A GRAPHIC SOLUTION FOR CERTAIN PROBLEMS OF LINEAR STRUCTURE

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ABSTRACT

The coordinates of linear structure in rocks without foliation or platy flow structure can be determined stereographically from field measurements of its traces on several outcrop surfaces. The method is based on a simple geometrical concept involving linear elements of ideal cylindrical shape, and its application is limited to minerals with roughly equi-dimensional cross-sections. The departure of the graphic results from the theoretical ideal is evaluated by statistical treatment. The Storm King granite at Bear Mountain, New York, furnishes illustrative examples.

INTRODUCTION

The problem of finding the coordinates of linear structure in rocks devoid of foliation or platy flow structure frequently presents difficulties in the field. The attitude of lineation on outcrop surfaces may be quite misleading, since it merely represents the visible elongation of intersection figures between random plane surfaces and three-dimensional rod or pencil shaped bodies.

The Storm King granite in the interior of the Bear Mountain intrusive in the Hudson Highlands of New York is a case in point. This medium-grained flow rock has a well-developed gneissoid structure produced by the linear alignment of hornblende elements of rather stout prismatic habit. Only outcrop surfaces oriented essentially perpendicular to this linear structure have an even granular appearance without noticeable mineral parallelism. On all other exposures the hornblende is drawn out in parallel streaks. If we assume this linear structure to be plunging 20° toward N45°E, an exposure surface striking approximately N45°W and dipping about 70°SW would fail to show any distinct elongation of the hornblende sections. On the other hand, a surface of the same strike but with a dip of 40°SW may produce elongate sections of the linear elements whose axes of elongation have a pitch of 90°. The true plunge of linear structure (20°), then, can be measured directly only on a vertical outcrop surface striking N45°E, i.e. parallel to the linear structure. Considering the thick stubby shape of the hornblende elements, it becomes even more difficult to determine the coordinates of linear structure accurately owing to insufficient variation in the shape of the streaks on the many exposure surfaces of widely different orientation.

Petrofabric analysis, though tedious and time-consuming, will give adequate results, provided a reasonably constant relation exists between the preferred dimensional orientation of the linear elements and their optical or crystallographic directions. Referring to the example cited

above, the larger quartz grains possess the same dimensional alignment as the hornblende, but their optic axes have essentially random orientation.[1] Standard petrofabric methods, therefore, cannot be employed successfully as far as the dimensional orientation of these quartz elements is concerned.

An acceptable solution of this problem can be obtained from field measurements on at least 6 outcrop surfaces by use of the stereographic projection. The accuracy of the results increases with the number of surfaces measured and the diversity of their space orientation. Strike and dip of each surface as well as pitch of lineation must be determined. The orientation of surfaces which do not show any preferred direction of mineral elongation serves as a useful check on the graphic solution.

Definitions

Structural geologic terminology has been used in a different sense by different authors. Some of the more controversial terms appearing in this paper are therefore defined.

The trend in space of a line is defined by two angles:

Bearing—the projection of a line onto the horizontal plane, measured from the geographical north (Bucher, 1944, p. 195), and

Plunge—the angle from the horizontal to the line, measured in a vertical plane (Billings, 1942, p. 44).

The coordinates of a line *within* a given plane are defined by the strike and dip of that plane, and by the

Pitch—the angle that the line makes with a horizontal line in that plane (Billings, 1942, p. 135).

Linear structure—parallel alignment of the long axes of elongate mineral grains or other linear elements in a rock.

Lineation[2]—parallel alignment of elongate traces of linear elements on an exposure surface.

Linear flow structure or flow lines—linear structure interpreted as the result of flow in igneous rocks.

Linear elements—in this case, the individual elongate mineral grains responsible for the linear structure.

[1] This unusual feature will be discussed in detail by the writer in a forthcoming paper on the Storm King granite at Bear Mountain, N.Y.

[2] The writer believes that a clear distinction should be made between the parallelism of the axes of linear elements in a rock and the parallel alignment of the elongate traces of such elements on an exposure surface. Although the terms "linear structure" and "lineation" have been used interchangeably by most authors (Billings, 1942, p. 300), it is proposed to limit the concept of "lineation" to the elongate traces of linear elements on exposure surfaces.

Planar structure or foliation—arrangement of mineral grains in parallel layers (planes or curved surfaces) with or without linear alignment of the elements.

Platy flow structure or flow layers—foliation interpreted as the result of flow in igneous rocks.

Geometrical Principle

The proposed graphic method is based on the application of a simple geometrical principle. Figure 1 illustrates the relationship between a cylinder E and a plane P cutting it at any given angle, except essentially parallel or perpendicular to the cylinder axis XX'. The intersection is seen to be an ellipse S of varying eccentricity depending on the angle of intersection. Plane N, perpendicular to plane P and containing the long diameter YY' of ellipse S, also includes the cylinder axis XX'. This plane of construction will henceforth be referred to as the N-plane. Two or more N-planes erected on cutting planes of different orientation, then, must intersect in a single line, namely the cylinder axis XX'.

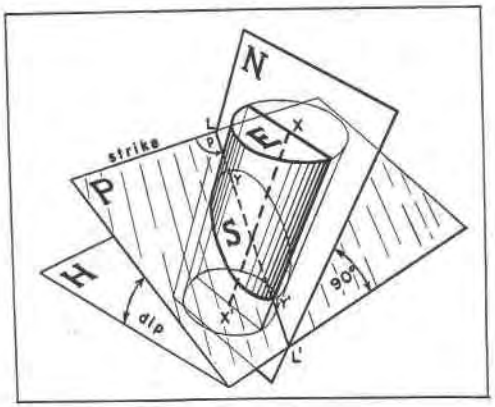

Fig. 1. Geometrical relations between an ideal cylindrical element and an outcrop surface.

XX' = axis of linear element
LL' = lineation on outcrop surface
p = pitch of lineation

Substituting geological concepts for the geometrical terminology we find that

E is a linear element of ideal cylindrical shape,
P is an outcrop surface,
S is the elongate trace of the linear element on the outcrop surface P,
YY' is the trend of mineral elongation or lineation LL' in surface P, having a pitch of p degrees,
XX' is the sought attitude of the linear structure in the rock.

The strikes and dips of several outcrop surfaces together with the angles of pitch defining the trends of lineation are known from field measurements. The corresponding N-planes are plotted on a stereographic projection net and their common line of intersection representing the desired attitude of linear structure is determined graphically.

The two principal limitations of this method as applied to geological field conditions now become quite evident. The presence of foliation or platy flow structure would render the basic concept inoperative, since the mineral elongation observed on the outcrop must be derived from sections of individual linear elements and cannot represent linear traces of mineral layers. Also the shape of the linear elements should approach, at least roughly, the shape of a cylinder. Minerals of prismatic habit with more or less equi-dimensional cross-sections, such as quartz, hornblende, augite, etc., are best for this purpose. Mica flakes and tabular feldspar crystals, on the other hand, are not suitable.

Stereographic Construction

A Wulff meridian stereonet (Bucher, 1944, p. 193, footnote) is used for the graphic solution of the problem. For field purposes a net of 4.5 inches diameter drawn to 2° intervals should be mounted on plywood or heavy cardboard. The data are plotted on a sheet of tracing paper or cellulose acetate placed over the stereonet. A 5×5 inch sheet of 10-point (.010 inches thick) transparent cellulose acetate is superior to tracing paper overlays. The acetate is not only more rigid and transparent than the best of papers, but also has less tendency to warp and wrinkle under diverse climatic conditions. Furthermore the field geologist need carry only a single sheet. An ordinary fountain pen, preferably with a fine writing point, is adequate for drawing all construction lines on the overlay. After obtaining the desired data the ink lines can be washed off with water and the acetate used over again.

The method of plotting follows the conventional procedure of lower hemisphere projection discussed by Bucher (1944). The consecutive steps of construction involved in this particular problem are reviewed for the benefit of those who do not have ready access to appropriate references such as the one cited above.

The acetate sheet is placed over the stereonet and attached by a pin piercing the center of the net about which the overlay must be free to rotate. A circle of the net diameter should be permanently inscribed on the acetate with a pair of dividers and reference ticks should be placed at the North and South poles in the same manner.

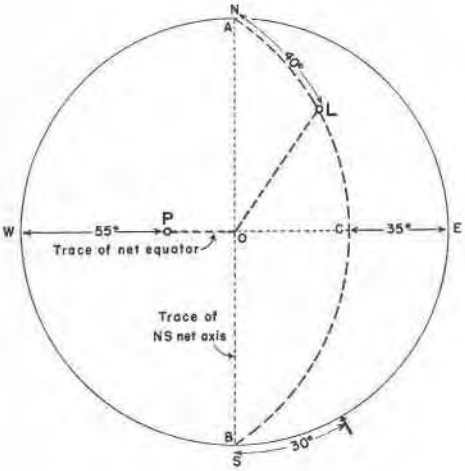

FIG. 2. Stereographic plot of an outcrop surface N30°E, 35°SE with lineation pitching 40°NE.

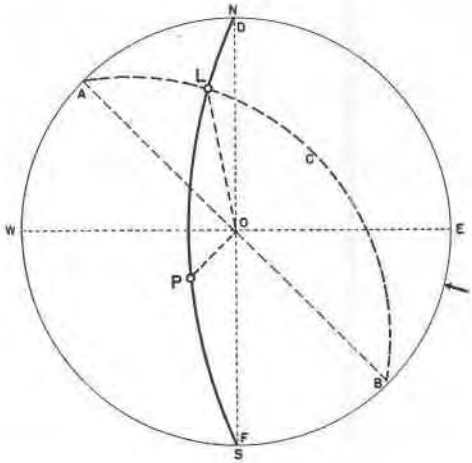

FIG. 3. Construction of the N-plane from data of Fig. 2.

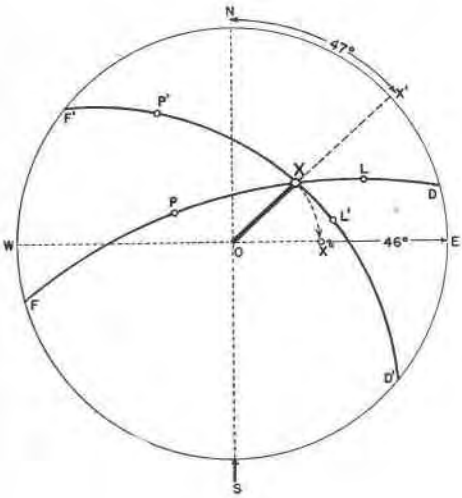

FIG. 4. Graphic solution for the coordinates of linear structure from 2 N-planes.

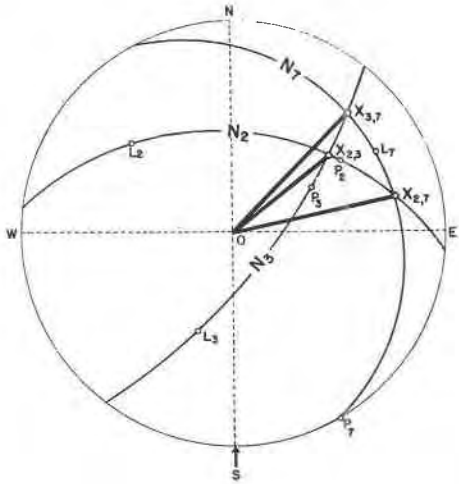

FIG. 5. Multiple graphic solutions for linear structure from 3 N-planes.

Subscripts to N, L, P and X (Fig. 5) refer to numbers of outcrop surfaces used as listed in Table 1.

It is well to remember at the outset that only great circle arcs representing N-planes and the projections of their intersections with each other should be drawn on the graphic plot. The dashed lines in the text figures have been added solely for instructive purposes and should not appear on the actual plot.

The illustrative example used in Figs. 2 and 3 involves an outcrop surface striking N30°E and dipping 35°SE. The pitch of lineation in this surface is 40°NE. With the acetate reference markers over the N and S poles of the net, plot the strike of the surface by placing ticks on the net circumference at N35°E and S35°W (this step is not shown in Fig. 2). Rotate overlay until strike line AB coincides with the NS net axis (Fig. 2). Plot the dip of 35°SE by counting 35° on the net equator from E toward center O to obtain point C. The great circle arc ACB is the projection of the outcrop surface.

The pole P of the surface, i.e. the point at which the normal to plane ACB pierces the projection hemisphere, is then found by counting 55° or the complement of the dip angle from W toward O. Finally the projection of the lineation OL pitching 40°NE is determined by locating L, which lies 40° from N along ACB. As previously indicated only points P and L should appear on the acetate overlay at this time.

Now rotate plotting sheet until P and L come to lie on one of the great circle projections (meridians) of the net and draw in the entire arc between N and S (Fig. 3). $DLPF$ represents the stereographic projection of the N-plane which is perpendicular to surface ACB, since it contains the plane-normal OP, and also includes the trace of lineation OL. Repeat this procedure for the remaining outcrop surfaces to obtain the requisite number of N-planes.

Figure 4 illustrates the last step of construction. To the N-plane $DLPF$ of Fig. 3 has been added N-plane $D'L'P'F'$ derived from a N60°E, 70°SE exposure surface with lineation pitching 42°NE. X is the projection pole of the line of intersection OX between the two N-planes. Assuming ideal cylindrical shape of the linear elements, OX then is the trend of linear structure in the rock. To find its coordinates, rotate plotting sheet into the starting position, i.e. with the N and S reference markers over the corresponding net poles. Extend OX to the net circumference and read the bearing $NX'=$N47°E. Rotate again until X lies on the net equator to determine the plunge of linear structure $EX''=46°$.

As actual field conditions fall short of the theoretical ideal, the N-planes constructed from field measurements will, on the whole, not intersect in a single line, but will give multiple solutions for linear structure coordinates. Figure 5 shows three possible results obtained from

three N-planes constructed from field data of outcrop surfaces 2, 3 and 7 of Table 1.

The number of possible graphic solutions increases in arithmetical proportion with the number of exposure surfaces measured and can be expressed mathematically as follows:

$$s = \frac{n(n-1)}{2}$$

whereby s = maximum number of possible graphic solutions
and n = number of N-planes constructed from field data on n outcrop surfaces.

Statistical Treatment

The multiple answers obtained from the graphic plot are resolved by finding the statistical median. The two variables involved, namely the bearing of linear structure including the concept of plunge direction and the angle of plunge, are treated separately. It is convenient to express the directional variable (bearing) in terms of 0° to 360° starting at the S pole and counting clockwise. Thus 90° indicates a linear structure trending EW and plunging W, while 270° represents the same bearing with a plunge to the E.

Field data from a road cut in Storm King granite at Bear Mountain, New York are listed in Table 1, and are used to illustrate the statistical procedure.

Table 1. Field Measurements from Road Cut in Storm King Granite at Bear Mt., N. Y.

No. of outcrop surface	Strike and dip of outcrop surface	Pitch of lineation
1	N. 50° W., 35° N.E.	81° N.
2	N. 35° W., 62° S.W.	26° N.
3	N. 30° W., 45° S.W.	60° S.
4	N. 5° E., 90°	30° N.
5	N. 15° E., 90°	26° N.
6	N. 30° E., 90°	26° N.
7	N. 60° E., 90°	15° E.

In accordance with the mathematical formula the maximum number of 21 possible answers was realized from the 7 N-planes. Table 2 gives the results of the graphic plot and the statistical solution. Column 1 shows the projection poles of N-plane intersections on the stereogram with subscripts indicating the planes involved. The corresponding co-

ordinates of linear structure are listed in columns 2 and 3. Bearings and plunges thus obtained are arranged in rising numerical order in columns 4 and 5 respectively.

TABLE 2. STATISTICAL TREATMENT OF STEREOGRAM RESULTS

N-plane intersection poles	Coordinates of linear structure from *N*-plane intersections		Coordinates arranged in statistical order		
	Bearing*	Plunge	Bearing*	Plunge	
1	2	3	4	5	
$X_{1,2}$	210°	36°	34°	8°	
$X_{1,3}$	34°	8°	111°	8°	
$X_{1,4}$	211°	28°	205°	9°	
$X_{1,5}$	211°	26°	210°	13°	
$X_{1,6}$	211°	25°	211°	13°	
$X_{1,7}$	212°	13°	211°	14°	
$X_{2,3}$	230°	30°	211°	14°	
$X_{2,4}$	111°	9°	212°	15°	
$X_{2,5}$	269°	8°	218°	15°	
$X_{2,6}$	250°	21°	223°	21°	
$X_{2,7}$	258°	14°	227°	21°	Median
$X_{3,4}$	228°	23°	228°	22°	
$X_{3,5}$	227°	22°	228°	23°	
$X_{3,6}$	228°	25°	230°	25°	
$X_{3,7}$	223°	14°	234°	25°	
$X_{4,5}$	234°	22°	248°	26°	
$X_{4,6}$	218°	26°	250°	26°	
$X_{4,7}$	248°	15°	252°	26°	
$X_{5,6}$	205°	26°	258°	28°	
$X_{5,7}$	252°	15°	269°	30°	
$X_{6,7}$	272°	13°	272°	36°	

* Bearings are given from 0° to 360° clockwise from *S* pole and indicate direction of plunge as well.

The median values of the statistical sequence (columns 4 and 5), then, give a bearing of 227° or N47°E and a plunge of 21°NE for the average linear structure in this road cut.

To test the accuracy of this method, certain additional field information pertaining to the above outcrop has been withheld from the reader. Lineation on a horizontal exposure surface had an average strike of

N45°E which is equivalent to the bearing of the linear structure. Adding the *N*-plane constructed from this horizontal surface to the graphic plot, 7 more possible sets of coordinates are found making a total of 28. The resulting median of N45°E, 22° shows but a minor refinement of the values obtained without field data from a horizontal surface.

Furthermore no visible mineral elongation was observed on a N70°W, 52°SW exposure, even though this surface evidently does not cut the linear structure of the rock exactly at right angles. The answer is to be found in the fact that the cross-sections of the linear hornblende elements are approximately, but not truly equi-dimensional. In this case, an exposure oriented exactly perpendicular to the axes of the linear elements would actually show crude but distinct parallelism in the direction of the crystallographic *b* axis. Generally speaking, therefore, coordinates of linear structure can be determined only in the crudest fashion from the orientation of outcrop surfaces showing a lack of visible lineation.

The solution of this problem by the use of the statistical median also has the advantage of minimizing the effects of certain inaccurate field measurements due to inadequate size of outcrop surface, indistinct lineation or sudden local variations of structure. Obviously impossible results will automatically appear at the extreme ends of the statistical column where they will have little influence on the median. Experience has shown that 7 to 8 adequate exposure surfaces give optimum statistical results, taking into account all factors including time involved in performing the necessary operations.

Conclusion

The coordinates of linear structure can be determined with reasonable accuracy from field data obtained from 6 to 8 different outcrop surfaces by the use of the stereographic projection followed by simple statistical treatment of the graphic results. This method is limited to rocks which do not show any visible platy flow structure or foliation, and whose linear elements are minerals with roughly equi-dimensional cross-section, such as quartz, hornblende, augite, etc. It is particularly useful when the bearing of the linear structure is not known owing to the lack of data from a horizontal exposure surface. The coordinate values represent the average for the entire outcrop.

This method has been applied by the writer with good results to numerous road cut exposures of the Storm King granite in the Hudson Highlands of New York. It is readily adaptable to field use, since little equipment is necessary and the time factor is not prohibitive.

In conjunction with petrofabric analysis of the Storm King granite

this procedure was also used for finding the preferred dimensional orientation of quartz grains lacking similar space lattice arrangement from oriented thin sections. The pitch of preferred quartz elongation in these sections of various known orientations was determined by azimuth counts (Haff, 1938, p. 571).

Acknowledgment

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