ALIGNMENT CHART FOR CALCULATION OF SPECIFIC GRAVITY

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ABSTRACT

An alignment chart is presented in the form of three scales arranged along the lines of a "Z" which solves the equation for determination of the specific gravity of a substance in terms of its weight in air and its weight when submerged in water. The construction of the chart is easy for any range of weights. If constructed on 8×10-inch paper, readings may be made to better than ±1%. A larger chart would permit proportionately better accuracy.

INTRODUCTION

With a few notable exceptions, geologists and mineralogists have made little use of nomographic methods, and almost no use of the particular device known as the alignment chart. An alignment chart consists of three or more scales constructed along straight or curved lines and related to one another in such a way that any straight line drawn across them (called an index line) will intersect the scales at points that correspond to a solution of the equation represented. Textbooks describing many types of alignment charts may be found in most engineering libraries.[1]

The necessity for repeated solution of the well known formula for determining specific gravity in terms of the weight of a substance in air, divided by the difference between its weight in air and its weight in water, led the writer to construct a "Z" alignment chart for the purpose. The formula is as follows:

$$d = W_a/[W_a - W_w], \tag{1}$$

where d = density or specific gravity, W_a = weight in air, W_w = weight when submerged in water. This fits the type equation for the "Z" chart, which may be given as follows:[2]

$$af_3(w) = f_1(u)/[f_1(u) + bf_2(v)]. \tag{2}$$

Setting $a=1$, $f_3(w)=d$, $f_1(u)=W_a$, $b=-1$, and $f_2(v)=W_w$, we obtain exact correspondence between (1) and (2).

THE CHART

For the problem at hand, the two parallel scales of the "Z" represent the two observed weights W_a and W_w. They have uniform spacing and equal moduli, i.e., equal distances represent equal increments of weight.

[1] Two recently published books on charts and scales are C. O. Mackey, *Graphical Solutions*, New York, 1944; and A. S. Levens, *Nomography*, New York, 1948.

[2] Mackey, *op. cit.*, p. 68.

Graph paper with millimeter divisions, or with 20 divisions per inch, is readily available; a sheet 8.5×11 inches in over-all dimensions serves to construct a chart that will give satisfactory accuracy for many purposes.

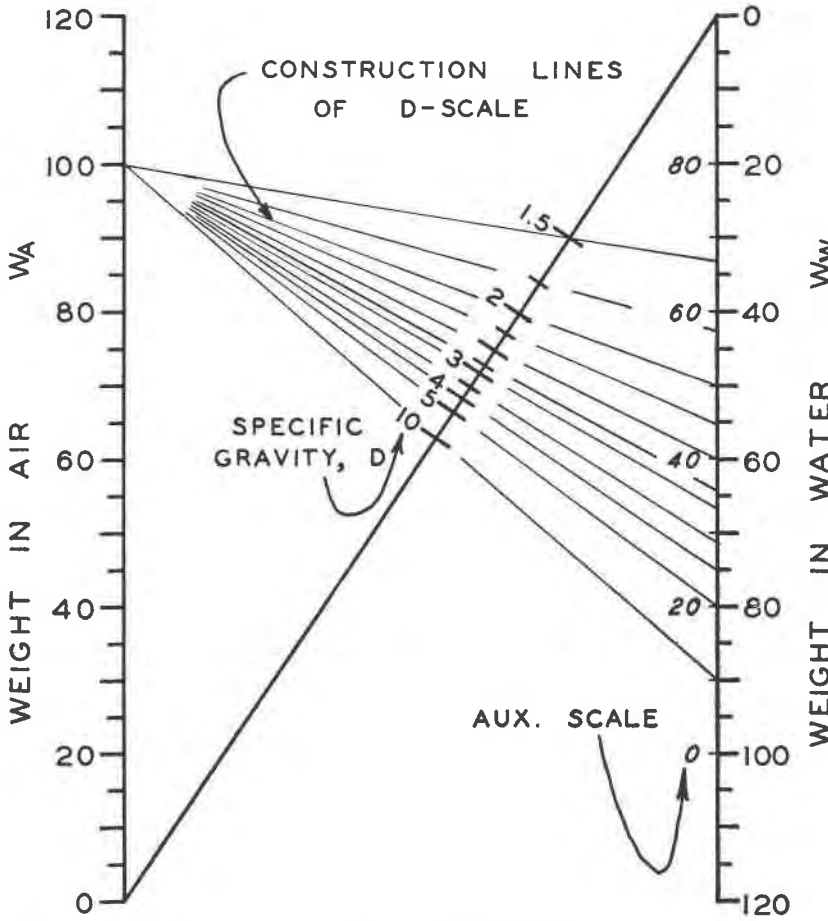

FIG. 1. Layout and construction of alignment chart for determination of specific gravity. Construction lines from point M on W_A scale to M/D on auxiliary scale.

A larger sheet will give proportionately greater accuracy. Along opposite edges of the ruled paper, lay off the scales for W_a and W_w, starting at diagonally opposite corners as in Fig. 1 and using any convenient scale modulus (e.g., 1 inch = 10 weight units) for both scales. The scale modulus may be chosen as to cover any range desired, or the scales may be num-

bered so as to show more than one range. The unit of weight (milligram, gram, pound, etc.) is immaterial provided that the same unit is used for both weighings.

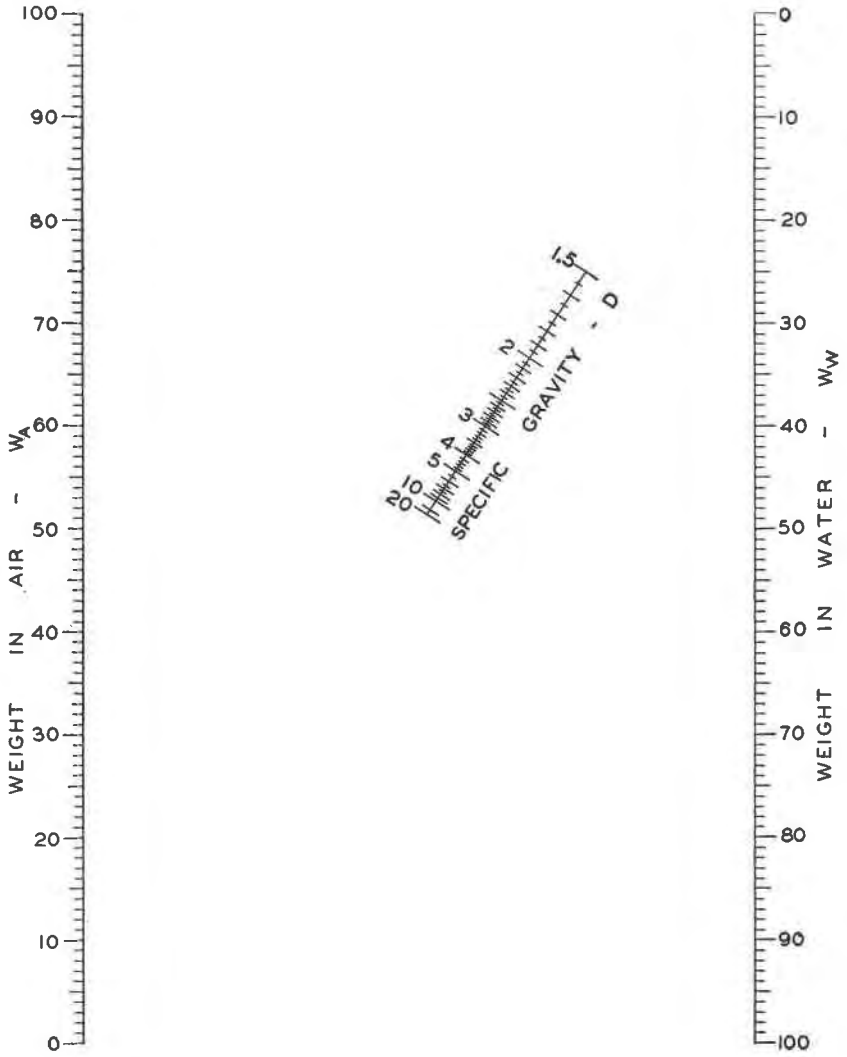

FIG. 2. Alignment chart for determination of specific gravity
$D = W_A/[W_A = W_W]$
Note: *A straight index line from* W_A *to* W_W *cuts diagonal scale at* D.

The diagonal scale is calibrated by a simple graphical method. Assuming some constant value, say 100, for W_a, equation (1) is rearranged as follows:

$$100 - W_w = 100/d. \tag{3}$$

Equation (3) gives the clue to the calibration of the diagonal scale, for by laying off in pencil an auxiliary scale $(100 - W_w)$ along the W_w axis, as shown in Fig. 1, any desired point on the diagonal scale may be found by drawing a line from 100 on the W_a scale to 100/d on the auxiliary scale. Tables of reciprocals are included in many books containing mathematical tables,[3] and should be used for convenience and accuracy in evaluating the quantity 100/d which is found on the auxiliary scale. Figure 1 shows the steps in the layout of the whole chart, especially detailing the procedure for the diagonal scale. A small copy of the completed chart is shown in Fig. 2.

Use of the Chart

The "Z" chart is very simple to use. The observational data are W_a, the weight of a sample in air, and W_w, its weight when submerged in water. To calculate specific gravity the larger of these numbers is divided by their difference; but the subtraction and division are simultaneously performed by the chart (Fig. 2). A straight-edge is laid from the observed value W_a on the left scale to the observed value of W_w on the right scale; this intersects the diagonal d-scale at the proper value for the specific gravity with reference to the liquid used (water). For example, suppose that a specimen weighs 30.0 grams in air and 20.0 grams in water. One can mentally divide the larger (30.0), by the difference (10.0) obtaining a specific gravity of 3.00: note that a ruler laid across the chart from $W_a = 30.0$ to $W_w = 20.0$ intersects the d-scale at 3.00.

Modification for Use with Toluene

A factor that has not been mentioned so far is the density of the water or other liquid used in the submerged weighing. The density of water does not vary enough to affect results more than about 0.3% in the extreme range of room temperatures. This is below the limit of accuracy of the d-scale if the chart is constructed on paper of notebook size, and can therefore be ignored.

However, when toluene or other liquid is used instead of water, the density of the liquid must be used as a multiplier to correct the apparent

[3] *Handbook of Chemistry and Physics*, Chemical Rubber Publishing Co., Cleveland, Ohio, any edition; also many other similar handbooks.

specific gravity with reference to the liquid, to the true specific gravity with reference to water. Two alternatives are presented. First, the diagonal d-scale may be recalibrated for a new, constant value of the density of the liquid to be used. (There is room on the chart for this to be done once without confusion, since only the upper half of the diagonal axis has been used: to use the lower half, interchange the designations W_a and W_w on the parallel scales and calibrate the lower half of the diagonal axis for the new density.) An error of less than 0.5% will be introduced if toluene is the liquid used and a standard temperature of 24° C. assumed, since

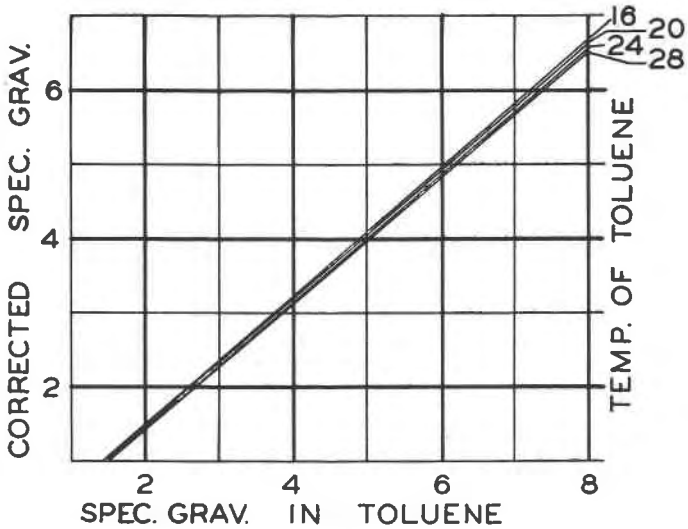

FIG. 3. Intersection chart of $D_{tol.} = k_T D_{tol.}$ where k_T = correction factor depending upon temperature and $D_{tol.}$ is specific gravity with respect to toluene.

the range of densities of toluene from 20° to 30° is about .0067 and the density of toluene is 0.86 at room temperatures of 23° or 24°. This maximum error due to temperature variation may be too great for some purposes, but is not excessive for many uses, and is near the maximum error to be expected for "Z"-chart constructed on paper of note-book size. A larger chart, as stated above, would improve the accuracy of chart readings in direct proportion to the size of the chart, and would necessitate making allowance for temperature separately.

A second alternative, necessary if several liquid densities are to be considered (i.e., several liquids, or several temperatures in one liquid), is to make the density-of-liquid correction separately after first determining the specific gravity with respect to the liquid and noting the tempera-

ture. Since this correction involves only a multiplication, it can be done by slide rule or it can be done with an auxiliary chart consisting of a graph made up with uncorrected specific gravities along the abscissa axis, corrected specific gravities along the ordinate axis, and a family of straight lines through the origin for the several densities of liquid desired. Figure 3 illustrates the plan of such an auxiliary chart, for correction of observed specific gravities in toluene at various temperatures. In actual use, a larger one should of course be made. The closeness of the diagonal lines illustrates clearly the smallness of the error inherent in assuming a constant density of toluene. The uncorrected value of specific gravity derived from readings using toluene instead of water is obtained from the "Z"-chart; this value is found on the abscissa scale of a chart like Fig. 3, and followed upward to the appropriate temperature line; thence horizontally to the left to the correct value of specific gravity on the ordinate scale. Figure 3 may be designed for use in connection with the basic "Z"-chart for water, or with the modification described in the preceding paragraph.

Discussion

The accuracy of the chart method in general is dependent mainly upon the size of the chart used. Accuracy of reading charts varies in different parts depending upon the angle of intersection of two lines; in the present chart the acuteness of the angle between the straight edge and the diagonal scale is indeed a limitation of the accuracy with which the reading can be made: but data which produce such an acute angle are in general the very data that cannot give reliable answers. Thus, if the specimen weighs only 10 milligrams, and its weight in liquid is 7.04 milligrams, the acuteness of the angle is such that a reading of the density may be anywhere from 3.4 to 3.5 on the diagonal scale. This serves immediately to warn that the data, though possibly more accurate than this result seems to indicate, must still be used with great caution. Since the two parallel scales are not calibrated for any particular weight units, the results of both weighings may be multiplied by 2 or 10 or any other number, and the resulting figures used with the chart, thereby improving the accuracy of the computation by making the index line intersect the diagonal scale at a larger angle. On the other hand, if the weight in air is 40.0 milligrams and in liquid is 36.0 milligrams, the index line crosses the diagonal scale at a large angle and the resulting determination of $d = 10.0 \pm 0.3$ is probably as accurate as the data warrant. The diagonal scale is compressed in this vicinity, reflecting the fact that the *difference* (small in this example) between the observed weights enters the denominator of the formula; its

relative accuracy limits the resulting accuracy of the quotient severely. This factor is much less serious with substances of lower density, and for them the diagonal scale is accordingly more extended.

At the scale of reproduction, it is not intended that the "Z"-chart be of much practical use except perhaps to give an approximate result for checking the order of magnitude of the results obtained by other means. On a sheet of graph paper 7.5×10 inches, accuracy of about $\pm .01$ may be obtained in the density range from 2.0 to 3.0, $\pm .02$ from 3.0 to 4.0, and $\pm .05$ from 4.0 to 5.0. If the length of the diagonal scale of the chart is doubled, the accuracy is improved proportionately.