After this article was in press, Dr. Fred A. Hildebrand of the U. S. Geological Survey, informed the authors that he found eudialyte in tinguaite dikes, probably of Cretaceous age, which have intruded the northwest edge of a 400 square mile province of phacolithic syenite intrusives in central Arkansas.

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A STEREOGRAPHIC CONSTRUCTION FOR DETERMINING OPTIC AXIAL ANGLES

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The derivation of the optical axial angle from the three chief refractive indices is a task often carried out graphically with the aid of nomograms. Of these several have been described in the course of time. A good example is the diagram constructed by H. Waldmann (1945) who also quotes and discusses the work of previous authors. A more recent paper on the same subject is that by C. P. Gravenor (1951). The purpose of the nomogram in all these cases is to supply a solution to a problem which in essence can be stated as follows: Given an ellipse, the main axes of which are proportional in length to $n_x(n_\alpha)$ and $n_z(n_\gamma)$ of a given optically biaxial crystal; required the position within the ellipse of the two radius vectors having a length proportional to $n_y(n_\beta)$. These radius-vectors of the ellipse are also radii of one of the circular sections through the optical indicatrix and hence are perpendicular to one of the optical axes. The position of the latter within the ellipse and the angle between them therefore follows from that of the radius vectors n_y.

The construction of ellipses is not usually thought of as one that can easily be based on a stereographic projection. But this is actually the case as the following considerations show. An ellipse having the major axis a and the minor axis b can be thought of as the intersection of a right circular cylinder of radius b with a plane inclined at a certain angle to the horizontal. If $ABCD$ in Fig. 1 be four points on a random axial section through such a cylinder and PQ be the trace on this section of the plane containing the ellipse, then OP is a radius-vector of the ellipse and has the length $b/\cos\theta = b \sec\theta$ in which b, as stated, is the radius of the cylinder and θ the angle between the trace and the horizontal. When the axial section is perpendicular to the plane of the ellipse, θ then equals the inclination angle of the plane and assumes the greatest possible value θ_{max} such that $b \sec\theta_{max} = a$. The connection between the axial ratio of the ellipse and the inclination of the plane is thus very simply given by $\sec\theta_{max} = a/b$. In a stereogram the axis of the cylinder would emerge in Z, the center of the diagram, while the plane containing the ellipse would be a great circle such as GHK in Fig. 2. The axial section shown in Fig. 1

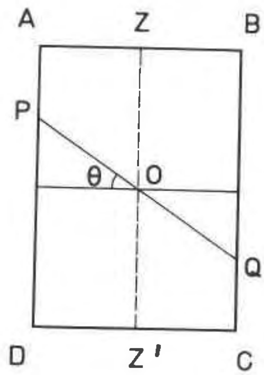

FIG. 1

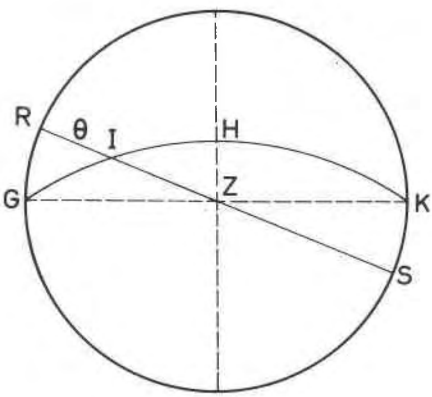

FIG. 2

would appear as a diameter RS of the stereogram while $\angle\theta$ is the arc RI between the trace which emerges at I and R on the horizon. To construct the ellipse one has merely to draw diameters of the stereogram at regular angular intervals* and then to measure the θ-angle on each of these. If the values obtained be called $\theta_1, \theta_2, \cdots, \theta_n$, one has merely to plot the values of $\sec\theta_1, \sec\theta_2, \cdots, \sec\theta_n$ along the radii of a circle and at angular intervals equal to the spacing between the I-points in the stereogram. The ellipse is then obtained by joining the plotted points and can, of course, be given any size by suitably choosing the scale for plotting the secants.

* "Regular angular intervals" may be interpreted as meaning equal angles between consecutive sections through the cylinder. In this case the angles must be measured at Z, or on the ground circle of the projection. The angles between consecutive traces in the plane of the ellipse (angles between consecutive I-points on the great circle) are unequal under these circumstances. It is, however, preferable to arrange for equal spacing between the traces and this can easily be done by so drawing the diameters that the angles between consecutive I-points are equal.

The application of this construction to the optical problem entails the following steps:

(i) Calculate the quotients n_z/n_x and n_y/n_x. This must be done because the ellipse to be constructed has the axial ratio $n_z:n_x=n_z/n_x:1$. On the same scale the radius-vector whose location is sought for has the length n_y/n_x.

(ii) Draw a great circle *GHK* in the stereogram such that *H* is at a distance $\theta_z=\text{arc sec } n_z/n_x$ from the ground circle or $90°-\theta_z=\text{arc cosec } n_z/n_x$ from the center *Z* of the projection.

(iii *a*) Draw a small circle about *Z* having an angular radius of $90°-\theta_y=\text{arc cosec } n_y/n_x$ (Fig. 3). Then the points at which it intersects the great circle are *I*-points on the diameters of the projection correspond-

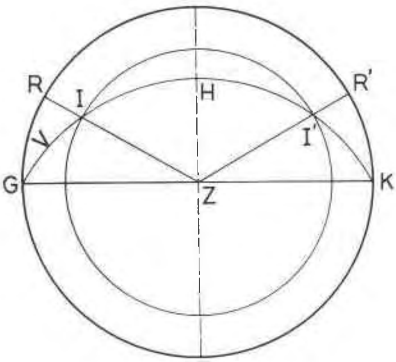

Fig. 3

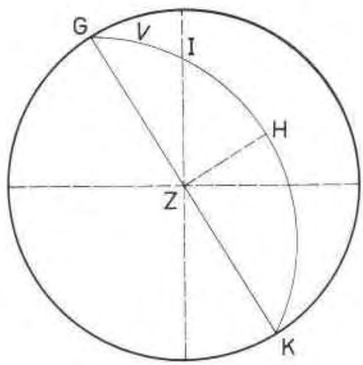

Fig. 4

ing to the radius-vectors n_y/n_x. It follows, therefore, that the arc *GI* is the angle between such a radius-vector and the minor axis of the ellipse. The value obtained by reading this angle on the net equals that between the major axis of the ellipse and the optical axis. Hence $GI=V$ and furnishes the required solution of the problem.

(iii *b*) In practice the construction of the small circle can be avoided by merely rotating the stereogram over the net until the great circle *GHK* intersects one of the diameters of the net at a distance $\theta_y=\text{arc sec } n_y/n_x$ from the ground circle (Fig. 4); This point of intersection is identical with *I* as obtained under (iii *a*) and again the angle *GI* equals *V*.

(iv) Determine the optical sign as follows: If the arc *GI* is less than 45° the corresponding optical axis will lie at the same distance from *H* the point of emergence of n_z. The sign will, therefore, be positive. If, on the contrary, the angle *GI* is greater than 45° the optical axis will form an acute angle with n_x and the optical sign will be negative.

If we wish to judge the usefulness of this or any other graphical method of determining the optical axial angle, account must be taken of the general convenience of the technique involved, of the accuracy obtained and of the time saved as compared with that required for a numerical computation with the basic formula. With regard to the first of these points the present method, though sharing with that of H. Waldmann (1945) the rather elaborate preamble of calculating the ratios of the refractive indices has the great advantage of requiring no pre-constructed diagram beyond an ordinary stereographic net. The actual graphical construction is a matter of a few moments and under favorable circumstances produces results of a high degree of accuracy. The condition for obtaining good results is mainly that double refraction must be high. The two ratios are then markedly different and in consequence the great and small circles intersect at a comparatively steep angle which allows the common point to be located with precision. When double refraction is low and the two ratios very similar, a blurred intersection of the two circles results and leads to much uncertainty as to the true position of the point in common.

The stereographic construction belongs to the type offering a graphic solution of the equation

$$\tan^2 V = \frac{n_z^2(n_y^2 - n_x^2)}{n_x^2(n_z^2 - n_y^2)}. \tag{1}$$

As this expression is awkward to calculate and ill adapted to logarithmic treatment, it may be said that the graphic solution offers a real saving of time over the numerical computation.* It is, however, an interesting fact that the stereographic construction points the way to a great simplification of the calculation. Thus in Fig. 4 the points IZH are the corners of a right-angled spherical triangle in which the sides $IZ=90-\theta_y$ and $ZH=90-\theta_z$ are known and $HI=90-V$ remains to be calculated. From formulae we have

$$\cos IZ = \cos HI \cdot \cos ZH$$

* Compare this with the fact that Mallard's approximate formula

$$\tan^2 V = \frac{n_y - n_x}{n_z - n_y}$$

strictly speaking requires no graphical treatment at all. For the expression to be worked out namely

$$\text{arc tan} \sqrt{\frac{n_y - n_x}{n_z - n_y}}$$

may be read from a single setting of an ordinary slide rule when the numerator and denominator of the fraction have been measured or calculated from the refractive indices.

and therefore

$$\cos HI = \frac{\cos IZ}{\cos ZH}.$$

This leads to

$$\sin V = \frac{\sin \theta_y}{\sin \theta_z}$$

which can be expanded to

$$\sin V = \frac{\sin\left(\operatorname{arc\,sec}\frac{n_y}{n_x}\right)}{\sin\left(\operatorname{arc\,sec}\frac{n_z}{n_x}\right)}. \tag{2}$$

This equation can be transformed into the following which is better adapted for use with most logarithm tables:

$$\sin V = \frac{\sin\left(\operatorname{arc\,cos}\frac{n_x}{n_y}\right)}{\sin\left(\operatorname{arc\,cos}\frac{n_x}{n_z}\right)}. \tag{3}$$

This expression is most convenient for logarithmic calculation and can be worked out so quickly when three refractive indices or two refractive indices and V are given, that it seems questionable whether much is gained by the use of this or indeed any other graphic method. The calculation in the case of sulfur (to which also correspond the angles used in Figs. 3 and 4) is as follows: (The values of the refractive indices are quoted after Dana's System of Mineralogy, 1944, p. 142)

	n	*log*	Δ=*log cos*	*log sin*	Δ	*arc sin*=*V*
x	1.9579	029179				
y	2.0377	030914	998265	944253		
z	2.2452	035126	994053	968965	975288	34°28½′

$$2\,V = +68°57'$$

Like equation 1 the new ones provide accurate and not merely approximate values of V. It is also possible to deduce equations 2 and 3 from 1 by suitable transformation of the classic formula without having recourse to the stereographic projection at all. The latter, however, very considerably aids in visualizing the change of approach.

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