

TRICLINIC GNOMONOSTEREOGRAMS

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ABSTRACT

The preparation of a combined gnomonic and stereographic projection of a triclinic crystal is a relatively simple procedure if carried out by the technique herein outlined. The method is suitable not only for the standard orientation, but for the first and second permutations; these latter are likely to be met when using the Buerger *x*-ray precession camera. Such a projection is useful because: (1) it enables one to make graphical calculations of crystal constants avoiding any gross errors; (2) it clearly shows which angles are obtuse, which acute; (3) it serves as a base from which to derive useful formulae. The principles herein elucidated may be applied readily to non-triclinic crystals, which are always simpler to handle.

INTRODUCTION

The preparation of a combined gnomonic and stereographic projection (a gnomonostereogram) of a triclinic crystal offers definite advantages in the interpretation of its geometry. Such a diagram can be made with both projections drawn to the same scale if the stereographic plane is taken as a central one of the unit sphere and if after drawing the gnomonic plane, it is projected orthographically to the stereographic plane; of course the two projection planes are originally parallel to one another, but offset by unit distance. This matter has been discussed previously by Fisher (1941, pp. 315-319.)

When the goniometry of crystals was limited to the optical technique, there was perhaps no advantage in preparing gnomonostereograms in any but the standard orientation, that with the *c*-axis at the center of the primitive circle. But when the goniometry is carried out with the *x*-ray precession camera (Buerger, 1944), it becomes desirable to handle both permutations of this orientation; first, with the *a*-axis at the center, second with the *b*-axis at the center; compare the orthorhombic cases (Palache *et al.* 1944, Fig. 11, p. 20); see Table 1. This matter is further discussed in the following paper.

TABLE 1. TRICLINIC ORIENTATIONS

Orientation	At center	$\phi = 0^\circ$	$\phi = 90^\circ$	Unit value† (polar axial ratio)	Fig.
Standard	[<i>c</i>]	[<i>b</i> [*]]	[<i>a</i>] [†]	<i>r</i> ₀	1
First Permutation	[<i>a</i>]	[<i>c</i> [*]]	[<i>b</i>] [†]	<i>p</i> ₁	2
Second Permutation	[<i>b</i>]	[<i>a</i> [*]]	[<i>c</i>] [†]	<i>q</i> ₂	3

† The radius of the fundamental sphere; see Palache *et al.* (1944, Fig. 1, p. 9).

The graphical technique herein described has the big advantage that it leaves no doubt as to which angles are obtuse and which acute. Thus in handling the formulae given, one need not be greatly concerned with negative trigonometric functions, or with the question of whether a given ϕ -angle is greater or less than 90° . These details all show up clearly in the projections, unless one of the interaxial angles is very close to 90° .

In preparing the gnomonostereograms it is time-saving to work on protractor paper with the aid of a scale for plotting (polar) ρ -angles in either projection; such scales are available as the top one of the projection protractor¹ (Fisher, 1941, Fig. 1) or of the crystallographic protractor (Barker, 1922, Fig. 1). Failing these, one may use an ordinary metric scale with the r. tangent tables (Goldschmidt, 1934, p. 162; or Wherry, 1920, p. 119), remembering that when one plots x° gnomonic, this is equivalent to plotting $2x^\circ$ stereographic under the conditions stated.

The three figures used to illustrate this paper are based on chalcantite with elements $a:b:c$ of 0.5693:1:0.5548 and $\alpha=97^\circ34'$, $\beta=107^\circ17'$, $\gamma=77^\circ26'$ (see Fisher, 1952). Had an example been chosen with obtuse γ , some of the conditions would be changed, but these should cause no difficulty to one employing the graphical techniques here outlined.

DEFINITIONS AND SYMBOLS

It is intended that this section may be consulted as one carries out the graphical procedure outlined in the remainder of the paper. Further definitions appear in an earlier paper (Fisher, 1941, pp. 296-300). First it may be desirable to make clear what is meant by the cyclographic projection (Fisher, 1941, pp. 315-316); the contrast between this and the stereographic is brought out in Table 2. The cyclographic is a *direct*

TABLE 2. CYCLOGRAPHIC AND STEREOGRAPHIC PROJECTIONS

Element to project	Type of Projection	
	Cyclographic (direct)	Stereographic (reciprocal)
Zone Axis (or other <i>direction</i>)	ZONE POLE (a point) Symbol [uvw] ^o	ZONE LINE (a great circle) Symbol [uvw] ^s
Crystal face (or other <i>plane</i>)	FACE LINE (a great circle) Symbol (hkl) ^o	FACE POLE (a point) Symbol (hkl) ^s

projection; that is, the element itself is projected (after moving it parallel to itself so that it goes through the center of the fundamental sphere).

¹ Obtainable from Ward's Natural Science Establishment, Rochester, N. Y.

On the other hand, the stereographic is a *reciprocal* projection; a normal to the element is projected. Thus in the stereographic a *plane* is represented by a point (a face pole) obtained by erecting a line through the center of the fundamental sphere normal to the plane. Where this line cuts the fundamental sphere establishes the face pole in the spherical projection. If this point is joined to the center of projection (typically the south pole) the resulting line cuts the plane of the stereogram at the point of the desired face pole. A *direction* appears in the stereographic as a line (a zone line) which is derived in a similar manner from the trace of the plane normal to the direction through the center of the fundamental sphere. Otherwise the stereographic and cyclographic projections are identical. In the past many writers have shown an axis or direction as a point in a so-called stereographic projection. This has, of course, confused the student and militated against clear thinking. This is particularly true when indicatrix directions and planes are added to those of morphology or structure. On the projection itself, the point (needle-prick) representing the pole of a face or other *plane* (stereographic) is surrounded by a small circle, one standing for the pole of a zone axis or some other *direction* (cyclographic) is in a small square. Thus \square^a on the projection corresponds to $[a]^o$ in the text; either one refers to the cyclographic projection of the *a*-axis.² The symbol $[a]^t$ is used to represent the *trace* of the *a*-axis; that is, the line in which a plane including $[a]$ and the axis whose direct projection lies at the center of the primitive (divided) circle, cuts the plane of the projection.

Elements of the reciprocal lattice are indicated by the use of an asterisk*. Since the gnomonic projection of a crystal is nothing but a scaled enlargement of the 1-level of the reciprocal lattice (with the addition of "fractional" face poles, etc.), the polar axes of Goldschmidt (P, Q, & R) are closely related to a^* , b^* , and c^* . Goldschmidt (1934, Fig. 102, p. 67) shows his polar axes as going through the "center" of the crystal and also lying in the plane of the gnomonogram. In this paper the reciprocal axes are taken to go through the center of the crystal while two of the unprimed polar axes lie in the polar elements plane and two of the primed polar axes lie in the gnomonic plane. Elements in the polar elements plane are unprimed; those in the gnomonic plane are primed. The gnomonic plane in the standard orientation, is tangent to the unit sphere where it is cut by the *c*-axis; the polar elements plane is parallel the gnomonic plane, but goes through the point where the R = the c^* -axis cuts

² If one is also plotting in the gnomonic (reciprocal) and euthygraphic (corresponding direct) projections, the superscripts *g* and *e* may be employed; see Fisher, 1941, p. 318. As here used, the superscript *o* refers to a direct (i.e., non-reciprocal) version of the orthographic projection.

the unit sphere; thus r_0 is equal to unit distance; see Palache *et al.*, 1944, Fig. 1, p. 9. Goldschmidt's polar interaxial angles λ , μ , and ν , are the optical goniometric equivalents of the reciprocal lattice angles α^* , β^* , and γ^* , respectively, measured by x -ray goniometry.

A given face-pole or other *point* in the stereographic projection may be spoken of as a *ster*-point; a similar point in the gnomonic projection is a *gnom*-point. A *ster*-triangle is one whose sides are great circles which join any three non-colinear *ster*-points; it is the stereographic projection of a spherical triangle. The *primitive gnomonogram* (PG) gives the mesh of the reciprocal lattice at the scale of the projection; for standard orientation, it is the parallelogram made by joining the following gnomonic face-poles: (001), (101), (111), and (011). This excepts the hexagonal; see Barker, 1922, Figs. 27-34. The corresponding figure in the stereographic projection is the *primitive stereogram* (PS)

CASE 0. STANDARD ORIENTATION. FIG. 1

Given α , β , and γ (or γ^*); also for #7 the axial ratio $a:b:c$, or ρ_0' and q_0' .

1. Draw $[a]^t$, the x -axis, as a radius of the primitive circle of $r=1$ at $\phi=90^\circ$ (Fig. 1a). Plot $[b]^t$ anticlockwise $(180-\gamma^*)^\circ$ from $[a]^t$. [γ^* can be obtained from (3) below].

2. Plot the *gnom*-point E' along $[b]^t$ at $(\alpha-90)^\circ$ to the right of $[c]$. The line cE' may be designated s_0' . If desired (see #5) also plot $[b]^c$ along $[b]^t$ at α° *ster.* to the right of $[c]$.

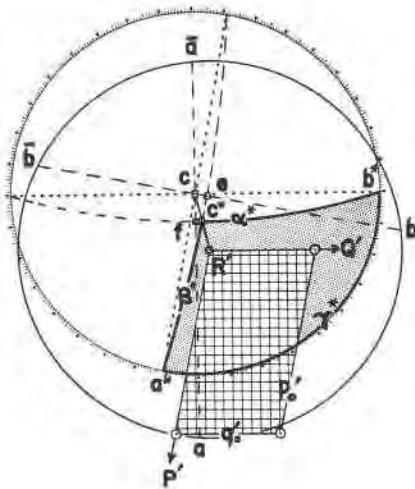
3. Plot the *gnom*-point F' along $[a]^t$ at $(\beta-90)^\circ$ down from $[c]$. Then $cF'=x_0'$. If desired (see #5) also plot $[a]^c$ along $[a]^t$ at β° *ster.* down from $[c]$.

4. Erect normals through *common points*³ E' and F' to $[b]^t$ and $[a]^t$ respectively, intersecting at R' , the gnomonic face pole (001). These normals are the P' and Q' axes of Goldschmidt, and E' and F' are their zone centers. The point R' has cartesian coordinates x_0' and y_0' and polar coordinates ϕ_0 and d_0' , where $d_0' = \tan \rho_0$.

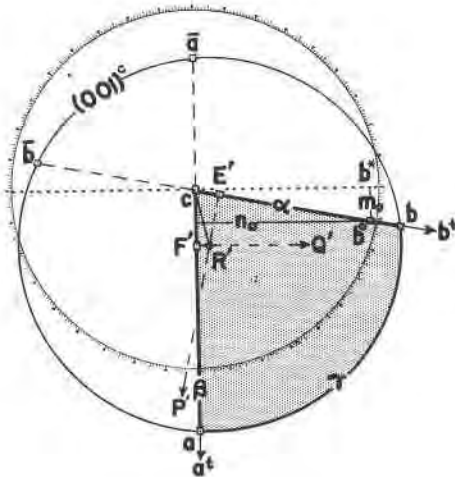
5. If desired, one may draw the great circle $(001)^c$ using R' as center and as radius the distance R' to $[a]^c$ or R' to $[b]^c$. This circle contains the cyclographic projection of γ , and also serves to locate $[\bar{a}]^c$ and $[\bar{b}]^c$. The *ster*-triangle $a b c$ may be designated the *primitive octant*.

6. The *ster*-point (Fig. 1b) corresponding to R' is $[c^*]^c$. A great circle (which may be traced from a stereonet) from here to $[b^*]^t$ (which lies on the

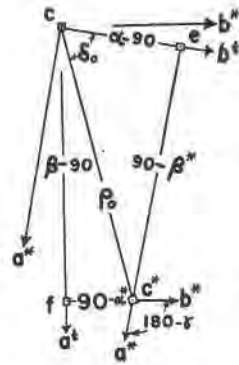
³ So-called because each one lies on both a polar axis and the trace of a (direct, primitive) crystal axis. Each of these points (geometrically) bisects the line joining the cyclographic projections of the (+) and (-) ends of the crystal axis on whose trace it lies.



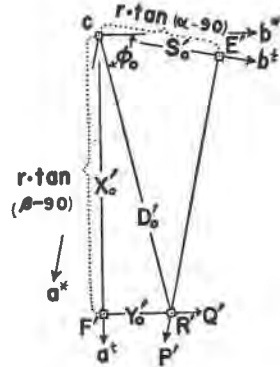
(b) Polar octant with primitive gnomonogram



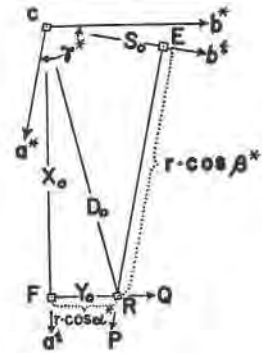
(a) Primitive octant



(c) Stereographic



(d) Gnomonic



(e) Polar elements

FIG. 1. Gnomonostereogram of chalcantite, standard orientation. a. The entire projection with primitive circle (graduated) of $r=1$. The primitive octant is stippled. b. The entire projection with primitive of $r=1$. The polar octant is stippled and the primitive gnomonogram is reticulated. c. Central portion of stereoprojection (see b) with $r=10$. d. Central portion of gnomonogram with $r=5$. Note that distances D_o' , S_o' , X_o' , and Y_o' are capitalized to emphasize $r \neq 1$. e. Central portion of polar elements plane with $r=5$. Here also D_o , S_o , X_o , and Y_o are capitalized, since $r \neq 1$.

primitive at $\phi=0^\circ$) is the cyclographic projection of α^* . [a^*] is γ^* degrees clockwise around the primitive from [b^*]. A great circle from [a^*] to [c^*] is the cyclographic projection of β^* . The ster-triangle $a^*b^*c^*$ may be called the *polar or reciprocal octant*. Its vertices are the face-poles of the three pinacoids in the stereographic projection.

7. The primitive gnomonogram (PG) may be plotted from the values of p_0' and q_0' ; these may be obtained from the elements of crystallization by using (9) and (10) below. It should be noted that Figs 1a and 1b can be combined to advantage in a single diagram if colored inks (or sharp, colored crayons) are used to outline the primitive gnomonogram (blue), polar octant (green), and primitive octant (red).

Useful formulae—Standard Orientation

$$\cot \alpha^*/2 = \sqrt{\frac{\sin(\sigma - \beta) \sin(\sigma - \gamma)}{\sin \sigma \sin(\sigma - \alpha)}} \quad (1)$$

$$\cot \beta^*/2 = \sqrt{\frac{\sin(\sigma - \alpha) \sin(\sigma - \gamma)}{\sin \sigma \sin(\sigma - \beta)}} \quad (2)$$

$$\cot \gamma^*/2 = \sqrt{\frac{\sin(\sigma - \alpha) \sin(\sigma - \beta)}{\sin \sigma \sin(\sigma - \gamma)}} \quad (3)$$

Where $\sigma = (\alpha + \beta + \gamma)/2$

Note: Exactly analogous formulae permit one to compute α , β , or γ from α^* , β^* , and γ^* . Graphical solutions are given by Bond (1950, p. 239).

$$\cos \rho_0 = \sin \alpha^* \sin \beta = \sin \alpha \sin \beta^* \quad (4)$$

$$\tan \phi_0 = \cos \beta / \cot \alpha^* \quad (5)$$

$$s_0' = \cot \alpha = cE' \quad (6)$$

$$x_0' = \cot \beta = cF' \quad (7)$$

$$y_0' = \cot \alpha^* / \sin \beta = F'R' \quad (8)$$

$$q_0' = c / (\sin \alpha^* \sin \gamma) = c / (\sin \alpha \sin \gamma^*) \quad (9)$$

$$p_0' = c / (a \cdot \sin \beta \cdot \sin \gamma^*) = c / (a \cdot \sin \beta^* \sin \gamma) \quad (10)$$

Note: Any vector in the gnomonic plane (Fig. 1d) may be converted to one in the polar elements plane (Fig. 1e) by multiplying it by $\cos \rho_0$; thus $p_0' \cos \rho_0 = p_0$, etc. Thus the polar elements plane is like a gnomonogram at a slightly reduced scale; it does not represent an orthographic projection, in spite of the fact that $d_0 = \sin \rho_0$. Distances in these two diagrams (where $r=5$) are represented by capital letters (D, S, X, and Y) to emphasize that $r \neq 1$.

The denominator of (9) is called v_2 by Bond (1946, p. 33) and by Evans (1948, p. 61). It is here designated n_0 ; it is the y-coordinate

(Fig. 1a) of the orthographic projection of one unit on $[b]$; that is, of where $[b]$ cuts the unit sphere. The corresponding x-coordinate, called v_1 by Bond and by Evans, is here designated m_0 , where:

$$m_0 = \sin \alpha \cos \gamma^* = n_0 / \tan \gamma^* \quad (11)$$

These coordinate values are useful in triclinic calculations according to Evans. They may be shown in the projection if one plots $[b]^\circ$ (the orthographic projection of unit distance on $[b]$) along $[b]^t$ to the right of $[c]$ a distance of $\sin \alpha$. Then n_0 is parallel to $[b]^*$ and extends from $[b]^\circ$ to $[a]^t$, while m_0 is parallel to $[a]^t$ and extends from $[b]^\circ$ to $[b]^*$.

Derivation of equations

The first three are given in Buerger (1942, p. 355).

To get (4) apply Napier's rules (Phillips, 1947, p. 184) to right-angle triangles $c e c^*$ and $c f c^*$ of Fig. 1c where $r=10$. Here e and f are stereopoints corresponding to gnom-points E' and F' , respectively.

To obtain (5) note from Fig. 1e that $y_0 = \cos \alpha^*$; thus $y_0' = \cos \alpha^* / \cos \rho_0$. But from Fig. 1d $\cot \bar{\phi}_0 = \tan \phi_0 = \cot \beta / y_0'$. Thus $\tan \phi_0 = \cot \beta / (\cos \alpha^* / \cos \rho_0)$. Substitute in this the value of $\cos \rho_0$ from (4), and get (5).

(6) and (7) are self evident.

To get (8) substitute the value of $\cos \rho_0$ from (4) in the equation $y_0' = \cos \alpha^* / \cos \rho_0$ obtained in deriving (5).

(9) and (10) are modified from Palache *et al.* (1944, p. 13) equations [24] and [25], where the latter should be written $c = (q_0' \cos \rho_0 \sin \nu) / \sin \mu$.

CASE 1. FIRST PERMUTATION. FIG. 2

Given: β , γ , and α (or α^*) also for #7 the axial ratio $a:b:c$, or q_1' and r_1' .

1. Draw $[b]^t$, the y-axis, as a radius of the primitive circle at $\phi = 90^\circ$ (Fig. 2a). Plot $[c]^t$ anticlockwise $(180 - \alpha^*)^\circ$ from $[b]^t$. [α^* can be obtained from (1)].

2. Plot gnom-point F_1' along $[c]^t$ at $(\beta - 90^\circ)$ to the right of $[a]$. The line aF_1' may be designated s_1' . If desired also plot $[c]^\circ$ out β° ster. to the right of $[a]$.

3. Plot gnom-point G_1' along $[b]^t$ at $(90 - \gamma)^\circ$ up from $[a]$. (This assumes γ is acute). Then $aG_1' = y_1'$. If desired also plot $[b]^\circ$ down γ° ster. from $[a]$.

4. Erect normals through common points F_1' and G_1' to $[c]^t$ and $[b]^t$ respectively, intersecting at P' , the gnomonic face-pole (100). These normals are the Q' and R' axes of Goldschmidt, and F_1' and G_1' are their zone centers. The point P' has cartesian coordinates y_1' and z_1' , and polar coordinates ϕ_1 and d_1' , where $d_1' = \tan \rho_1$.⁴

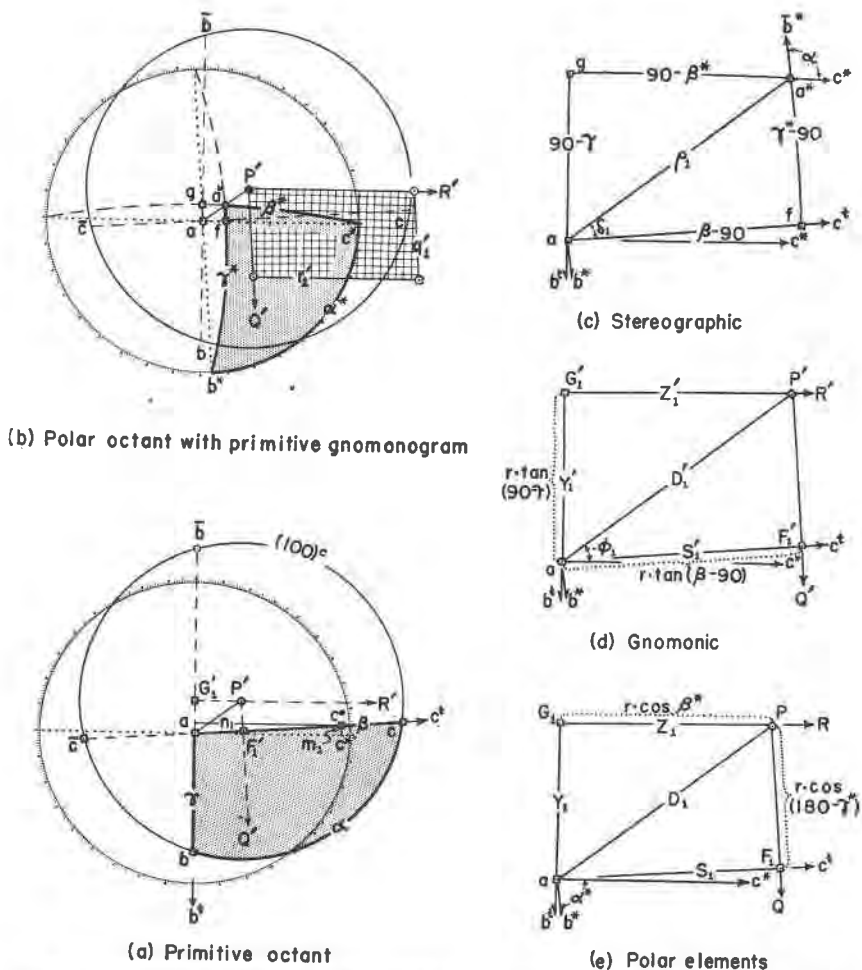


FIG. 2. Gnomonostereogram of chalcantite, first permutation. a. The entire projection with primitive circle (graduated) of $r=1$. The primitive octant is stippled. b. The entire projection with primitive of $r=1$. The polar octant is stippled and the primitive gnomonogram is reticulated. c. Central portion of stereographic (see b) with $r=10$. d. Central portion of gnomonogram with $r=5$. Note that distances D_1' , S_1' , Y_1' , and Z_1' are capitalized to emphasize that $r \neq 1$. e. Central portion of polar elements plane with $r=5$. Here also D_1 , S_1 , Y_1 , and Z_1 are capitalized, since $r \neq 1$.

⁴ ϕ_1 and ρ_1 are used here as the polar coordinates of (100). Palache *et al.* (1944, p. 20) use these in the orthorhombic system as the position angles of any given face in the first permutation.

5. If desired, draw great circle $(100)^\circ$ with center P' and radius from here to $[c]^\circ$ or $[b]^\circ$; this circle contains $(\alpha)^\circ$ and also serves to locate $[b]^\circ$ and $[\bar{c}]^\circ$.

6. The *ster*-point (Fig. 2*b*) corresponding to P' is $[a^*]^\circ$. A great circle from here to $[c^*]$ (which lies on the primitive at $\phi=0^\circ$) is the cyclographic projection of β^* . Clockwise around the primitive α^* degrees from $[c^*]$ is $[b^*]$. The great circle from $[b^*]$ to $[a^*]$ is the cyclographic projection of γ^* .

7. The primitive gnomonogram may be plotted from the values of q_1' and r_1' ; these may be obtained from the elements of crystallization by using (17) and (18) below.

Useful formulae—First permutation

$$\cos \rho_1 = \sin \beta^* \sin \gamma = \sin \beta \sin \gamma^* \quad (12)$$

$$\tan \phi_1 = \cos \gamma / \cot \beta \quad (13)$$

$$s_1' = \cot \beta = aF_1' \quad (14)$$

$$y_1' = \cot \gamma = aG_1' \quad (15)$$

$$z_1' = \cot \beta^* / \sin \gamma = G_1' P' \quad (16)$$

$$q_1' = a / (\sin \alpha \sin \gamma^*) = a / (\sin \alpha^* \sin \gamma) \quad (17)$$

$$r_1' = a / (c \cdot \sin \alpha^* \cdot \sin \beta) = a / (c \cdot \sin \alpha \cdot \sin \beta^*) \quad (18)$$

Note: Any vector in the gnomonic plane (Fig. 2*d*) may be converted to one in the polar elements plane (Fig. 2*e*) by multiplying it by $\cos \rho_1$; thus $q_1' \cos \rho_1 = q_1$, etc.

Further note: In a manner similar to that described under "standard orientation," one (Fig. 2*a*) may plot $[c]^\circ$ out on $[c]^\circ$ to the right of $[a]$ a distance of $\sin \beta$. The z -coordinate of $[c]^\circ$, called n_1 , is given by (19). It is the distance to $[b]^\circ$ going parallel to $[c^*]$. The y -coordinate of $[c]^\circ$, called m_1 , is given by (20). It is the distance to $[c^*]$ going parallel to $[b]^\circ$.

$$n_1 = \sin \alpha^* \sin \beta = \sin \alpha \sin \beta^* \quad (19)$$

$$m_1 = \cos \alpha^* \sin \beta = n_1 / \tan \alpha^* \quad (20)$$

It is clear that the denominator of (18) may be replaced by $c \cdot n_1$.

CASE 2. SECOND PERMUTATION. FIG. 3

Given α , γ , and β (or β^*); also for #7 the axial ratio $a:b:c$, or p_2' and r_2' .

1. Draw $[c]^\circ$, the z -axis, as a radius of the primitive circle at $\phi=90^\circ$ (Fig. 3*a*). Plot $[a]^\circ$ anticlockwise $(180-\beta^*)^\circ$ from $[c]^\circ$. [β^* can be obtained from (2)].

2. Plot *gnom*-point G_2' along $[a]^\circ$ at $(90-\gamma)^\circ$ to the left of $[b]$. (This assumes that γ is acute). The line bG_2' may be designated s_2' . If desired, also plot $[a]^\circ$ out γ° *ster.* to the right of $[b]$.

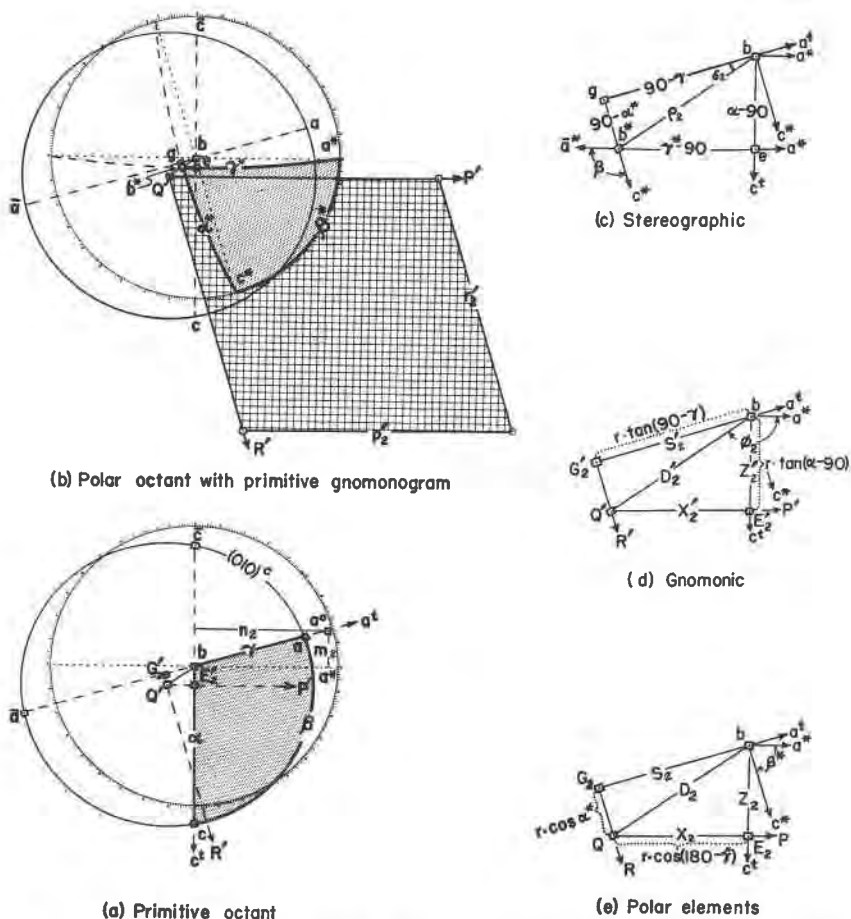


FIG. 3. Gnomonostereogram of chalcantite, second permutation. a. The entire projection with primitive circle (graduated) of $r=1$. The primitive octant is stippled. b. The entire projection with primitive of $r=1$. The polar octant is stippled and the primitive gnomonogram is reticulated. c. Central portion of stereoprojection (see *b*) with $r=10$. d. Central portion of gnomonogram with $r=5$. Note that distances D_2' , S_2' , X_2' , and Z_2' are capitalized to emphasize that $r \neq 1$. e. Central portion of polar elements plane with $r=5$. Here also D_2 , S_2 , X_2 , and Z_2 are capitalized, since $r \neq 1$.

3. Plot *gnom*-point E_2' along $[c]^t$ at $(90-\alpha)^\circ$ down from $[b]$. Then $bE_2' = z_2'$. If desired, also plot $[c]^c$ down α° *ster.* from $[b]$.

4. Erect normals through common points G_2' and E_2' to $[a]^t$ and $[c]^t$ respectively, intersecting at Q' , the gnomonic face-pole (010). These normals are the R' and P' axes of Goldschmidt, and G_2' and E_2' are their zone centers. The point Q' has cartesian coordinates x_2' and z_2' and polar coordinates ϕ_2 and d_2' , where $d_2' = \tan \rho_2$.⁵

5. If desired, draw great circle $(010)^{\circ}$ with center Q' and radius from here to $[a]^{\circ}$ or $[c]^{\circ}$; this circle contains $(\beta)^{\circ}$ and also serves to locate $[\bar{a}]^{\circ}$ and $[\bar{c}]^{\circ}$.

6. The *ster*-point (Fig. 3*b*) corresponding to Q' is $[b^*]^{\circ}$. A great circle from here to $[a^*]$ (which lies on the primitive at $\phi=0^{\circ}$) is the cyclographic projection of γ^* . Clockwise around the primitive β^* degrees from $[a^*]$ is $[c^*]$. The great circle from $[c^*]$ to $[b^*]$ is the cyclographic projection of α^* .

7. The primitive gnomonogram may be plotted from the values of p_2' and r_2' ; these may be obtained from the elements of crystallization by using (26) and (27) below.

Useful formulae—Second permutation

$$\cos \rho_2 = \sin \alpha \sin \gamma^* = \sin \alpha^* \sin \gamma \quad (21)$$

$$\tan \phi_2 = \cos \alpha / \cot \gamma^* \quad (22)$$

$$s_2' = \cot \gamma = bG_2' \quad (23)$$

$$x_2' = \cot \gamma^* / \sin \alpha = E_2'Q' \quad (24)$$

$$z_2' = \cot \alpha = bE_2' \quad (25)$$

$$p_2' = 1/(a \cdot \sin \beta \cdot \sin \gamma^*) = 1/(a \cdot \sin \beta^* \cdot \sin \gamma) \quad (26)$$

$$r_2' = 1/(c \cdot \sin \alpha \sin \beta^*) = 1/(c \cdot \sin \alpha^* \sin \beta) \quad (27)$$

Note: Any vector in the gnomonic plane (Fig. 3*d*) may be converted to one in the polar elements plane (Fig. 3*e*) by multiplying it by $\cos \rho_2$; thus $p_2' \cos \rho_2 = p_2$, etc.

Further note: In a manner similar to that described under "standard orientation," one (Fig. 3*a*) may plot $[a]^{\circ}$ out on $[a]^{\circ}$ to the right of $[b]$ a distance of $\sin \gamma$. The x-coordinate of $[a]^{\circ}$, called n_2 , is given by (28). It is the distance to $[c]^{\circ}$ going parallel to $[a^*]$. The z-coordinate of $[a]^{\circ}$, called m_2 is given by (29). It is the distance to $[a^*]$ going parallel to $[c]^{\circ}$.

$$n_2 = \sin \beta^* \sin \gamma = \sin \beta \sin \gamma^* \quad (28)$$

$$m_2 = \cos \beta^* \sin \gamma = n_2 / \tan \beta^* \quad (29)$$

It is clear that the denominator of (26) may be replaced by $a \cdot n_2$.

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⁵ ϕ_2 and ρ_2 are used here as the polar coordinates of (010). Palache *et al.* (1944, pp. 18, 20) use these in the monoclinic and orthorhombic systems as the position angles of any given face in the second permutation.

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