immersion liquid and convenience in changing liquids may be improved by decreasing the area of the plate, and for an extended series of index measurements, the slotted plate may be cemented in a centered position on the water cell. For measurements on universal stages without water cell, the slotted plate may be cemented to a glass slide to provide the proper total thickness of the mount between hemispheres.

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THE SYMMETRY OF THE COMPLETE TWIN

H. CURIEN AND J. D. H. DONNAY*
Laboratoire de Minéralogie et Cristallographie, Faculté des Sciences, Paris, France

In a recent paper (Curien & Le Corre, 1958, to be referred to as CLC) it was shown how the symbolism of the (black-white) Shubnikov groups can be used to designate the various twin laws, in the case of twinning by merohedry or by reticular merohedry.

Every such *twin law* gives the geometrical relationship between two crystals, a "black" one and a "white" one. It may usually be expressed by any one of several possible *twin operations*, which (as is well known) are all the symmetry operations that are deficient in the merohedral crystal symmetry but are present in the next-higher merohedry. If the crystal symmetry is a hemihedry, say $4/m$, only one twin law is possible: the next-higher merohedry, in this case, is the holohedry $4/m\ 2/m\ 2/m$, and the twin operation may be chosen at will from four 180° rotations or four reflections; the corresponding *twin elements* are primed in the *twin*

* Fulbright Lecturer 1958–59. Permanent address: The Johns Hopkins University, Baltimore 18, Md.

symbol $4/m\ 2'/m'\ 2'/m'$. Another hemihedry, say 4 2 2, gives another twin law, with the symbol $4/m'\ 2/m'\ 2/m'$ (as written by CLC); the *twin center* need not be explicitly denoted by $\bar{1}'$, to stress the fact that inversion too is a possible twin operation, because $2/m'$ implies $\bar{1}'$. In the case of a tetartohedry, there will be as many possible twin laws as there are hemihedries that have the given tetartohedry as a subgroup. Example: the polar tetartohedry 4 leads to three twin laws: $4/m'$, $4\ 2'\ 2'$, and $4\ m'\ m'$. Finally the ogdohedry 3 gives seven twin laws (listed by CLC, *loc. cit.*, Table I).

The concept of *twin symmetry* or *symmetry of the complete twin*, recently revived by M. J. Buerger (1954), was already discussed by G. Friedel (1904; 1926). In twinning by merohedry or by reticular merohedry, the *complete twin* is defined in each case as the edifice that comprises, in addition to an original crystal, as many twinned crystals as there are possible twin laws. In the case of low-temperature quartz, of symmetry 3 2, the four individual crystals[1] are shown by CLC (*loc. cit.*, Fig. 1),[2] with an indication of the twin law that holds foi any two crystals: $6'\ 2\ 2'$ for I–II and III–IV, $\bar{6}'\ 2\ m'$ for I–III and II–IV, $\bar{3}'\ 2/m'$ for I–IV and II–III. The notation of the black-white groups also brings out the fact that the product of any two of these three twin laws is equal to the third one.

By an immediate generalization the notation can be made to express the symmetry of the complete twin. Let each of the four crystals I to IV have its own color; let the twin operations that bring I to coincidence with II, III, and IV be designated, respectively, by the signs prime, double prime, and triple prime attached to the corresponding twin elements. The twin symmetry receives the symbol $6'/m''\ 2/m'''\ 2'/m''$. The crystal-symmetry elements are unprimed; the twin axes of the Dauphiné law are primed ($6'$ implies $2'$ and 3); the twin planes of the Brazil law are double-primed; the product of the two laws is shown by triple primes attached to the twin center (both $2/m'''$ and $2'/m''$ imply $\bar{1}'''$) and twin planes (mirrors of the first kind of the lattice).

It may also be remarked that, in the most complicated case, that of twining by ogdohedry, the notation can be simplified. Only one kind of prime sign is needed ($6'/m'\ 2'/m'\ 2'/m'\ \bar{1}'$) as every primed element stands for a different twin law. Note that, in this case, it is advisable to symbolize the twin center explictly, so that all seven twin laws are shown in the symbol.

Let us now consider a species that can twin both by merohedry and by reticular merohedry; for instance, dolomite, of symmetry $\bar{3}$ (CLC, p.

[1] Often called *individuals* for short.

[2] Note that, in order to comply with the convention current in U.S.A., Fig. 1 of CLC should be rotated in its own place 30° counter-clockwise.

128–9; Fig. 2).[3] Only one twin law exists in twinning by merohedry; it is correctly written $\bar{3}\ 2'/m'$, a symbol that expresses both the twin law and the symmetry of the twin. The two additional twin laws that are made possible by reticular merohedry, $\bar{3}\ 1\ 2''/m''$ and $6'''/m'''$, will lead to the complete twin, comprising four crystals in all, the symmetry of which is written $6'''/m'''\ 2'/m'\ 2''/m''$, in which $6'''$ implies $2'''$ so that $6'''/m'''$ implies $\bar{3}$. We note that this symbol would apply equally well if the crystal had an hexagonal lattice and all twin laws were due to merohedry.

How does the generalized notation work in the case of cubic crystals? Consider a twin by hemihedry, with crystal symmetry 4 3 2. The twin law is denoted by $4/m'\ \bar{3}'\ 2/m'$ (CLC, Table I), a symbol which also expresses the symmetry of the twin. A crystal that belongs to the tetartohedry, 2 3, can have three twin laws: $2/m'\ \bar{3}'$, $4''\ 3\ 2''$, $\bar{4}'''\ 3\ m'''$ (symbolized by CLC); the complete twin comprises four crystals. The task of symbolizing the complete twin symmetry is here complicated by the fact that we are trying to represent a *symmetry operation* by a symbol that denotes either a set of equivalent symmetry elements or at least one symmetry element. (A *symmetry element* is a cyclic group, standing for all the powers of the operation.) The *twin operation* in the twin law $4''\ 3\ 2''$, is a 180° rotation about any one of the six 2-fold axes of the lattice or a ±90° rotation about any one of its three 4-fold axes. CLC have circumvented the difficulty for one twin law at a time by pointing out that $4''$ implies 2 ($4''\supset 2$). To be more explicit we would have to write the symbol as $4''[\supset 2]3\ 2''$. Likewise $\bar{4}'''\ 3\ m'''$, or $\bar{4}'''[\supset 2]3\ m'''$, should serve as a reminder that the twin operation is a reflection in any one of the six mirrors of the lattice or a ±90° rotatory-inversion about any one of its three $\bar{4}$-axes. Again the CLC symbol $2/m'\ \bar{3}'$ immediately indicates that the twin operation is the inversion through the symmetry center of the lattice or a reflection in any one of its three mirrors; although not so obvious from the symbol, other possible twin operations are ±120° rotatory-inversions about the four $\bar{3}$-axes of the lattice.

The symbol for the symmetry of the complete twin, in its expanded form,

$$\frac{4''}{m'}\supset[\bar{4}'''][\supset 2]3\,\frac{2''}{m'''}\,\bar{1}',$$

will look somewhat cumbersome. The abbreviated symbol, $m'\ 3\ m'''$, which implies $2''$, is probably more convenient; each twin law is defined by an operation of order 2, namely a reflection or a 180° rotation.

[3] To comply with current American convention, Fig. 2 of CLC should be rotated 180° in its own plane, so that a face of the rhombohedron will slope toward the observer.

From the group-theory viewpoint,[4] a twin is a *representation* of an abstract *factor group* (C_2, $S_{2\times2}$, $S_{2\times2\times2}$ in twinning by hemihedry, tetartohedry, ogdohedry, respectively). In the above example the 48 symmetry operations of the holohedral point group *m* 3 *m* are divided into one subgroup and three cosets; the operations of the *subgroup* describe the crystal symmetry 2 3, those in each *coset* are all the possible twin operations for one of the three twin laws. The factor group is a four-color group that differs in structure from the four-color space groups described by Belov and Tarkhova (1956). In our example all the elements of the factor group that correspond to changes of color are of order 2 ("element" being given its group-theory meaning, corresponding to "operation" in crystallographic groups); in the four-color space groups studied by Belov and Tarkhova, the factor group is the cyclic group C_4.

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[4] To readers unfamiliar with the subject of groups, we take pleasure in recommending Bryan Higman's "Applied group-theoretic and matrix methods" (Oxford, 1955). We have found this book illuminating, although we must say that not all its statements about crystals can be taken at face value. Mineralogists and crystallographers will not agree that the study of the external forms of crystals is "metaphorically, as well as literally, superficial" (p. 90); they will not be easily persuaded that the five trigonal point groups are possible only with a rhombohedral lattice (p. 110), nor will they see why the space groups of $\bar{4}$ 2 *m* are distributed among *four* lattice modes (p. 132) when only *two* are possible (p. 121)!

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NOTE ON "REVOREDITE" AND RELATED LEAD-SULFUR-ARSENIC GLASSES*

Charles Milton and Blanche Ingram
U. S. Geological Survey, Washington 25, D. C.

Recently Amstutz, Ramdohr, and de las Casas (1957) have described as a new mineral, "revoredite," from Cerro de Pasco, Peru, which is

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