DETERMINATION OF THE MODE OF A METAMORPHIC ROCK

DENIS M. SHAW AND W. D. HARRISON*
Hamilton College, McMaster University, Hamilton, Ontario.

ABSTRACT

An outcrop of a sillimanite gneiss was sampled and its mode estimated by the point-counter. Statistical analysis shows that foliation does not bias the results, which are independent of the orientation of the slides to a reasonable probability. Considerable lack of homogeneity in the rock sampled is indicated by the muscovite and quartz figures, stressing the need for careful sampling of metamorphic rocks where any geochemical work is anticipated.

INTRODUCTION

In studies of metasomatism too little attention is usually given to the material which has been metasomatized. In particular it does not seem to be realized that it is necessary to know the range of composition of the original rock before metasomatism can be proved, unless there are very far-reaching changes in composition. This is especially necessary when the rocks are sedimentary and when the metasomatism does not involve wholesale replacement by minerals of the heavy metals.

The first author has been investigating the effects of metamorphism on the composition of the Littleton pelitic rocks of New Hampshire, which can be traced from fossiliferous (Devonian) shales to sillimanite gneisses. The results on trace elements are now complete (Shaw, 1954) and work on the major elements is under preparation. To supplement chemical analyses it was decided to investigate the variation in mode, which would at the same time cast light on the problem of mode determination in foliated rocks.

The point-counter analyses were carried out by the second author (Harrison, 1954). Since the work was initiated Chayes has published a theoretical study of thin-section analysis (Chayes, 1954) in which it is shown that it is immaterial whether thin sections are cut parallel or perpendicular (or at any other angle) to the foliation. One of the purposes of this paper is to present experimental evidence bearing on the subject.

The first author is indebted to the Penrose Bequest Fund of the Geological Society of America and the Geological Survey of Canada for grants which aided this research. The advice and assistance of Professor J. D. Bankier, Department of Mathematics, Hamilton College, McMaster University is gratefully acknowledged.

---

* Now at Queen's University, Kingston, Ontario.

The Problem

To calculate the composition of a rock unit it is first necessary to estimate the mineralogical mode and reliability of the estimate. Apart from difficulties in identification the chief sources of variation are as follows:

(a) variability of hand specimens
(b) variability of thin sections from any hand specimen
(c) operator and mechanical error in the point-counter method

One outcrop was studied in detail, taking several hand specimens. From each of three rocks three sections were cut, parallel to three perpendicular planes, one of which was parallel to the foliation. The operator and mechanical error was estimated by replicate analyses, sixfold for one rock and twofold for the others.

The rock chosen was a sillimanite gneiss located 1 mile south-east of West Rumney, P.O., N.H. Major constituents are muscovite, biotite, quartz and sillimanite, with garnet, opaques, chlorite, tourmaline, apatite and plagioclase in minor amounts. All the major and most of the minor constituents are present in each slide, so the outcrop shows no major inhomogeneities in composition. The grain-size averages about 0.5 mm., but sillimanite and the minor constituents (except garnet) are frequently much smaller. Much of the sillimanite occurs as fine needles embedded in quartz and muscovite grains. Some is also present as dark masses of fibrolite. Garnet is present as sporadic porphyroblasts. The rock has a well-developed schistosity and foliation,[1] the latter due to bands alternately rich and poor in muscovite or sillimanite. The combination of these textural features illustrates well the difficulties likely to be encountered in the estimation of homogeneity in metamorphic rocks, and is illustrated in Figures 1, 2 and 3.

Inhomogeneity in a Single Thin Section

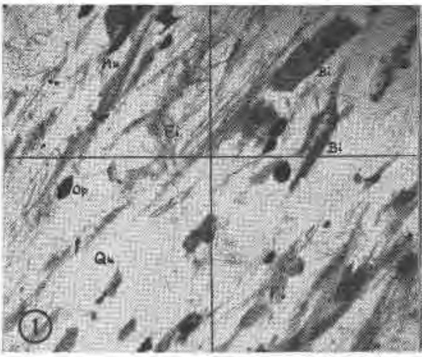
Before presenting the results of the experiment it is necessary to indicate the effect of two textural features on the results for a single thin section.

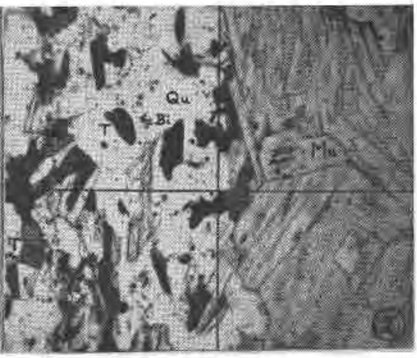
Porphyroblasts will always be a problem in modal determinations. Table 1 (left half) shows the result of analyzing the two halves of a slide separately.[2] Garnet occurs as porphyroblasts and in this section one garnet crystal was located in one half and none in the other half. It is evident that the abundance of porphyroblasts cannot be estimated with any precision. Probably the only satisfactory solution of this

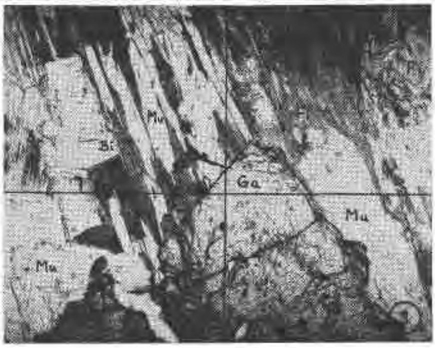
---

[1] Foliation is used in Harker's sense, referring to segregational banding.

[2] Spacing of microtraverses was 0.50 mm, except for analyses of about 4000 points, where the spacing was 0.25 mm.







FIGS. 1–3. Photomicrographs of slides analyzed. Magnification 50×. Abbreviations as follows: Mu, muscovite; Bi, biotite; Qu, quartz; Fi, fibrolite; Ga, garnet; Op opaques; T, tourmaline.

FIG. 1. L29A3. The schistosity and weak foliation are shown by the subparallel crystals of fibrolite, biotite and muscovite.

FIG. 2. L29C3. Well-developed foliation is apparent, with muscovite in a segregated band.

FIG. 3. L29A1. A garnet porphyroblast in a foliated rock consisting of alternate bands of muscovite-biotite and fibrolite-muscovite. Fractures have developed in the garnet, perpendicular to the schistosity.

problem would be to estimate these crystals separately on large polished sections.

Secondly, foliation banding affects the estimate in the same manner. Table 1 (right) shows the effect of varying the count length on a single foliated section. In column A are the results for about a quarter of the slide area and in column B the results for the whole slide (including A). Foliation in this case was parallel to the traverse direction. The variation is most noticeable for quartz and muscovite, and could obviously be avoided by traversing at right angles to the foliation. No specific attempt was made to do this, in order to avoid a favorable orientation and to emphasize possible variability.

TABLE 1. INHOMOGENEITIES

Inhomogeneity due to porphyroblasts			*Inhomogeneity due to foliation*		
Mineral	Slide L29B3		Mineral	Slide L29C3	
	First half	Second half		A	B
Muscovite	21.5	18.6	Muscovite	45.6	21.4
Sillimanite	3.4	3.4	Sillimanite	7.0	5.0
Biotite	11.8	22.5	Biotite	25.3	15.2
Quartz	52.9	51.3	Quartz	18.3	56.7
Garnet	5.2	nil	Opaques	2.6	1.2
Opaques	1.5	1.1	Tourmaline	1.1	.5
Chlorite	.9	.2	Plagioclase	.2	.1
Tourmaline	2.2	2.8			
Apatite	.2	nil			
Plagioclase	.6	.1			
Total	100.2	100.0	Total	100.1	100.1
Points tallied	2000	2022	Points tallied	1124	3484

Results and Discussion

Table 2 gives the results of the 30 analyses of 9 thin sections of 3 hand specimens of the rock (L29). The orientations are indicated by letters A, B and C where B indicates the foliation plane and A and C the planes mutually perpendicular to each other and to B. The hand specimens are numbered 1, 2 and 3 in the final figure of the slide symbol.

It is evident that reproducibility is satisfactory, and in few cases is there much disagreement among replicate analyses of the same slide (e.g., sillimanite, L29B3). However there are notable differences between different slides. This may arise in several ways: (a) variation in estimates of one hand specimen but without any foliation bias, (b) variation in estimates of one hand specimen due to bias caused by the foliation, (c) variation in composition between hand specimens irrespective of foliation bias. Any separate estimates of a population (in this case the composition of the outcrop) will differ, and the differences will reflect the homogeneity of the population. We require to find however the relative importance of (a), (b) and (c), i.e., whether it matters in what direction the slides are cut and whether we take one or more hand specimens. These factors can be assessed, together with reproducibility, by variance analysis. For the reader unfamiliar with this procedure the methods and symbols of Dixon & Massey (1951,

TABLE 2. ANALYSES OF HAND-SPECIMENS

Mineral	*Specimen* 1—L29A1						*Specimen* 1—L29B1					
Muscovite	39.5	39.8	44.6	44.1	44.3	41.8	30.4	34.1	33.1	31.2	29.5	33.2
Sillimanite	19.3	19.8	16.0	17.9	18.0	17.1	8.5	5.7	4.7	7.3	8.1	7.4
Biotite	15.6	14.9	14.5	14.9	14.3	15.3	23.6	22.0	22.6	23.8	23.5	23.2
Quartz	17.3	17.3	16.1	14.3	15.6	16.7	30.5	31.6	34.1	30.3	32.0	29.5
Garnet	5.3	5.4	5.3	6.0	5.5	5.5	3.4	3.5	2.6	3.3	3.2	3.6
Opaques	1.5	1.0	1.5	1.2	.9	1.7	1.8	1.5	1.2	1.8	1.3	1.4
Chlorite	.9	1.0	1.3	1.0	.9	1.0	.7	.9	1.0	1.3	1.4	.9
Plagioclase	.1	.4	.4	.2	.2	.3	.9	.6	.5	.6	.7	.7
Tourmaline	.5	.5	.3	.4	.2	.5	.2	.1	.2	.3	.2	.1
Total	100.0	100.1	100.0	100.0	99.9	99.9	100.0	100.0	100.0	99.9	99.9	100.0
Points Tallied	1946	1870	1951	1656	1771	1917	2100	3558	2500	2500	2200	2200

	Specimen 1—L29C1					
Muscovite	38.8	38.5	36.0	37.2	37.6	37.2
Sillimanite	10.6	10.4	10.9	11.0	11.1	11.2
Biotite	19.0	18.1	19.4	19.6	18.8	19.2
Quartz	23.1	24.8	24.6	24.4	23.9	24.9
Garnet	5.0	5.3	5.2	5.0	5.0	4.6
Opaques	1.4	1.2	1.6	1.2	1.3	1.1
Chlorite	1.5	1.0	1.7	1.1	1.8	1.3
Plagioclase	.3	.2	.3	.2	.3	.5
Tourmaline	.4	.6	.4	.4	.3	.1
Total	100.1	100.1	100.1	100.1	100.1	100.1
Points Tallied	2010	1900	1896	2079	2046	1941

	Specimen 2						*Specimen* 3					
	L29A2		L29B2		L29C2		L29A3		L29B3		L29C3	
Muscovite	22.5	24.0	33.5	31.5	32.2	28.4	13.0	11.1	20.1	20.1	21.4	21.7
Sillimanite	11.6	11.8	14.8	15.2	16.8	18.5	8.2	11.0	3.4	11.5	5.0	4.3
Biotite	28.1	26.3	23.9	24.2	23.6	23.7	17.0	16.5	17.1	15.5	15.2	13.6
Quartz	30.4	29.1	14.4	16.4	21.8	23.2	60.6	60.4	52.1	42.6	56.7	59.0
Garnet	3.1	3.6	6.6	7.1	3.0	2.9	nil	nil	2.6	2.7	nil	nil
Opaques	3.1	3.6	4.6	3.5	1.6	2.2	1.2	1.0	1.3	3.7	1.1	1.0
Chlorite	1.1	1.6	1.4	1.2	.7	.7	nil	nil	.5	1.4	nil	.1
Plagioclase	.1	.1	.1	.1	nil	.1	nil	nil	.4	nil	.1	.1
Tourmaline	nil	.1	.7	.8	.2	.2	.1	.1	2.5	2.2	.5	.4
etc.	nil	nil	nil	nil	nil	nil	nil	.1	.1	.4	nil	nil
Total	100.0	100.2	100.0	100.0	99.9	99.9	100.1	100.2	100.1	100.1	100.0	100.2
Points Tallied	2213	2000	2034	2062	1799	1736	2000	2000	4022	1933	3484	2000

pp. 134–138) are used in the following, where variance analysis is carried out for each of the major constituents muscovite, sillimanite, biotite and quartz. The procedure for muscovite will be outlined first.

The muscovite measurements are set out in Table 3, together with the sums of the individual measurements from each category (T_{ij}), and the

TABLE 3. MUSCOVITE IN L29

		Orientations			*Totals*				
		A	B	C	A	B	C	$T_{i\cdot}$	$n_{i\cdot}$
Rocks	1	39.5	30.4	38.8	254.1	191.5	225.3	670.9	18
		39.8	34.1	38.5					
		44.6	33.1	36.0					
		44.1	31.2	37.2					
		44.3	29.5	37.6					
		41.8	33.2	37.2					
	2	22.5	33.5	32.2	46.5	65.0	60.6	172.1	6
		24.0	31.5	28.4					
	3	13.0	20.1	21.4	24.1	40.2	43.1	107.4	6
		11.1	20.1	21.7					
				$T_{\cdot j}$	324.7	296.7	329.0	950.4	
				$n_{\cdot j}$	10	10	10		30

sums of all the measurements on a particular hand specimen ($T_{i\cdot}$) and the number of measurements ($n_{i\cdot}$), and similar quantities for each orientation ($T_{\cdot j}$ and $n_{\cdot j}$). The bottom figure in column 9 is $T_{\cdot\cdot}$, the sum of all the muscovite measurements, and the bottom figure in column 10 is n, the total number of measurements. The steps in computing the variance figures are as follows:

(1) $C = \dfrac{T_{\cdot\cdot}^2}{n}$

(2) Total sum of squares: $\sum X^2 - C$ (X is an individual measurement in Table 5).

(3) Subclasses: $\sum_{ij} \dfrac{T_{ij}^2}{n_{ij}} - C$ (T_{ij} is an individual total from Table 3, comprising n_{ij} measurements.)

(4) Residual: (2)–(3)

(5) Orientations: $\sum_j \dfrac{T_{\cdot j}^2}{n_{\cdot j}} - C = \dfrac{1}{10} \sum_j T_{\cdot j}^2 - C$

(6) Rocks: $\sum_i \dfrac{T_{i\cdot}^2}{n_{i\cdot}} - C$

(7) Interactions: (3)–(5)–(6)

The variance figures are listed in Table 4, together with the appropriate degrees of freedom (*d.f.*), the quotients giving the mean squares in column 4. Following the usual procedure the ratio of interactions and residual mean squares is computed first (column 5). This is found to exceed the value of the F-distribution for 99% probability and the

TABLE 4. ANALYSIS OF VARIANCE OF MUSCOVITE

Source	Sum of Squares	*d.f.*	Mean square	s_3^2/s_4^2	s^2/s_3^2
Orientations	61.526	2	$30.763=s_1^2$		.27
Rocks	1756.124	2	$878.062=s_2^2$		7.57
Interactions	463.905	4	$115.976=s_3^2$	40.2	
Residual	60.633	21	$2.887=s_4^2$		
Total	2342.188	29	80.765		

$F_{.99}$ (4, 21) = 4.37
$F_{.95}$ (2, 4) = 6.94
$F_{.99}$ (2, 4) = 18.0

Mean $\bar{X}=T_{..}^2/n=31.68$

TABLE 5. ANALYSIS OF VARIANCE FOR QUARTZ, BIOTITE, SILLIMANITE

	Source	Sum of Squares	*d.f.*	Mean Square	s_3^2/s_4^2	s^2/s_3^2
Quartz	Orientations	71.429	2	35.715		.14
	Rocks	4813.222	2	2406.611		9.49
	Interactions	1014.538	4	253.635	72.1	
	Residual	73.865	21	3.517		
	Total	5973.054	29	205.967		
	$\bar{X}=29.92$					
Biotite	Orientations	92.683	2	46.342		1.42
	Rocks	264.780	2	132.390		4.06
	Interactions	130.384	4	32.596	73.6	
	Residual	9.300	21	.443		
	Total	497.147	29	17.143		
	$\bar{X}=19.57$					
Sillimanite	Orientations	210.662	2	105.331		1.85
	Rocks	177.317	2	88.659		1.56
	Interactions	227.392	4	56.848		
	Residual	59.412	21	2.829	20.1	
	Total	674.783	29	23.268		
	$\bar{X}=11.57$					

appropriate number of degrees of freedom (4,21) as given in Table 4 (from Dixon & Massey, 1951, Appendix, Table 7). The significance of this is difficult to assess and its discussion is relegated to the Appendix. In the last column of Table 4 are found the ratios of orientations-to-interactions and rocks-to-interactions mean squares. These are compared with the values of the *F*-distribution for 95% and 99% probability (2, 4 *d.f.*) as given in Table 4. From this it is concluded that there is no significant difference between means for different orientations. For rocks there is a significant difference at the 95% level only. In other words, any of the slides from a given rock gives an equally valuable estimate of the composition of the rock, regardless of its orientation.

Table 6. Significance Tests

Mineral	Interactions—Residual	Orientations—Interactions		Rocks—Interactions	
	99%	99%	95%	99%	95%
Muscovite	HS	—	—	—	S
Quartz	HS	—	—	—	S
Biotite	HS	—	—	—	—
Sillimanite	HS	—	—	—	—

HS = Highly significant.
S = Significant.
— = Not significant.

However it seems unlikely that a single rock is a good estimate of the composition of the outcrop.

Results for quartz, biotite and sillimanite are set out in Table 5, omitting the computations and omitting the *F*-values which are the same as in Table 4. The significance tests are summarized in Table 6. It is seen that the interactions term is highly significant in every case, but otherwise the means are only significantly different for rocks when the muscovite and quartz figures are tested at the 95% level.

In every case the variance components may be interpreted as follows. The residual mean square (s_4^2) measures the experimental error and its square root expresses this as a standard deviation. By subtracting the residual sum of squares from the total sum of squares (or by adding the orientations, rocks and interactions sums of squares) a figure is obtained which estimates the variability of the outcrop and which has 29 minus 21 or 8 degrees of freedom. The mean square (s_o^2) is calculated and s_o is

TABLE 7. AVERAGE COMPOSITION OF ALL SLIDES

Mineral	%	s_e	s_o
Muscovite	31.7	1.7	16.9
Quartz	29.9	1.9	27.2
Biotite	19.6	0.7	7.8
Sillimanite	11.6	1.7	8.8
Garnet	3.8		
Opaques	1.8		
Chlorite	1.0		
Tourmaline	0.5		
Plagioclase	0.3		
Total	100.2		

then the standard variation. Table 7 contains the values of s_4 ($=s_e$) and s_o, together with the estimated average composition of the outcrop.

CONCLUSIONS

The results of the experiment may be summarized as follows. If it is desired to measure the modal composition of a metamorphic formation numerous slides should be analyzed. The orientation of the slides with respect to any foliation planes is irrelevant, but to get a good estimate several rocks should be used. Whereas the latter is intuitively obvious to any geologist, the former is not and the writers have come across considerable criticism along the lines that it is "wrong" to use a section cut parallel to foliation.

The large figures in Table 12 for s_o in the case of muscovite and quartz suggest that the original sedimentary formation consisted of beds rich in sand and beds rich in aluminous material, or alternatively that metamorphic differentiation has simulated this state of affairs. The much lower experimental errors (s_e) for all four minerals studied indicate again the value of the point counter as an analytical tool.

APPENDIX

Significance of the Interactions Mean Square

In Table 7 and successive Tables it is seen that the interactions-to-residual mean squares exceeds the appropriate F-value. In other words the interactions term is significant. The implication of this is discussed by Dixon & Massey (1951, p. 137) who say:

> "Several factors may cause the interaction to be significant.
>
> 1. There is no interaction, but we have obtained a value which we declare significant. This will occur 5 per cent of the time if we use a 5 per cent level of significance.

2. The two variables of classification are interacting, i.e., producing effects together which would not be produced separately.
3. Another uncontrolled factor is of sufficient importance to include in the experiments.
4. The items in the subgroups are not randomly drawn."

In view of the fact that four minerals all show an interactions term which appears highly significant, it seems improbable that this is due to chance. Of the remaining three factors the writers can make no choice. An uncontrolled factor which may possibly be important is lineation, but this is very weak in these rocks and might be expected to affect sillimanite more than quartz, whereas the reverse seems the case. A true interaction might be produced by the confusion of basal muscovite with quartz in the foliation plane. This was detected early in the work however, and in any case should not affect sillimanite and biotite.

REFERENCES

CHAYES, F. (1954): The Theory of Thin-Section Analysis. *Jour. Geol*, **62,** 92–101.

DIXON, W. J., & MASSEY, F. J. (1951): *Introduction to Statistical Analysis.* New York.

HARRISON, W. D. (1954): Determination of the Composition of Metamorphic Rocks by Use of the Point Counter. *M.Sc. thesis, McMaster University.*

SHAW, D. M. (1954): Trace Elements in Pelitic Rocks: Part I, Variations During Metamorphism. *Geol. Soc. Am., Bull.* **65,** 1151–1166.