches have been carried out on the twin-gliding and translation-gliding of metal crystals. Most of these studies have as their foundation the early work of the mineralogists. Consequently most of the modern theories of crystal deformation have been developed by the metallurgists (4, 13). These theories are limited by the material on which the metallurgists have worked, and also by lack of aid from the mineralogical data on twin-gliding, which for the most part had not even been compiled.

Morphological Changes in a Crystal Due to Mechanical Twinning as Shown by Calcite

In spite of the fact that mechanical twinning has been produced in at least fifty different minerals, calcite is still used as the type example of mechanical twinning. This is due to several factors: the mineral is very common; in the second place, it can be twinned easily; in the third place, it is the only common mineral in which an easily obtainable form (the cleavage rhombohedron) develops both morphological and structural symmetry as a result of mechanical twinning.

The familiar technique of producing twinning in a calcite rhombohedron by pressing a knife blade into the crystal was first described by Baumhauer (1) in 1879. Figure 1 copied from Johnson (5) illustrates this procedure. A wedge-shaped opening of constant angle develops under the knife blade. At the same time a triangular prismatic section of the crystal is displaced to the right. This displacement causes an apparent

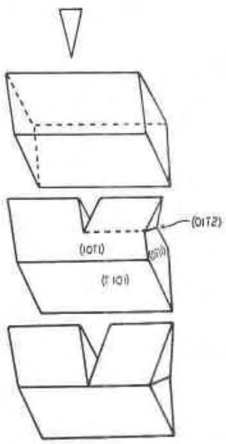

FIG. 1. Formation of a "knife blade" twin on a cleavage rhombohedron of calcite (modified after Johnson).

tipping of a section of the right hand face of the rhombohedron, forming a reentrant angle of constant value for that face. The tipped section of the right hand face remains an optically flat surface. The other two faces of the cleavage rhombohedron which bound the displaced triangular prism show no effects of the deformation. The contact between the displaced prism and the rest of the cleavage rhombohedron is a horizontal plane equivalent to the flat rhombohedron $(01\bar{1}2)$ in the original crystal. It is often possible to develop a good parting parallel to this plane and thereby determine its attitude on the goniometer.

If the faces on the deformed and undeformed parts of this cleavage rhombohedron are measured, and plotted on a stereographic projection, it is found that the displaced portion of the crystal and the undisturbed portion are symmetrical about this plane of separation $(01\bar{1}2)$. Crystallographically this relationship may be thought of as a 180° rotation of the displaced part about the normal to this plane or about an axis parallel to the horizontal edges of the cleavage rhombohedron (Fig. 2).

Optical studies show that this relationship is the same for the interference figures of the two parts of the crystal. It is, of course, obvious that the mechanically twinned portion of the crystal was not actually cut loose and turned 180° about either of these axes, just as natural twins are not actually rotated.

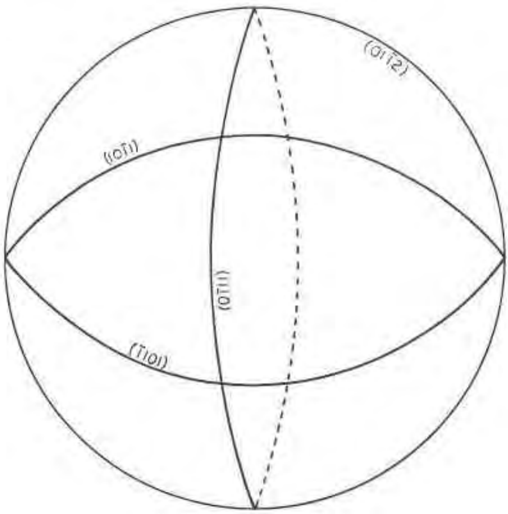

Fig. 2. Cyclographic projection of the "knife blade" twin illustrated in Fig. 1.

Fig. 3. (011̄2) twins in calcite; left: "knife blade" twin similar to those illustrated in Figs. 1 and 2. Center: twin-gliding lamellae on a crystal of calcite from Egremont, England. The crystal is bounded by cleavage rhombohedrons and prism faces. Compare with the projection Fig. 4. Right: growth twin from Guanajuato, Mexico.

If the calcite crystal is not bounded by the planes of the cleavage rhombohedron but for example by six prism faces terminated by the unit rhombohedron, as shown in Fig. 3, the twinning resulting from deformation is not so obvious. After deformation the crystal is cut by a series of lamellae. These lamellae are present in all of the faces except two of the unit rhombohedron. Close examination shows that these lamellae bound tipped portions of the faces on which they appear. Although all of these tipped planes are optically flat, various physical tests have shown that in many cases they are not the same as their undeformed equivalents. The cyclographic projection* of this crystal showing all of the faces present after deformation (Fig. 4) does not show any symmetry in re-

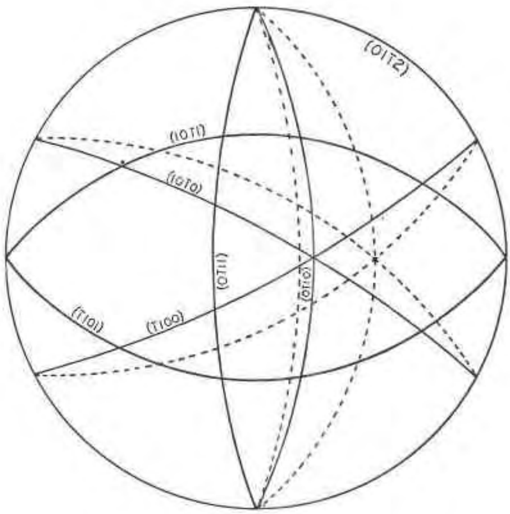

FIG. 4. Cyclographic projection of the faces present on the crystal shown in Fig. 3 (center). Note the lack of symmetry about $(01\bar{1}2)$.

spect to the plane of the lamellae, although the projections of the rhombohedral planes are the same as they were in the case of the deformed cleavage rhombohedron (Fig. 2). The cleavage rhombohedron, therefore, must represent some unique form in calcite which shows mechanical twinning morphologically. Muegge (7, 8) recognized the presence of these unique forms in calcite as well as in other minerals. He realized that the displacement of any face on a crystal due to mechanical twinning must in some way be related to this so-called "grundform" and he thought that the mechanical twinning could probably be characterized by this "grundform." However, his attempt in 1885 (8) to set up analytical expressions for these relationships was unsuccessful.

Application of the Theory of Simple Shear to Mechanical Twinning

Simple shear is a homogeneous deformation which takes place by movements along a series of parallel planes (in a definite direction)

* A projection in which planes are represented by great circles on a stereographic net.

whereby each plane has exactly the same amount of displacement in respect to its neighbors as every other plane does in respect to its neighbors. A prosaic analogy would be the deformation of a deck of cards in which each card slides over its neighbor in a definite direction and by the same amount as every other card.

In a deformation of this kind there is no volume change, planes remain planes, straight lines remain straight lines and a sphere will become an ellipsoid. Let us imagine a plane perpendicular to the slip planes and parallel to the slip direction, through a sphere and the ellipsoid which is its deformed equivalent. Such a plane is called the plane of deformation and it is the plane of projection of the drawing in Fig. 5, which may be used to discuss this process of simple shear.

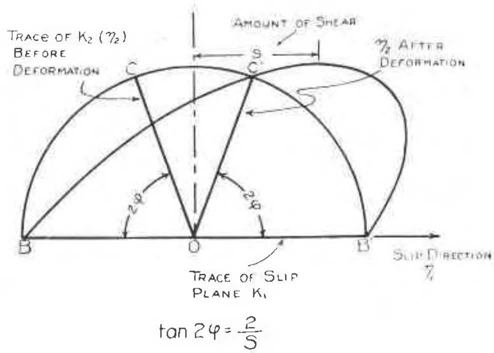

FIG. 5. Diagram illustrating simple shear. The plane of deformation is the projection plane.

The circle $BCC'B'$ with a radius equal to unity may for example be deformed into the ellipse $BC'B'$ by slip along a set of lines parallel to BB'. The ratio of the amount of displacement along any line to its perpendicular distance from BB' is a constant for any particular deformation and is called the "amount of shear." This is designated by the letter s and is equal to the displacement along a line at a unit distance above BB'. It is obvious that any simple shear could be uniquely defined by the slip plane, the slip direction and the amount of shear.

There are two planes in the ellipsoid, the circular cross-sections, which do not change in shape or size as a result of this deformation. These are represented by the lines BB' and OC. BB' does not change its position. OC changes its position to that of OC' and this change is a function of the amount of shear. The angle 2ϕ between the slip plane BB' and the other plane of no distortion OC bears the following relation to the amount of shear: (13)

$$\tan 2\,\phi = \frac{2}{s}$$

The intersection of the two planes of no distortion is perpendicular to the slip direction. Therefore, it is also possible to uniquely define the simple shear by means of the two planes of no distortion and their included angle.

It is seen that the relationship of the two planes of no distortion before and after deformation can be geometrically described by a rotation of 180° of the original positions about an axis normal to the slip plane or about an axis parallel to the slip direction.

In 1889, Liebisch (6) realized that this theory of simple shear could be used as a geometrical means of describing the end result of mechanical twinning. In the knife blade twin of calcite, the deformation could be considered as the result of slip along a series of planes parallel to $(01\bar{1}2)$ with the slip direction parallel to the horizontal edge of the cleavage rhombohedron. In this case the right-hand face of the cleavage rhombohedron would be the other plane of no distortion and its symmetry in respect to the slip plane and the slip direction is explained. The homogeneity of this kind of deformation also explains the fact that all of the tipped planes remain optically flat. It also explains the fact that the tipped portions of planes which are not symmetrical to their original positions in respect to the slip plane no longer have the same properties as they had originally (etch figures). This theory of simple shear explains all of the facts which can be observed on mechanically twinned crystals. However, it should be emphasized that this may not represent the actual movements of atoms, molecules, ions or other units in the crystal structure.

Transformation of Indices of a Face as a Result of Mechanical Twinning

After it was realized that the development of mechanical twins could be geometrically described as a simple shear, the next important problem was to find some way of using the tipped faces of a deformed crystal to define the twinning process. We have seen that a process of simple shear can be uniquely defined by the two planes of no distortion and their included angle. Therefore, if we know the indices of these two planes in a certain crystal we can describe the twin. It has also been shown that this twin will be geometrically equivalent to a 180° rotation about the normal to the slip plane or about the axis which is parallel to the direction of slip.

In 1889 Liebisch (6) and Muegge (10) developed the transformation formulae which relate the indices of an original crystal face to the indices of its tipped portion in the twinned individual in terms of the indices of the planes of no distortion of the strain ellipsoid of simple shear. These transformation formulae can be developed in the following manner:

In crystallographic literature the slip plane is designated by K_1. The intersection of this plane with the plane of deformation is parallel to the slip direction and is designated by N_1. The other plane of no distortion is designated by K_2 and its intersection with the plane of deformation is designated by N_2.

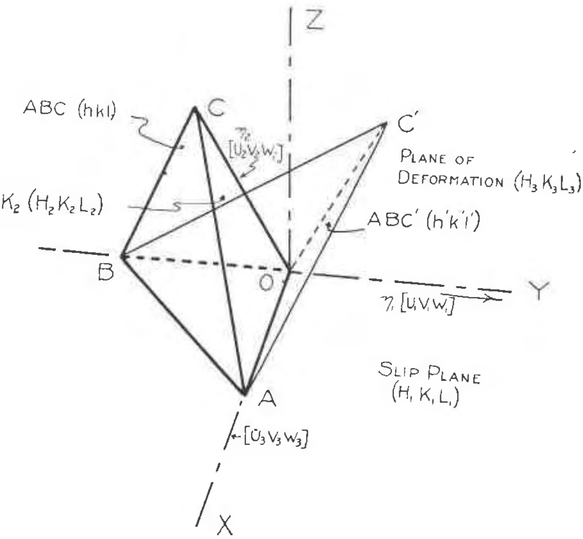

FIG. 6. Diagram illustrating the derivation of the transformation formulae.

Let us refer a crystal which shows twin-gliding to a set of rectangular coordinates, X, Y, Z, Fig. 6, in such a manner that the XY plane coincides with K_1, and the YZ plane coincides with the plane of deformation. OY is then parallel to N_1 and OX is perpendicular to N_1 in the slip plane K_1. Let OC and OC', which lie in the plane of deformation, be parallel to N_2 before and after deformation respectively. OCX and $OC'X$ then represent K_2 before and after deformation. Let the plane ABC represent any plane in the crystal before deformation. After deformation it will occupy a position and attitude indicated by ABC'.

Let the indices in respect to the undeformed crystal axes of the elements which we have just described be:

ABC,	any plane before deformation	(hkl)
ABC',	any plane after deformation	$(h'k'l')$
K_1,	the slip plane	$(H_1K_1L_1)$

K_2,	the other plane of no distortion	$(H_2K_2L_2)$
N_1,	the slip direction	$[u_1v_1w_1]$
N_2,		$[u_2v_2w_2]$
OX,	intersection of K_1 and K_2	$[u_3v_3w_3]$
YZ,	plane of deformation	$(H_3K_3L_3)$

It is now possible to refer the plane ABC with indices (h, k, l) to a new set of axes. Let these new axes be OX, OY, and OC as the a, b, and c-axes respectively.

By cross multiplication we find that the indices of the new axial planes are:

$$(1)\quad \begin{aligned} (H_3K_3L_3) \text{ of } YZ &= ((v_1w_2-v_2w_1)(u_2w_1-u_1w_2)(u_1v_2-u_2v_1)) \\ (H_2K_2L_2) \text{ of } K_2 &= ((v_2w_3-v_3w_2)(u_3w_2-u_2w_3)(v_1w_3-u_3w_1)) \\ (H_1K_1L_1) \text{ of } K_1 &= ((v_3w_1-v_1w_3)(u_1w_3-u_3w_1)(u_3v_1-u_1v_3)) \end{aligned}$$

The transformation formulae for this change are:

$$(2)\quad \begin{aligned} \rho\cdot h_0 &= \frac{hu_3+kv_3+lw_3}{\theta_3}, & \theta_3 &= (u_3+v_3+w_3) \\ \rho\cdot k_0 &= \frac{hu_1+kv_1+lw_1}{\theta_1}, & \theta_1 &= (u_1+v_1+w_1) \\ \rho\cdot l_0 &= \frac{hu_2+kv_2+lw_2}{\theta_2}, & \theta_2 &= (u_2+v_2+w_2) \end{aligned}$$

where h_0, k_0, l_0 are the indices of the plane (hkl) referred to new axes $[u_3v_3w_3]$, $[u_1v_1w_1]$ and $[u_2v_2w_2]$ and ρ is a proportionality factor.

By solving the three equations (2) for h, k and l we get:

$$(3)\quad \begin{aligned} \rho\cdot h &= \theta_3h_0(v_1w_2-v_2w_1)+\theta_1k_0(v_2w_3-v_3w_2)+\theta_2l_0(v_3w_1-v_1w_3) \\ \rho\cdot k &= \theta_3h_0(u_2w_1-u_1w_2)+\theta_1k_0(u_3w_2-u_2w_3)+\theta_2l_0(u_1w_3-u_3w_1) \\ \rho\cdot l &= \theta_3h_0(u_1v_2-u_2v_1)+\theta_1k_0(u_2v_3-u_3v_2)+\theta_2l_0(u_3v_1-u_1v_3) \end{aligned}$$

These formulae give the original indices of the faces in terms of the indices referred to the axes OX, OY, and OC.

By substituting values from (1) equations (3) may be expressed:

$$(3a)\quad \begin{aligned} \rho\cdot h &= \theta_3h_0H_3+\theta_1k_0H_2+\theta_2l_0H_1 \\ \rho\cdot k &= \theta_3h_0K_3+\theta_1k_0K_2+\theta_2l_0K_1 \\ \rho\cdot l &= \theta_3h_0L_3+\theta_1k_0L_2+\theta_2l_0L_1 \end{aligned}$$

The discussion of the strain ellipsoid has shown that the deformed equivalents of the axes OX, OY and OC will be $-OX$, $-OY$ and OC' or $-OX$, OY and $-OC'$ depending upon whether we choose an axis of rotation normal to K_1 or parallel to n_1 in describing the deformation. In either case, the relationship of the crystallographic axis in the new position to the new positions of the axes OX, OY and OC will be the same as the relationship between these two sets of axes in the original positions.

Therefore, the formulae

$$\rho\cdot h'=\theta_3h_0'H_3+\theta_1k_0'H_2+\theta_2l_0'H_1$$
$$(4)\qquad \rho\cdot k'=\theta_3h_0'K_3+\theta_1k_0'K_2+\theta_2l_0'K_1$$
$$\rho\cdot l'=\theta_3h_0'L_3+\theta_1k_0'L_2+\theta_2l_0'L_1$$

express the relationship between the indices $h'k'l'$ of the plane ABC' referred to the new crystallographic axes and its indices referred to the deformed equivalents of OX, OY and OC.

Let us first consider the deformation as equivalent to a rotation of the axes 180° about the normal to K_1. In this case, OX is equivalent to $-OX$, OY is equivalent to $-OY$ and OE is equivalent to OE' in the new positions. Therefore the indices of the plane ABC' are:

$$(5)\qquad h_0=-h_0',\ k_0=-k_0' \text{ and } l_0=l_0'.$$

Substituting the values of (5) in equation (4) we find:

$$\rho\cdot h'=-\theta_3h_0H_3-\theta_1k_0H_2+\theta_2l_0H_1$$
$$(6)\qquad \rho\cdot k'=-\theta_3h_0K_3-\theta_1k_0K_2+\theta_2l_0K_1$$
$$\rho\cdot l'=-\theta_3h_0L_3-\theta_1k_0L_2+\theta_2l_0L_1$$

If we now substitute the values of h, k and l from (2) in (6), we find that:

$$\rho\cdot h'=2H_1(u_2h+v_2k+w_2l)-h(u_2H_1+v_2K_1+w_2L_1)$$
$$(6a)\qquad \rho\cdot k'=2K_1(u_2h+v_2k+w_2l)-k(u_2H_1+v_2K_1+w_2L_1)$$
$$\rho\cdot l'=2L_1(u_2h+v_2k+w_2l)-l(u_2H_1+v_2K_1+w_2L_1)$$

We have now expressed the transformation of indices of any plane due to twin-gliding in terms of the original indices of the face, the indices of N_2 and the indices of K_1.

If on the other hand we consider the deformation as equivalent to a 180° rotation of the axes about N_1 then the indices of the plane ABC' are:

$$(7)\qquad h_0=-h_0',\ k_0=k_0' \text{ and } l_0=-l_0'.$$

Substituting these values in equation (4) we find:

$$\rho\cdot h'=-\theta_3h_0H_3+\theta_1k_0H_2-\theta_2l_0H_1$$
$$(8)\qquad \rho\cdot k'=-\theta_3h_0K_3+\theta_1k_0K_2-\theta_2l_0K_1$$
$$\rho\cdot l'=-\theta_3h_0L_3+\theta_1k_0L_2-\theta_2l_0L_1$$

If we now substitute the values of h_0, k_0 and l_0 from (2) in (8), we find that:

$$\rho\cdot h'=2H_2(u_1h+v_1k+u_1l)-h(u_1H_2+v_1K_2+w_1l_2)$$
$$(8a)\qquad \rho\cdot k'=2K_2(u_1h+v_1k+u_1l)-k(u_1H_2+v_1K_2+w_1l_2)$$
$$\rho\cdot l'=2L_2(u_1h+v_1k+u_1l)-l(u_1H_2+v_1K_2+w_1l_2)$$

Here we have expressed the transformation due to twin-gliding of the indices of any plane in terms of the original indices of the plane, the indices of N_1 and the indices of K_2.*

* The use of these transformation equations can be illustrated by using calcite as an example. Let K_1 be $(01\bar{1}2)$. The corresponding K_2 is $(0\bar{1}11)$ and S is $(2\bar{1}\bar{1}0)$. The intersection of S with K_1 is N_1 $[12\bar{3}1]$ and with K_2 is N_2 $[12\bar{3}2]$. For use in the transformation equations only 3 indices are used. For these elements they are: $K_1=(01.2)$, $K_2=(0\bar{1}.1)$, $S=(2\bar{1}.0)$, $N_1=[12.\bar{1}]$, $+N_2=[12.2]$.

If we now investigate the transformation of the plane $(02\bar{2}1)$ due to twin-gliding with the above-mentioned elements we find:

By means of these transformation formulae we can express the morphological changes which take place in a crystal as a result of mechanical twinning. It has been shown that these twins can be described by a rotation about one of two axes. Observers have found that in some cases the slip plane has simple indices, and then it can be considered as the twin plane or its normal as the twin axis. This is known as *TWINNING OF TYPE I*. In other cases the slip plane is irrational, but the slip direction, N_1, is rational and can be considered as the twin axis. This is known as *TWINNING OF TYPE II*. It should be emphasized that from a morphological standpoint these two types of twinning are identical except for the so-called rationality or irrationality of certain elements used in our geometrical description of mechanical twinning. In some cases all of the elements used to describe a mechanical twin are rational. If this is the case, it often happens that K_2 may also function as a slip plane, whereby two varieties of twins will be formed. One variety can be described as twinning by rotation about the normal to K_1 or about N_1. The other can be described by rotation about the normal to K_2 or about N_2. This is called *RECIPROCAL TWINNING*.

There are hundreds of measurements on record which show the validity of the transformation formulae which have been developed. If the indices of two faces which do not lie in the same zone are known before and after the deformation, it is possible to solve these formulae for K_1 and N_2 or for K_2 and N_1. However, if we do not know what the twin plane in a crystal is, it is impossible to index any faces with respect to the twinned axes. Most investigators have been able by close observation and shrewd guessing to arrive at probable values for K_1, K_2, N_1, and N_2. After doing this they substituted these values in the transformation formulae and found what the indices of the displaced faces should be. It was then a simple matter to measure the angles between the displaced faces

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By substituting the values for $K_1(H_1=0,\ K_1=1,\ L_1=2)$, $N_2(u_2=1,\ v_2=2,\ w_2=2)$ and $h=0$, $k=2$, $l=1$ in equation (6*a*) we find that:

$$\rho\cdot h'=0(0+4+2)-0(0+2+4)=\ 0$$
$$\rho\cdot k'=2(0+4+2)-2(0+2+4)=\ 0$$
$$\rho\cdot l'=4(0+4+2)-1(0+2+4)=18$$

Dividing through by the proportionality factor, $\rho=18$, $h'k'l'$ become (001).

By substituting the values for $K_2(H_2=0,\ K_2=-1,\ L_2=1)$, $N_1(u_1=0,\ v_1=2,\ w_1=-1)$ and $h=0$, $k=2$, $l=1$ in equation (8*a*) we find that:

$$\rho\cdot h'=0(0+4-1)-0(0-2-1)=0$$
$$\rho\cdot k'=-2(0+4-1)-2(0-2-1)=0$$
$$\rho\cdot l'=2(0+4-1)-1(0-2-3)=9$$

Dividing through by the proportionality factor, $\rho=9$, $h'k'l'$ become (001).

This shows that with these deformation elements $(02\bar{2}1)$ becomes (0001) in the twinned crystal. This fact was established experimentally by Muegge in 1883 (7).

and their equivalent faces in the undeformed crystal. These values could be compared with those values which were calculated for the angles between the known indices of the original crystal face and the assumed indices in the twinned individual. The agreement between the calculated and the measured values served to substantiate or disprove their guess as to the values of K_1, K_2, N_1, and N_2. The transformation formulae, therefore provide an excellent means of checking, but only an indirect solution for the twinning elements.

Graphic Solution of the Twinning Elements in Deformed Crystals with the Aid of the Stereographic Projection

It has been shown that a simple shear and thereby a mechanical twin can be completely described by three elements, namely:

(1) The slip plane (K_1)
(2) The slip direction in the slip plane (N_1)
(3) The amount of slip or shear (s).

If these three elements are known and their relation to the crystallographic axes of a crystal are known, then the twinning elements which developed during the deformation of a crystal are completely defined.

These three elements can be determined graphically. In order to do this one must know the positions with respect to the *original* crystal axes of two planes of a crystal before and after the twinning deformation. These two planes must not lie in the same zone and neither of them can be in the zone of the slip direction. The planes may be crystal faces or they may be artificially prepared planes. The only further restriction is that they give reflections which can be measured accurately on the reflection goniometer, or that they are suitable for measurement by other means.

From the data described above and illustrated in Fig. 7, the steps in the graphic solution are as follows:

(1) Plot the two planes before deformation (planes A and B) and the corresponding planes after deformation (planes A' and B') as great circles on the stereographic net along with the crystallographic axes of the original crystal.

(2) The intersection of planes A and A' (point 1) and the intersections of planes B and B' (point 2) must both lie in the slip plane. Therefore the slip plane can be constructed as a great circle passing through these two points. The fact that the intersections of these planes lie in the slip plane is clear when we remember that in mechanical twinning the slip plane separates the deformed and undeformed parts of the crystal (see Fig. 1).

(3) The line of intersection of planes A and B (point 3) and the line of intersection of planes A' and B' (point 4) must lie in a plane which contains the slip direction. Therefore a great circle drawn through these two points will intersect the slip plane in a point which represents the slip direction (point 5).

(4) A great circle is then constructed parallel to the slip direction and perpendicular to the slip plane. This great circle represents the plane of deformation. The plane of deformation will intersect planes A, A' and B' in four points (points 6, 7, 8, and 9).

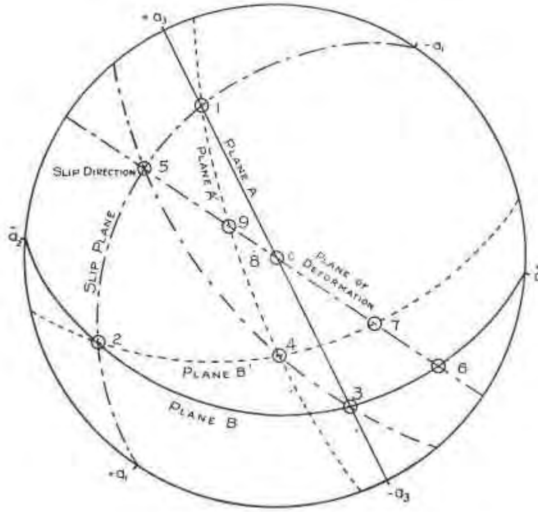

Fig. 7. Cyclographic projection of a mechanical twin in calcite illustrating the graphic solution for twin-gliding elements.

(5) The amount of shear can be calculated from the angles between the slip direction and the intersection of the planes of deformation with one of the planes before deformation (θ_1) and the angle between the slip direction and the intersection of the plane of deformation with the same plane after deformation (θ_2). The formula used in this calculation is:

$$s = \text{ctg}\,\theta_1 - \text{ctg}\,\theta_2.$$

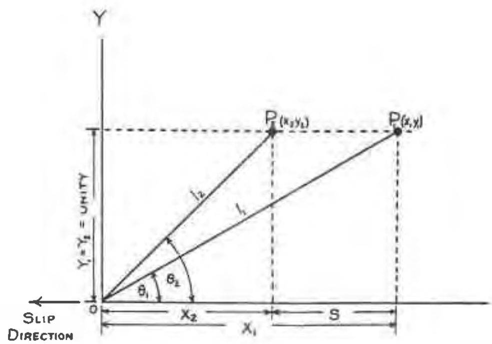

Fig. 8. Diagram illustrating how the amount of shear (s) may be derived from the graphic solution.

The derivation of this formula can be explained with the aid of Fig. 8. The plane of deformation is again the plane of the projection. The slip direction is parallel to OX. By simple shear, the point P_1 is transferred to the position of point P_2. If we consider that their y coordinate is equal to unity then the difference between their x coordinates is equal to s. It follows that:

$$s = x_1 - x_2$$
$$x_1 = l_1 \cos \theta_1$$
$$l_1 = \frac{1}{\sin \theta_1}$$

so
$$x_1 = \text{ctg } \theta_1$$
similarly
$$x_2 = \text{ctg } \theta_2$$
$$\therefore \quad s = \text{ctg } \theta_1 - \text{ctg } \theta_2.$$

From the value of s we can calculate the angle between K_1 and K_2 and then we can plot K_2 and N_2 on the projection. We have now determined all of the elements necessary to describe a mechanical twin. The accuracy of our determination is limited only by the accuracy of the measurements and the accuracy of plotting. This graphic solution of the twinning elements is advantageous because it is direct and rapid and it helps to visualize the deformation. For final determinations the graphic solution can be checked by using the transformation formulae.

Conclusions

From a review of morphological studies it has been shown that mechanical twinning in crystals may be described as a process of simple shear. However, it is to be emphasized that at present this process of simple shear cannot be applied to an analysis of the actual paths of movement of atoms, ions or molecules in the crystal during the deformation by twinning.

Formulae relating the indices of a crystal face before and after mechanical twinning to the elements of the deformation are reviewed. It has been pointed out that these transformation formulae cannot be used as a direct solution for the elements of the twinning deformation.

A new and direct graphic solution of the deformation elements has been presented.

Although much of the material presented in this paper is not original, it is the first time that it has been compiled in English in the present form. The writer feels that such a presentation is a necessary foundation for further studies of mechanical twinning.

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