NOTES AND NEWS

CRYSTALLOGRAPHIC PROJECTIONS NOMENCLATURE DILEMMA

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Abstract

The terms stereographic and gnomonic refer to direct projections in their original and current cartographic meanings. It is suggested they be used in this sense by crystallographers, and that the latter designate their polar (reciprocal) variants as verstereographic and antignomonic respectively.

The stereographic projection has been known and used by astronomers, cartographers, and navigators since Ptolemy (probably derived from Hipparchus). It was given its present name in 1613 by Aguilonius. The gnomonic projection[1] may have been known to Ptolemy (according to Delambre), but apparently it was described first a millennium and a half later, by Grienberger in 1612. Originally used for star maps, later it became important for terrestrial maps to facilitate great circle navigation and avigation. These historical remarks are abstracted from Hutchinson (1908, pp. 105–112).

Except in crystallography these projections were always *direct* ones; that is, the element itself was projected; not its normal. Thus a *line* on a sphere such as a meridian on the globe (a spherical projection of the earth) became a *line* on the map or projection. A central plane intersects a sphere in a great circle; the meridians on a globe are examples. In crystallography such a great circle is often represented by a "pole," or point where the end of a radius of the sphere of projection normal to the plane in question is projected. Such a projection is *reciprocal;* instead of projecting the element itself, one projects its normal.

These projections were first taken over into crystallography by Franz Neumann in 1823. Most of his *Beitraege zur Kristallonomie* is devoted to a development of "the projection" (he nowhere refers to it as gnomonic; it is unlikely that he knew this term) and its crystallographic value. He emphasizes the use of a "gerade Endflaeche" (we would say a plane normal to the *c*-axis) as the plane of projection (even in the

[1] Germain, A. (1865?), Traité des Projections, Paris, states (pp. 122–123): "The origin of this (gnomonic) projection, the oldest known, seems to go back before Thales of Miletus, the predictor of eclipses, who died in 548 B.C. Called successively by the names of 'horoscope' and 'analemme,' it was much used in astronomy under the name of 'gnomonic' to trace the course of celestial phenomena from the surface of the earth." This quotation was kindly furnished me by the Director of the U. S. Coast and Geodetic Survey (June 5, 1952). *Sp. Publ. No.* **68** of this organization on the "Elements of Map Projection" is a most successful elementary treatment of the subject.

monoclinic), but devotes the last portion of Section 4 of Part 1 to the projection of isometric forms on other planes. He considers face-directions (the normals to crystal faces; the directions of growth) and zone-planes (planes normal to zone-axes) as of far greater significance than crystal faces and zone axes. He thus projects what we would consider reciprocal elements, which is the antithesis of previous practice: "so wird *unsere* Betrachtungsweise gleichsam als die *invertierte* von der bisher ueblichen zu bezeichnen sein; sie ist im eigentlichen Sinn ihr Gegenstueck" (Carl Neumann, 1916, p. 349; slightly different in E. R. Neumann *et al.*, 1928, p. 287). Here indeed is the first recognition of the germ of the reciprocal lattice idea, a concept that lay fallow for a generation or more; until Bravais (1850) introduced his *polar* lattice (see Shaler. 1949, p. 92), and his pupil Mallard (1879) and later Goldschmidt (1886, 1887) tied this back to the gnomonic projection. Neumann speaks of the projected elements as zone-lines and face-points (Flaechenorte); indeed the latter are marked by symbols that are substantially the same as the Goldschmidt indices (1916, Pl. IV; 1928, Fig. 9, p. 204). In an appendix he states that one can also project crystal faces as face-lines, and a zone-axis as a zone-point (Zonenort); but this he obviously considers to be of little importance. He notes that all the face-lines of a single zone intersect at the zone-point, and all the zone-axes in a single plane have zone-points which lie on the face-line of that plane.

While Franz Neumann made only minor use of the stereographic projection (which he referred to as the spherical surface) in his 1823 work (see 1916, Pl. VI; 1928, Figs. 20 and 28), he emphasized it in his 1825 paper on axinite (1928, p. 368). However, according to Barker (1922, p. 90) this "projection was eventually taken up and adapted to a remarkably simple and symmetrical form of logarithmic treatment by Miller, with results that are familiar to all." Actually Miller made great use of spherical trigonometry in crystal calculations; he spoke of (face) poles on the surface of the sphere of projection in his 1863 "Tract," which does not have the word *stereographic* in it; though in the preface he refers to two of his papers in the *Philosophical Magazine* (**18,** 1859, 37–50; and **19,** 1860, 325–28) which treat the gnomonic and stereographic projections.

Thus while these two projections were introduced into crystallography by Neumann when still a poverty-stricken 25-year old student under Weiss at Berlin, and only four years after he started in this field of endeavor, others pushed them into crystallographic prominence. In fact Johannsen (1918, p. 5) notes that Neumann's book does not appear to have been appreciated, since only the first part of it was issued (during his lifetime). The book had three parts according to C. Neumann (1916), who included F. Neumann's doctorate thesis as Part III; this last is put as a separate paper by E. R. Neumann *et al.* (1928), thus leaving but two parts. In fact these latter two "reprints" differ from each other, and probably also from the original (which has not been seen by the present writer) in many respects. An exact reprint would be very difficult for the modern reader to interpret; even Groth (1926, p. 78) found this to be true. In Franz Neumann's youth the science was in its infancy.

Goldschmidt (1886, p. 3) carefully distinguished between polar (reciprocal) and direct projections, and introduced the term *cyclographic* for the direct form of the stereographic projection, which latter he regarded as a polar projection (as Neumann had indeed used it). At this time he spoke of *linear* as the direct version of the gnomonic, but later (1887, p. 3) introduced the term *euthygraphic* for this, to avoid confusion with Quenstedt's linear projection (see Rogers, 1937, pp. 30, 47). In general Goldschmidt's terminology has been employed up to the present see Fisher, 1952, pp. 84–85).

In handling these projections in a course on structural geology, where maps are so important, this fundamental misuse of the terms stereographic and gnomonic struck the writer very forcibly. Since the structural geologists (Bucher, 1944; Nevin, 1949, pp. 379–391) are now beginning to make considerable use of these projections, it is time to redefine the terms used in crystallography to fit the original cartographic meanings. Thus, we must employ the terms stereographic and gnomonic for direct projections, and coin new ones for their reciprocal variants.

We could speak of the polar stereographic/gnomonic, or we could use reciprocal, inverse, or anti- in place of polar. The word polar is not good, since it is too easily confused with face-poles or zone-poles (edge-poles), terms in standard use. For the same reason I am against costereographic and cognomonic, since just as we speak of *ster-* and *gnom*-points (Fisher, 1952, p. 86), it is often handy to refer to *coster-* and *cognom* points. The word reciprocal is too long. The terms *verstereographic* and *antignomonic* are thus suggested for the reciprocal (polar) variants of the stereographic and gnomonic projections. It is better not to use the same prefix to designate these two variants, because it is often handy to employ one-letter symbols on a combined projection (gnomonostereogram). Thus, on such a projection one may use ⊡ $[uvw]^{s}$ or ⊡ $[uvw]^{g}$ for zone-poles and $[uvw]^{v}$ or $[uvw]^{a}$ for zone-lines. Similarly ⊙ $(hkl)^{v}$ or ⊙ $(hkl)^{a}$ symbolizes a face-pole and $(hkl)^{s}$ or $(hkl)^{g}$ a face-line.

In view of these considerations the writer (see Fisher, 1952, p. 84) would like to submit a revised Table 2.

Goldschmidt's terms were unsatisfactory not only because they tended to freeze crystallographic usage into a terminology that was contrary to the historical meanings of the older words, but also because there was no obvious connection between euthygraphic-gnomonic and stereographic-cyclographic. The suggested new terminology avoids both of these defects. Professor J. D. H. Donnay[2] prefers to speak of direct and polar stereographic or gnomonic (or spherical), and not introduce new terms. After all, the projections remain what they are, regardless of whether an element or its normal is projected. "The chief trouble with the other names is that they make people believe that the projections are different in principle, whereas they differ only by *what we project.*"

Revised Table 2. Direct and Reciprocal Projections

Element to project	Type of Projection	
	Direct (*primitive*) Stereographic or Gnomonic	*Reciprocal* (*polar*) Verstereographic or Antignomonic
Zone axis (or other *direction*)	ZONE POLE (a point) Symbol $[uvw]^s$ or $[uvw]^g$	ZONE LINE (a great circle) Symbol $[uvw]^v$ or $[uvw]^a$
Crystal face (or other *plane*)	FACE LINE (a great circle) Symbol $(hkl)^s$ or $(hkl)^g$	FACE POLE (a point) Symbol $(hkl)^v$ or $(hkl)^a$

This is a most pertinent statement; it is true for the Goldschmidt terms. If one wishes to project a bedding plane or a fault plane, is it better to refer to the plane in the projection as verstereographic/stereographic or as polar stereographic/direct stereographic? In settling this question it should be remembered that crystallographers and structural geologists will likely be dealing with the same students; the choice of proper use of terms cannot be decided on satisfactorily with a limited point of view. Properly speaking the stereographic projection of a bedding plane is a projected great circle; not a "bedding-pole." Unfortunately it has become common crystallographic usage to speak of a face-pole (a point) as the stereographic (or gnomonic) projection of the face, and it may be difficult to change this. The thesis of this paper is that it is better to do it now rather than later. Clear thinking is essential, especially when we complicate our projections by adding cleavage and optical directions and planes to those of morphological crystallography; thus precise terminology without conflicting meanings becomes fundamental.

[2] Personal communication, June 7, 1952.

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UNIVERSAL COMPASS AND THE PLUNGE OF THE BEDDED CUPRIFEROUS PYRITIC ORE DEPOSIT IN JAPAN

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In 1942, Earl Ingerson[1] devised a universal compass in order to measure the angle and direction of pitch of lineation in the field. In 1943, Ingerson and Tuttle[2] drew a graph, which is convenient for the determination of a lineation in the plane of schistosity. Also, D. J. Fisher[3] proposed the stereographic and gnomonic solution of the lineation. K. E. Lowe[4] described another geometrical method of determination.

[1] Ingerson, Earl, Apparatus for direct measurement of linear structures: *Am. Mineral.*, **27**, 721–725 (1942).

[2] Ingerson, Earl, and Tuttle, O. F., A graph for determining angle and direction of pitch of lineations in the field: *Am. Mineral.*, **28**, 209–210 (1943).

[3] Fisher, D. Jerome, Measuring linear structures on steep-dipping surfaces: *Am. Mineral.*, **28**, 204–208 (1943).

[4] Lowe, K. E., A graphic solution for certain problems of linear structure: *Am. Mineral.*, **31**, 425–434 (1946).