A NEW EQUATION RELATING INDEX OF REFRACTION AND SPECIFIC GRAVITY*

ROBERT D. ALLEN, *U. S. Geological Survey, Claremont, Calif.*

ABSTRACT

A new equation relating index of refraction and specific gravity is proposed: $\alpha\beta\gamma/d=k$. The terms $\alpha\beta\gamma$ and k are referred to as refractive capacity and specific refractive capacity, respectively. The new equation is compared with 3 classical equations by means of 12 groups of polymorphous compounds. Specific refractive capacities are in closer agreement for Al_2O_3, SiO_2, $CaCO_3$, Al_2SiO_5, and $NaAlSiO_4$ than are the classical constants. All equations yield satisfactory agreement between modifications of $MgSiO_3$, $CaSiO_3$, ZnS, and Na_2SO_4. Specific refractive capacities for the respective polymorphs of TiO_2, As_2O_3, and Sb_2O_3 show gross discrepancies, considerably greater than those found with the older equations. Such discrepancies demonstrate that refraction of light is a constitutive as well as an additive property and are thought to result from variations in manner of aggregation.

Specific refractive capacities for the common oxides are tabulated. Average index of refraction is calculated for 26 compositions by means of (1) $\alpha\beta\gamma/d=k$ and (2) $(n-1)/d=k$; in the majority of these examples, the former expression yields indices closer to those experimentally measured.

INTRODUCTION

During the past century, three principal equations relating index of refraction to specific gravity have been proposed. The existing formulas (Larsen and Berman, 1934) are listed in Table 1. The Lorentz and Lorenz

TABLE 1. EQUATIONS RELATING INDEX OF REFRACTION AND SPECIFIC GRAVITY

Author(s)	Equation	Designation of Constant
Gladstone and Dale	$\frac{n-1}{d}=k$	Specific refractivity
Lorentz and Lorenz	$\frac{n^2-1}{n^2+2}\cdot\frac{1}{d}=k$	Specific refraction
Lichtenecker	$\frac{\log n}{d}=k$	———

equation is based on the electromagnetic theory of light, whereas the others are of empirical origin.

Although numerous articles dealing with relationships among chemical

* Publication authorized by the Director, U. S. Geological Survey.

composition, index of refraction, and density in glass have recently appeared, at present only approximate generalized formulas for wide ranges and accurate expressions for limited ranges of composition are possible (Sun, Safford, and Silverman, 1940).

The postulate upon which these relationships are based may be stated as follows: for a definite chemical composition there exists a constant k, which may be calculated from the index of refraction and density of any one of the composition's physical modifications (solid, liquid, gas; polymorphs). In other words, it is assumed that refraction of light is an additive property dependent solely on concentration and composition. Despite the limited validity of the postulate, this type of equation is useful for computations within restricted ranges where index of refraction is an additive property. Conversely, definite discrepancies may supply useful information about the nature of bonding.

Basis for Proposed Equation

The concept of the indicatrix has been fundamental in redefining refractivity, the capacity of a substance to refract light. The indicatrix (Fletcher, 1892), a geometrical abstraction utilized in optical mineralogy to depict the behavior of light in crystals, may be defined as a three-dimensional geometric figure so constructed that the three principal indices of refraction of light waves in their directions of vibration are equal to its three mutually perpendicular semiaxes. Thus, in isotropic substances, which have but one index of refraction, the indicatrix is a sphere. In uniaxial crystals, which have two indices of refraction, the indicatrix is either a prolate or oblate spheroid of revolution depending on whether the substance is optically positive or negative, respectively. For biaxial crystals, with three indices, the indicatrix takes the general form of a triaxial ellipsoid. In essence, then, the indicatrix is a vectorial representation of the refraction of light in three dimensions.

The refractivity of a substance is here considered to be proportional to the volume of its indicatrix. The formulas expressing volume for the three classes of optical indicatrix are as follows:

Sphere	$4/3\pi n^3$
Spheroid of revolution	$4/3\pi\omega^2\epsilon$
Triaxial ellipsoid	$4/3\pi\alpha\beta\gamma$

In the general case, therefore, refractivity is proportional to the product $\alpha\beta\gamma$. The product of the three indices of refraction of a substance—$\alpha\beta\gamma$ for biaxial crystals, $\omega^2\epsilon$ for uniaxial crystals, and n^3 for isotropic substances—may be defined as refractive capacity. A new synthetic constant, k, may be obtained from the formula $\alpha\beta\gamma/d = k$, where d is the specific gravity. This value k may be termed the specific refractive capacity.

Tabulation of Constants of Some Common Physical Modifications

The four equations are compared by computation of their respective constants for 12 groups of polymorphous or dimorphous compounds. Data for water-ice and for the isomers of chlorotoluene are also given. Tridymite is omitted from the table because of uncertainty in optical data. For uniaxial and biaxial crystals the mean index of refraction, n, equals $\sqrt[3]{\omega^2\epsilon}$ or $\sqrt[3]{\alpha\beta\gamma}$; this quantity must be determined for solution of the classical equations. Specific gravities are calculated (Schlecht, 1944) for crystal modifications of SiO_2 and $CaCO_3$ as well as for compounds which show gross lack of agreement between values of k when experimental specific gravities are employed. Sources of experimental data are indicated by numbers in parentheses which refer to the bibliography.

It should be pointed out that impurities in minerals may affect the calculations in some cases. For instance, minor amounts of iron, titanium, or chromium may be present in corundum. Columbium, tantalum, and iron have been reported in rutile; iron is usually present in anatase and brookite. It is considered likely, however, that the optical and specific gravity data cited would be essentially correct for the corresponding ideally pure compounds.

Discussion

Specific refractive capacities are in closer agreement than are the constants calculated by means of the classical equations for polymorphs of the following compositions: Al_2O_3, SiO_2, $CaCO_3$, Al_2SiO_5, and $NaAlSiO_4$. Each of these groups is characterized by appreciable differences in specific gravity and indices of refraction. In contrast, for those groups of substances whose polymorphs do not have widely different physical properties, such as, $MgSiO_3$, $CaSiO_3$, ZnS, and Na_2SO_4, all four equations yield comparable results.

The specific refractive capacities for TiO_2, As_2O_3, Sb_2O_3, and H_2O show gross discrepancies between the respective modifications of each composition. The differences are considerably greater than those arrived at through use of the older formulas. The immediate explanation lies in the mathematical nature of the various equations. In a series of polymorphous compounds whose indices of refraction are dependent upon variables other than chemical composition and specific gravity, the ratio $\alpha\beta\gamma/d$ would be subject to greater variations than the corresponding ratios of the previously introduced equations.

It is clear that index of refraction is a constitutive property as well as an additive property, that is, the manner of aggregation of chemical elements is a determining factor besides the kinds and concentrations of

TABLE 2. CONSTANTS OF SOME COMMON PHYSICAL MODIFICATIONS

Formula	Physical modification; crystal system	Specific gravity	Indices of refraction	$\frac{n-1}{d}$	$\frac{n^2-1}{n^2+2}\cdot\frac{1}{d}$	$\frac{\log n}{d}$	$\frac{\alpha\beta\gamma}{d}$
Al_2O_3 (6, 11)	Corundum; hexagonal (synthetic)	3.98 (calc.)	ω, 1.768 ϵ, 1.760	0.192	0.104	0.0620	1.38
	"Third phase"; isometric	3.47	1.696	0.201	0.111	0.0661	1.41
SiO_2 (5, 12, 13)	Low quartz; hexagonal	2.66 2.664 (calc.)	ω, 1.544 ϵ, 1.553	0.206 0.205	0.119 0.119	0.0712 0.0711	1.39 1.39
	Low cristobalite; tetragonal	2.33 2.342 (calc.)	1.486 (mean)	0.209 0.208	0.123 0.123	0.0738 0.0735	1.41 1.40
TiO_2 (6)	Rutile; tetragonal	4.23 4.26 (calc.)	ω, 2.612 ϵ, 2.899	0.403 0.400	0.160 0.159	0.102 0.101	4.68 4.64
	Brookite; orthorhombic	4.14 4.12 (calc.)	α, 2.583 β, 2.584 γ, 2.700	0.392 0.394	0.160 0.161	0.101 0.102	4.35 4.37
	Anatase; tetragonal	3.90 4.04 (calc.)	ω, 2.561 ϵ, 2.488	0.394 0.380	0.165 0.159	0.104 0.100	4.18 4.04
As_2O_3 (6)	Arsenolite; isometric	3.87 3.88 (calc.)	1.755	0.195 0.195	0.106 0.106	0.0631 0.0630	1.40 1.39
	Claudetite; monoclinic	4.15 4.26 (calc.)	α, 1.871 β, 1.92 γ, 2.01	0.225 0.219	0.115 0.112	0.0690 0.0672	1.74 1.70
Sb_2O_3 (6)	Senarmontite; pseudoisometric	5.50 5.56 (calc.)	2.087	0.198 0.196	0.0960 0.0950	0.0581 0.0575	1.65 1.64
	Valentinite	5.76 (exp. and calc.)	α, 2.18 β, 2.35 γ, 2.35	0.224	0.1017	0.0625	2.09
$CaCO_3$ (7)	Calcite; hexagonal	2.710 2.743 (calc.)	ω, 1.6585 ϵ, 1.4865	0.221 0.218	0.126 0.125	0.0752 0.0743	1.51 1.49
	Aragonite; orthorhombic	2.95 2.944 (calc.)	α, 1.530 β, 1.681 γ, 1.685	0.214 0.214	0.121 0.121	0.0721 0.0720	1.47 1.47
	Vaterite; hexagonal	2.645 (calc.)	ω, 1.550 ϵ, 1.645	0.220	0.126	0.0752	1.49
$MgSiO_3$ (11)	Enstatite; orthorhombic	3.175	α, 1.650 β, 1.653 γ, 1.658	0.206	0.115	0.0688	1.42

TABLE 2 (*continued*)

Formula	Physical modification; crystal system	Specific gravity	Indices of refraction	$\frac{n-1}{d}$	$\frac{n^2-1}{n^2+2}\cdot\frac{1}{d}$	$\frac{\log n}{d}$	$\frac{\alpha\beta\gamma}{d}$
	Clinoenstatite; monoclinic	3.19	α, 1.651 β, 1.654 γ, 1.660	0.205	0.115	0.0686	1.42
$CaSiO_3$ (11)	Wollastonite; monoclinic	2.915	α, 1.616 β, 1.629 γ, 1.631	0.215	0.121	0.0724	1.47
	Pseudo-wollastonite; monoclinic	2.905	α, 1.610 β, 1.610 γ, 1.654	0.215	0.122	0.0725	1.48
Al_2SiO_5 (5)	Kyanite; triclinic	3.60	α, 1.712 β, 1.720 γ, 1.728	0.200	0.110	0.0654	1.41
	Sillimanite; orthorhombic	3.23	α, 1.659 β, 1.660 γ, 1.680	0.206	0.115	0.0687	1.43
	Andalusite; orthorhombic	3.15	α, 1.634 β, 1.639 γ, 1.643	0.203	0.114	0.0681	1.40
$NaAlSiO_4$ (11)	Carnegieite; triclinic	2.51	α, 1.509 β, 1.514 γ, 1.514	0.204	0.120	0.0716	1.38
	Nephelite; hexagonal	2.619	ω, 1.537 ϵ, 1.533	0.205	0.119	0.0711	1.38
ZnS (6)	Sphalerite; isometric	4.00	2.369	0.343	0.151	0.0936	3.32
	Wurtzite; hexagonal	3.98	ω, 2.356 ϵ, 2.378	0.343	0.151	0.0938	3.32
Na_2SO_4 (7, 11)	Thenardite; orthorhombic	2.664	α, 1.471 β, 1.477 γ, 1.484	0.179	0.106	0.0636	1.21
	Na_2SO_4—III; orthorhombic	2.696	1.4825 (mean)	0.179	0.106	0.0634	1.21
H_2O (3, 6)	Ice; hexagonal	0.917 0.921 (calc.)	ω, 1.309 ϵ, 1.310	0.337 0.336	0.209 0.208	0.128 0.127	2.45 2.44
	Water	1.00	1.335	0.335	0.207	0.125	2.38
$ClC_6H_4CH_3$ (3)	O-Chlorotoluene	1.0817	1.5238	0.484	0.283	0.169	3.27
	M-Chlorotoluene	1.0722	1.5214	0.486	0.284	0.170	3.28
	P-Chlorotoluene	1.0697	1.5199	0.486	0.284	0.170	3.28

elements. No formula, therefore, which involves only index of refraction and density as variables can be expected to produce a constant for all phases and polymorphs of a given chemical composition.

The gross differences in specific refractive capacities of TiO_2, As_2O_3, and Sb_2O_3 may be a reflection of variation in bond type from one polymorph to another. Pauling (1948) has pointed out differences in bonding between the respective polymorphs of As_2O_3 and Sb_2O_3. Arsenolite (As_2O_3, isometric) and senarmontite (Sb_2O_3, pseudo-isometric), whose specific refractive capacities are relatively low, consist of small molecules of the type A_4X_6. On the other hand, claudetite (As_2O_3, monoclinic) and valentinite (Sb_2O_3, orthorhombic), whose specific refractive capacities are relatively high, have infinite molecules. A somewhat parallel situation is presented by water with discrete molecules (specific refractive capacity $=2.38$) and ice with infinite molecules (specific refractive capacity $=2.44$). It may be noted that anomalous values of molar refraction* (Glasstone, 1946, p. 528) have been useful in the determination of bond type in organic compounds. Variations in bond mechanism could account for anomalous values of k encountered in the modifications of TiO_2.

It might also be significant that Ti, As, and Sb are all elements capable of existing in more than one state of positive valence.

Specific Refractive Capacities of the Oxides

Specific refractive capacities of the oxides have been calculated (1) from the oxides themselves or (2) from compounds containing the oxides. The specific refractive capacity, k, of a compound of the type $mAX \cdot nBX$, where m is the weight percentage of AX and n is the weight percentage of BX, is given by the equation

$$\frac{mk_{AX} + nk_{BX}}{100} = K_{AX \cdot BX}.$$

If $K_{AX \cdot BX}$ and either k_{AX} or k_{BX} are known, the equation may be solved for the unknown k value. Many of the specific refractive capacities tabulated below have been calculated in this manner. Of course, this computation presupposes additivity of k values.

References in Table 3 are indicated in parentheses.

Calculation of Average Index of Refraction for Minerals, Artificial Compounds, and Glasses

Average index of refraction may be calculated for a compound whose specific refractive capacity and specific gravity are known. (Conversely,

* Molar refraction is defined as $(n^2-1)/(n^2+2) \cdot M/d$, where M is the molecular weight.

density may be calculated if average index of refraction and specific refractive capacity are known.) Specific refractive capacity is given by the earlier described equation

$$\frac{mk_{AX} + nk_{BX}}{100} = K_{AX \cdot BX}.$$

Calculated average indices of refraction for minerals, artificial compounds, and glasses, obtained from (1)$\alpha\beta\gamma/d = k$ and (2)$(n-1)/d = k$, are tabulated in Table 4. Experimental indices and data used in the calculations are included. Values of k for the Gladstone and Dale equation were taken from Larsen and Berman (1934). Chemical analyses are recalculated to 100 per cent. As before, references are indicated in parentheses.

In 15 of 26 compositions the average indices calculated by means of the new equation are substantially nearer to the experimental values than are those obtained from the Gladstone and Dale equation. In nine compositions the two equations yield calculated indices which differ from the experimental by similar increments. In two compositions indices calculated by the Gladstone and Dale equation are significantly nearer to the experimental. It must be pointed out that certain oxides, such as H_2O, CO_2, Fe_2O_3, TiO_2, are not characterized by constant specific refractive capacities for all compounds (see Table 3), and calculations involving these oxides should be carried out with caution.

Summary

A new equation relating index of refraction and specific gravity is proposed: $\alpha\beta\gamma/d = k$. The term $\alpha\beta\gamma$ is referred to as refractive capacity and is proportional to the volume of the indicatrix; k is designated as specific refractive capacity.

The new equation is compared with three earlier equations by computation of their respective constants for 12 groups of polymorphous compounds. Specific refractive capacities for the respective polymorphs of Al_2O_3, SiO_2, $CaCO_3$, Al_2SiO_5, and $NaAlSiO_4$ are in closer agreement than the classical constants; each of these groups is characterized by appreciable differences in specific gravity and indices of refraction. In contrast, every equation yields satisfactory agreement between modifications of $MgSiO_3$, $CaSiO_3$, ZnS, and Na_2SO_4; each of these groups is characterized by small differences in specific gravity and indices of refraction.

The specific refractive capacities for TiO_2, As_2O_3, Sb_2O_3, and H_2O show large discrepancies, greater than the constants calculated from the older formulas, between the respective modifications of each composition. These anomalies are attributed to variations in the manner of aggregation of atoms.

Specific refractive capacities for the common oxides are tabulated.

TABLE 3. SPECIFIC REFRACTIVE CAPACITIES OF THE COMMON OXIDES

Oxide	Specific refractive capacity	Source of data	
Ag_2O	1.07	$AgNO_3$	(11)
Al_2O_3	1.38	Al_2O_3	(11)
As_2O_3	1.40	Arsenolite	(6)
	1.74	Claudetite	(6)′
B_2O_3	1.60	CaB_2O_4	(11)
	1.65	$Mg_3B_2O_6$	(11)
	1.70	B_2O_3 glass	(3)
BaO	0.909	Celsian	(5)
BeO	1.71	BeO	(11)
Bi_2O_3	0.820	Bi_2O_3	(3)
CO_2	1.31	Calcite-aragonite	(7)
	1.40	Dolomite	(7)
CaO	1.63	Anorthite	(5)
CrO_3	2.36	K_2CrO_4	(11)
Cr_2O_3	2.09	$FeCr_2O_4$	(12)
	3.01	Cr_2O_3	(11)
Cs_2O	0.834	Cs_2SO_4	(11)
Cu_2O	3.31	Cu_2O	(11)
CuO	1.75	$CuSO_4$	(11)
FeO	1.42	$FeAl_2O_4$	(12)
	1.54	Fayalite	(5)
Fe_2O_3	2.49	Andradite	(5)
	5.80	Hematite	(6)
H_2O	2.25	Gypsum	(7)
	2.38	Water	(3)
	2.44	Ice	(6)
HgO	1.41	HgO	(3)

TABLE 3—(*continued*)

Oxide	Specific refractive capacity	Source of data	
K_2O	1.33	Orthoclase	(5)
Li_2O	2.20	Li_2O	(11)
MgO	1.46	MgO	(11)
MnO	1.44	Tephroite	(5)
N_2O_5	1.60	BaN_2O_6	(11)
Na_2O	1.28	Albite	(5)
P_2O_5	1.03	$Na_4P_2O_7$	(3)
	1.26	KH_2PO_4	(11)
PbO	1.08	$Pb(NO_3)_2$	(3)
Rb_2O	0.885	Rb_2SO_4	(11)
SO_3	1.14	Anhydrite	(7)
Sb_2O_3	1.65	Senarmontite	(6)
	2.09	Valentinite	(6)
SiO_2	1.39	Quartz	(5)
SnO_2	1.20	Cassiterite	(3)
SrO	1.04	Celestite	(7)
	1.11	$SrAl_2Si_2O_8$	(11)
ThO_2	1.03	ThO_2	(3)
TiO_2	4.18	Anatase	(6)
	4.35	Brookite	(6)
	4.68	Rutile	(6)
WO_3	1.05	$CaWO_4$	(11)
ZnO	1.16	$ZnAl_2O_4$	(11)
	1.45	Zincite	(6)
ZrO_2	1.73	Zircon	(5)

TABLE 4. CALCULATED AVERAGE INDICES OF REFRACTION FOR MINERALS, ARTIFICIAL COMPOUNDS, AND GLASSES

Substance	Composition	k values employed		Specific gravity	Average index of refraction		
					Experimental	Calculated	
		$\alpha\beta\gamma/d$	$(n-1)/d$			$\alpha\beta\gamma/d$	$(n-1)/d$
Tridymite (5)	SiO_2	1.39	0.207	2.30	1.471	1.473	1.476
Baddeleyite (5)	ZrO_2	1.73	0.201	5.7±	2.17	2.144	2.146
Spinel (5)	$MgO \cdot Al_2O_3$			3.60	1.718	1.716	1.702
	MgO 28.34%	1.46	0.200				
	Al_2O_3 71.66%	1.38	0.193				
	K_S	1.403	0.1950				
Mullite (5)	$3Al_2O_3 \cdot 2SiO_2$			3.23	1.644	1.646	1.636
	Al_2O_3 71.80%	1.38	0.193				
	SiO_2 28.20%	1.39	0.207				
	K_M	1.382	0.1969				
Rhodonite (3)	$MnO \cdot SiO_2$			3.72	1.739	1.740	1.738
	MnO 54.15%	1.44	0.191				
	SiO_2 45.85%	1.39	0.207				
	K_R	1.417	0.1983				
Pyrope (5)	$3MgO \cdot Al_2O_3 \cdot 3SiO_2$			3.51	1.705	1.704	1.707
	MgO 30.01%	1.46	0.200				
	Al_2O_3 25.29%	1.38	0.193				
	SiO_2 44.70%	1.39	0.207				
	K_P	1.409	0.2013				
Grossularite (5)	$3CaO \cdot Al_2O_3 \cdot 3SiO_2$			3.53	1.735	1.734	1.743
	CaO 37.35%	1.63	0.225				
	Al_2O_3 22.64%	1.38	0.193				
	SiO_2 40.01%	1.39	0.207				
	K_G	1.477	0.2106				
Anorthite (5)	$CaO \cdot Al_2O_3 \cdot 2SiO_2$			2.765	1.583	1.583	1.589
	CaO 20.16%	1.63	0.225				
	Al_2O_3 36.65%	1.38	0.214				
	SiO_2 43.19%	1.39	0.207				
	K_A	1.435	0.2065				
Iron barium silicate (11)	$FeO \cdot BaO \cdot 4SiO_2$			3.33	1.620	1.611	1.591
	FeO 15.44%	1.54	0.187				
	BaO 32.95%	0.909	0.127				
	SiO_2 51.61%	1.39	0.207				
	K	1.255	0.1776				
Thenardite (5)	Na_2SO_4			2.69	1.477	1.478	1.481
	Na_2O 43.64%	1.28	0.181				
	SO_3 56.36%	1.14	0.177				
	K_T	1.201	0.1788				
Lithium sulfate (11)	Li_2SO_4			2.23	1.465	1.471	1.475
	Li_2O 27.18%	2.20	0.310				
	SO_3 72.82%	1.14	0.177				
	K	1.428	0.2132				

TABLE 4—(*continued*)

Substance	Composition		k values employed		Specific gravity	Average index of refraction		
						Experimental	Calculated	
			$\alpha\beta\gamma/d$	$(n-1)/d$			$\alpha\beta\gamma/d$	$(n-1)/d$
Arcanite (7)	K_2SO_4							
	K_2O	54.05%	1.33	0.189	2.70	1.495	1.497	1.495
	SO_3	45.95%	1.14	0.177				
	K_A		1.243	0.1835				
Sodium nitrate	$NaNO_3$							
	Na_2O	36.46%	1.28	0.181	2.27	1.499	1.499	1.496
	N_2O_5	63.54%	1.60	0.240				
	K		1.483	0.2185				
Cesium nitrate (11)	$CsNO_3$							
	Cs_2O	72.29%	0.834	0.124	3.69	1.553	1.570	1.576
	N_2O_5	27.71%	1.60	0.240				
	K		1.046	0.1561				
Rubidium nitrate (11)	$RbNO_3$							
	Rb_2O	63.38%	0.885	0.129	3.12	1.518	1.529	1.529
	N_2O_5	36.62%	1.60	0.240				
	K		1.147	0.1697				
Borosilicate crown glass, Wright's no. 12 (11)	SiO_2	72%	1.39	0.207	2.37	1.4997	1.497	1.486
	B_2O_3	12%	1.70	0.220				
	Na_2O	11%	1.28	0.181				
	Al_2O_3	5%	1.38	0.193				
	K		1.415	0.2050				
Ordinary crown glass, Wright's no. 1 (11)	SiO_2	74.6%	1.39	0.207	2.50	1.5055	1.510	1.510
	As_2O_3	0.3%	1.57	0.214				
	Mn_2O_3	0.1%	1.44	0.300				
	CaO	5.0%	1.63	0.225				
	K_2O	11.0%	1.33	0.189				
	Na_2O	9.0%	1.28	0.181				
	K		1.386	0.2040				
Sodium potassium calcium glass (11)	SiO_2	61.02%	1.39	0.207	2.626	1.5528	1.555	1.545
	Na_2O	6.31%	1.28	0.181				
	K_2O	9.59%	1.33	0.189				
	CaO	22.78%	1.63	0.225				
	Al_2O_3 (plus Fe_2O_3)	0.30%	1.38	0.193				
	K		1.432	0.2077				
Barium crown glass, Wright's no. 45 (11)	SiO_2	31.0%	1.39	0.207	3.54	1.6098	1.619	1.600
	B_2O_3	12.0%	1.70	0.220				
	Al_2O_3	8.0%	1.38	0.193				
	As_2O_3	1.0%	1.57	0.214				
	BaO	48.0%	0.91	0.127				
	K		1.198	0.1691				
$MnO \cdot SiO_2$ glass (11)	MnO	54.15%	1.44	0.191	3.48	1.700	1.700	1.690
	SiO_2	45.85%	1.39	0.207				
	K		1.417	0.1983				

TABLE 4—(*continued*)

Substance	Composition		*k* values employed		Specific gravity	Average index of refraction		
						Experimental	Calculated	
			$\alpha\beta\gamma/d$	$(n-1)/d$			$\alpha\beta\gamma/d$	$(n-1)/d$
Flint glass, Wright's no. 53 (11)	SiO_2	59.3%	1.39	0.207	2.90	1.5537	1.554	1.537
	PbO	27.5%	1.08	0.137				
	K_2O	8.0%	1.33	0.189				
	Na_2O	5.0%	1.28	0.181				
	As_2O_3	0.2%	1.57	0.214				
	K		1.295	0.1850				
Beryllium glass, Lai and Silverman's no. B_3 (11)	SiO_2	73.40%	1.39	0.207	2.45	1.5193	1.506	1.501
	Na_2O	18.96%	1.28	0.181				
	BeO	7.64%	1.71	0.238				
	K		1.393	0.2044				
Gahnite Analysis no. 11 (6)	FeO	1.71%	1.42	0.187	4.57	1.818	1.819	1.820
	MnO	0.50%	1.44	0.191				
	ZnO	41.31%	1.16	0.153				
	Al_2O_3	53.28%	1.38	0.193				
	Fe_2O_3	2.51%	2.49	0.308				
	SiO_2	0.69%	1.39	0.207				
	K_G		1.318	0.1793				
Ferroan gahnite analysis no. 14 (6)	MgO	2.38%	1.46	0.200	4.38	1.782	1.793	1.789
	FeO	8.52%	1.42	0.187				
	MnO	0.10%	1.44	0.191				
	ZnO	31.32%	1.16	0.153				
	Al_2O_3	57.59%	1.38	0.193				
	Cr_2O_3	0.09%	2.09	0.270				
	K_G		1.317	0.1802				
Augite analysis no. 13 (4)	SiO_2	50.91%	1.39	0.207	3.394	1.703	1.714	1.713
	Al_2O_3	2.68%	1.38	0.193				
	Fe_2O_3	1.86%	2.49	0.308				
	FeO	10.05%	1.42	0.187				
	MgO	12.39%	1.46	0.200				
	CaO	20.55%	1.63	0.225				
	Na_2O	0.47%	1.28	0.181				
	K_2O	0.02%	1.33	1.189				
	$H_2O(+)$	0.19%	2.25	0.340				
	TiO_2	0.40%	4.18	0.397				
	MnO	0.48%	1.44	0.191				
	K_A		1.483	0.2100				
Hedenbergite analysis no. 18 (4)	SiO_2	48.42%	1.39	0.207	3.535	1.734	1.733	1.735
	Al_2O_3	0.30%	1.38	0.193				
	Fe_2O_3	1.50%	2.49	0.308				
	FeO	22.97%	1.42	0.187				
	MgO	1.06%	1.46	0.200				
	CaO	21.33%	1.63	0.225				
	Na_2O	0.14%	1.28	0.181				
	K_2O	0.03%	1.33	0.189				
	$H_2O(+)$	0.46%	2.25	0.340				
	TiO_2	0.08%	4.18	0.397				
	MnO	3.71%	1.44	0.191				
	K_H		1.473	0.2078				

The constants are calculated (1) from the oxides themselves or (2) from compounds containing the oxides.

Indices of refraction are calculated for 26 compositions (minerals, artificial compounds, and glasses) by means of (1) $\alpha\beta\gamma/d=k$ and (2)$(n-1)/d=k$. In the majority of these examples, the former expression yields indices closer to those experimentally obtained.

In some respects the new equation is more convenient than the others. It is not necessary to compute an average index of refraction when an anisotropic substance is involved. Furthermore, if two indices are known, the third can be calculated readily, provided that k and d are known.

Acknowledgments

Criticisms and suggestions by H. W. Jaffe, of the U. S. Geological Survey, enabled the writer to expand and improve the preliminary manuscript. Contributions were also made by F. M. Byers, G. J. Neuerburg, and G. I. Smith, all of the Survey. Assistance was rendered by George Tunell, of the University of California at Los Angeles.

References

1. Fletcher, L. (1892), The Optical Indicatrix and the Transmission of Light in Crystals, London.
2. Glasstone, S. (1946), Textbook of Physical Chemistry, 2d ed., p. 528–534, D. Van Nostrand Co., New York.
3. Handbook of Chemistry and Physics (1951), 33d ed., Chemical Rubber Publishing Co., Cleveland.
4. Hess, H. H. (1949), Chemical composition and optical properties of common clinopyroxenes, Part I: *Am. Mineral.*, **34,** 621–666.
5. Larsen, E. S., and Berman, H. (1934), The microscopic determination of the nonopaque minerals: *U. S. Geol. Survey, Bull.* **848.**
6. Palache, C., Berman, H., and Frondel, C. (1944), Dana's System of Mineralogy, 7th ed., v. 1, John Wiley & Sons, New York.
7. Palache, C., Berman, H., and Frondel, C. (1951), Dana's System of Mineralogy, 7th ed., v. 2, John Wiley & Sons, New York.
8. Pauling, L. (1948), The Nature of the Chemical Bond, 2d ed., pp. 247–250, Cornell Univ. Press, Ithaca.
9. Schlecht, W. G. (1944), Calculation of density from x-ray data: *Am. Mineral.*, **29,** 108–110.
10. Sun, K.-H., Safford, H. W., and Silverman, A. (1940), Review of the relation of density and refractive index to the composition of glass, II: *Am. Ceramic Soc. Jour.*, **23,** 343–354.
11. Winchell, A. N. (1931), Microscopic Characters of Artificial Minerals, 2d ed., John Wiley & Sons, New York.
12. Winchell, A. N. (1951), Elements of Optical Mineralogy, Part II. Descriptions of Minerals, 4th ed., John Wiley & Sons, New York.
13. Wyckoff, R. W. G. (1948), Crystal Structures, Sec. I, Interscience Publishers, New York.

Manuscript received March 25, 1955.