

The layout of the 60 degree splitter is similar to that of the 45 degree. The radius of the arcs at the corners being determined such that the chord C is equal to half the circumference of a circle of diameter D , the spacing of the chutes.

The design of the supports, collar and pans is optional. Our splitters are supported at each end with legs made from pure aluminum, fastened together with a single bolt passing under the crotch formed by the chutes. Collars are machined from one piece of aluminum to fit on top of the supported chute assembly. Pouring and receiving pans are also machined from one piece of aluminum to eliminate the need for joining metal. This work was performed under *AEC Contract AT(11-1)-208*.

VALIDITY OF "VEGARD'S LAW"

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Vegard and Dale (1928), in their work on binary solid solutions in the cubic system, state as an empirical rule a linear relation between unit cell edge length and composition. Since then, however, this "law" has become accepted by many as of general validity, its soundness being assumed even in non-cubic crystal systems (Chave, 1952, p. 192; Hess, 1952, p. 182). It is taken by some to hold for all ideal solid solutions; and Wasastjerna (1951, p. 2) has employed it as basis for his theory on the heat of formation of binary solid solutions.

In view of this wide publicity, it would be desirable to examine the basis of this "law" theoretically. Consider a binary solid solution in which the volume effects of the two end-members are additive, i.e., the solution shows no volume change upon mixing. Let V_1 and V_2 denote the molar volumes of the two end-members, and N the mole fraction of species 1. We then have for the volume of the solid solution,

$$\begin{aligned} V &= NV_1 + (1 - N)V_2 \\ &= N(V_1 - V_2) + V_2. \end{aligned} \quad (1)$$

In the cubic system and in terms of the unit cell edges R_1 and R_2 , equation (1) becomes

$$R^3 = R_2^3 \left[1 - \frac{R_2^3 - R_1^3}{R_2^3} N \right] \quad (2)$$

and

$$R = R_2 \left\{ 1 - \left[1 - \left(\frac{R_1}{R_2} \right)^3 \right] N \right\}^{1/3}. \quad (3)$$

Now suppose we write "Vegard's Law" in the form

$$R = R_2(1 + \beta N) \quad (4)$$

and compare with equation (3), we see that they are not equal and cannot be made equal by any rearrangement of the factors. The exact equation (3), in fact, is proportional to the cube root of the mole fraction, whereas "Vegard's Law," equation (4), is proportional to the mole fraction itself.

This problem can be formulated in a slightly different way. Writing $\alpha = (R_1/R_2)^3 - 1$ in equation (3) and expanding the cube root in a series, we get

$$R = R_2(1 + \frac{1}{3}\alpha N - \frac{1}{3}\alpha^2 N^2 + \dots) \quad (5)$$

Comparing with (4), we see that $\beta = \frac{1}{3}\alpha$, and the two expressions can be reconciled only when terms higher than the first power in α are negligible, i.e., if

$$\left(\frac{V_1}{V_2} - 1\right) \ll 1,$$

which means that the pure end-members do not have very different molar volumes. This fact is intuitively obvious, but equation (5) expresses it analytically and gives a precise relation between allowable difference in end-member volumes and any desired degree of approximation to linearity for the solution (when the other conditions are also satisfied).

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THE CURVED-CRYSTAL X-RAY SPECTROMETER, A MINERALOGICAL TOOL*

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Birks and Brooks (1955) have recently described the application of a curved-crystal, reflection-type, focusing x-ray spectrometer, using fluorescent x-rays, to the determination of microgram quantities of a number of elements. They have reported that the intensities, resolution and line-background ratios for small samples of the order of 1 mg. are comparable to those obtained from a 10-g. sample on a commercial flat-crystal spectrometer.

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