

GRAPHS FOR THE ELIMINATION OF THE HARTMANN NET IN THE DETERMINATION OF REFRACTIVE INDICES IN HIGH DISPERSION MEDIA

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ABSTRACT

Direct reading charts are presented which eliminate graphical solutions (Hartmann Nets) required for index of refraction determinations in high dispersion media. Geometric proofs and construction procedures are explained. A special chart adapted to high dispersion liquids based on Tsuboi's classic work determines the composition of plagioclase feldspars from a single determination of N_x' .

INTRODUCTION

The determination of indices of refraction in high dispersion liquids requires a certain amount of graphing to obtain a solution to the problem. A single chart can be made which gives directly a simple solution to the problem.

A discussion of the entire method of determination of indices using high dispersion liquids is beyond the scope of this paper and the reader who is unfamiliar with the techniques is referred to a discussion by Emmons and others (1928, 1948, 1953) which covers the subject in detail.

Essentially, a properly oriented mineral is immersed in a liquid having an index for sodium light near that of the mineral in question. Since the change of index with respect to the change in wavelength is much larger in the liquid than in the mineral, it is possible to find some wavelength of light where the indices of mineral and liquid are the same. This will be subsequently referred to as the "wave length of agreement," (λ^*).

If this process is repeated with a second liquid, thus finding another wave length of agreement, the index of the mineral has been established for two different wave lengths. If the indices of the liquids are plotted on a Hartmann Net as a function of wave length, the wave lengths of agreement can be used to determine the sodium light index as shown diagrammatically in Fig. 1.

The values $N_D(\text{mineral})-N_D(\text{liquid No. 1})$ have been derived directly by computation, and are expressed in a chart for all possible values of the two wave lengths of agreement (Fig. 2). Two considerations must be satisfied before the chart is valid. (1) The lines representing the dispersion of liquids on the Hartmann Net must be essentially parallel. (2) The value read is a function of the difference in index for sodium light. Although the chart can be constructed to yield valid answers for any difference in index, this difference must be the same for all adjacent

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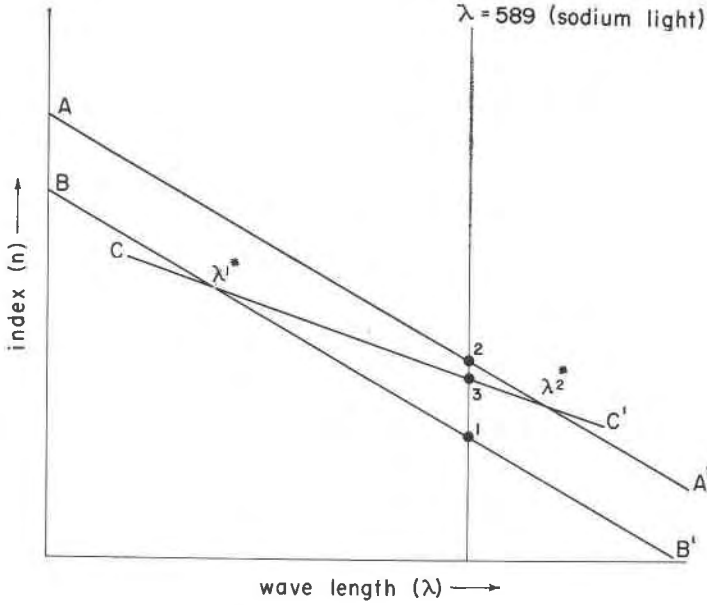


FIG. 1. Graphical solution for N_D of a mineral, using high dispersion liquids. AA' = plot of upper liquid dispersion; BB' lower liquid. λ_1^* and λ_2^* are wave lengths of agreement (1) and (2) respectively, where (1), (2) and (3) are N_D of lower liquid, upper liquid and mineral, respectively. N_D of mineral is determined by drawing a straight line from λ_1^* to λ_2^* .

liquids or the chart loses much of its usefulness. Standard immersion media are uniformly spaced, usually with intervals of 0.005, e.g., 1.500, 1.505,

USE OF THE DIAGRAM

No changes in microscope procedure are necessary for the use of the diagram. Two wave lengths of agreement are found using two adjacent liquids. The index for sodium light is known for both liquids. If the wave length of agreement for the liquid with the lower index of refraction is 510 $m\mu$. and that for the upper liquid is 610 $m\mu$., one reads up from 510 $m\mu$. on the abscissa and across from 610 $m\mu$. on the ordinate to the intersection of these two lines (Y, Fig. 2). This intersection falls between the correction lines 0.004 and 0.005 and interpolation gives in this case about 0.0042. Adding this last value to the sodium line index of the lower liquid, one obtains the correct sodium line index of the mineral.

For example, if the lower liquid has an index of 1.500 and the wave lengths of agreement are as above, then the index of the mineral for sodium light is 1.5042. If another mineral is tested and it is found to have wave lengths of agreement of 490 $m\mu$. and 570 $m\mu$. respectively (X,

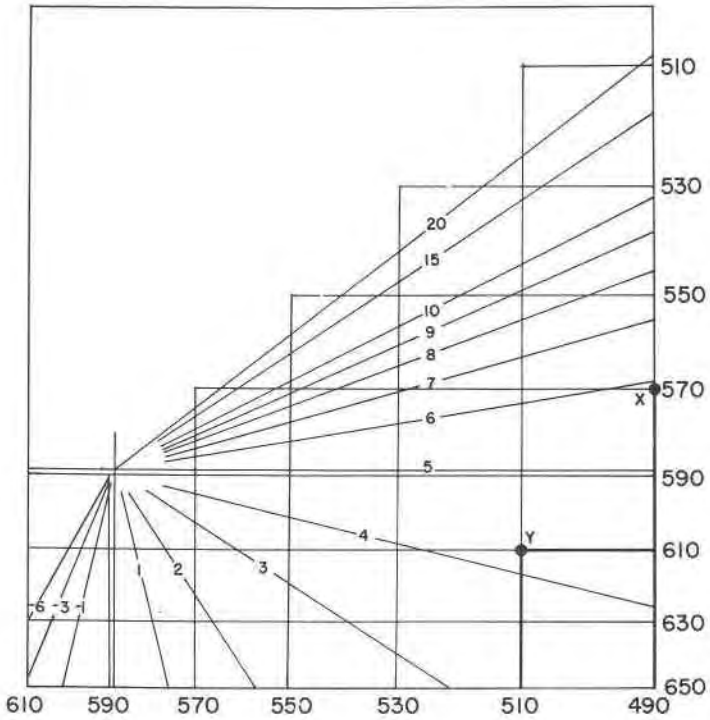


Fig. 2. N_D correction chart. Points X and Y refer to values used in examples. Correction $\times 10^3$. Thus number 4 represents a correction of 0.004.

Fig. 2), the liquids remaining the same, the correction is approximately 0.006 and the mineral has an index of 1.506 for sodium light. Figure 3 shows the same results as obtained on a Hartmann Net. It must be remembered that the examples are based on differences in index of 0.005 for the liquids. A second chart is shown in Fig. 4. This chart is read the same way as the first, except that the correction when subtracted from the mean dispersion of the two liquids gives the dispersion of the mineral, i.e., $N_F - N_C$. If the dispersion slopes of the two liquids are very nearly the same, either can be used with little error. The same conditions apply to this chart as applied to the first. (Fig. 4 as shown is not as accurate as Fig. 2; see construction below.) For example, if the dispersion of the lower liquid is 0.020 and that of the upper liquid is very nearly the same, and if the wave lengths of agreement are 510 $\mu\mu$. and 610 $\mu\mu$., then the correction is about 0.0084 and the dispersion of the mineral is equal to $(N_F - N_C) \text{ liquid} - (\text{Correction}) = 0.020 - 0.0084 = 0.0116$. This relation can be verified in Fig. 3.

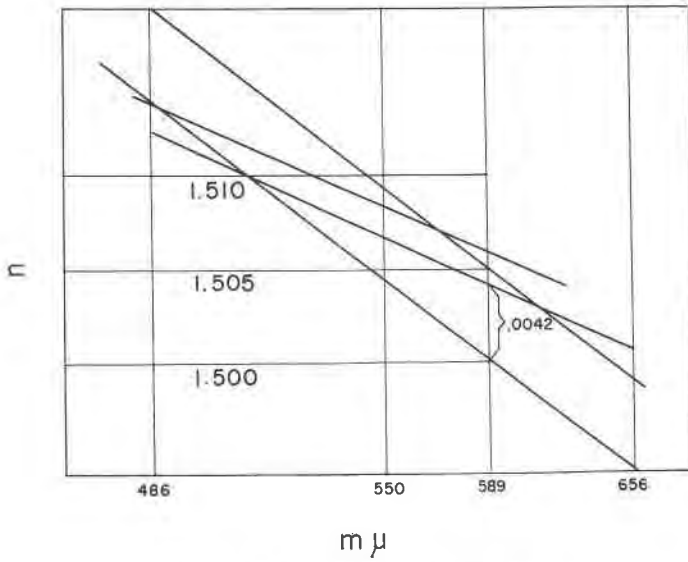


FIG. 3. Graphical solution of examples.

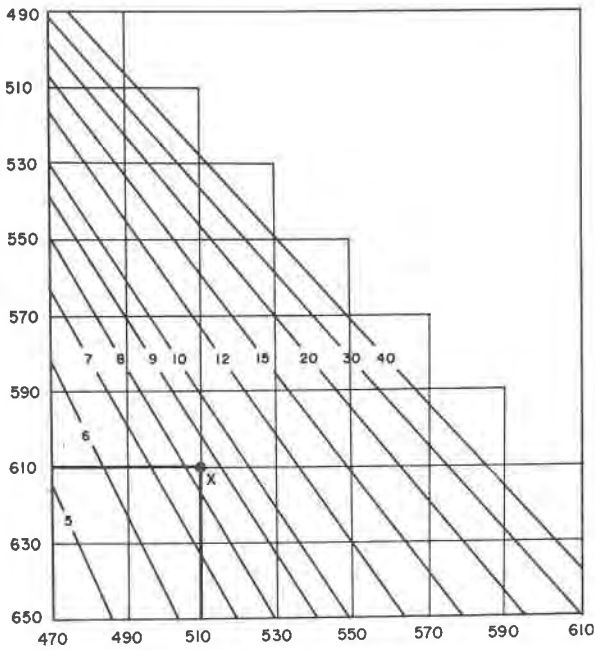


FIG. 4. Dispersion correction chart. Corrections $\times 10^3$. The point X refers to example.

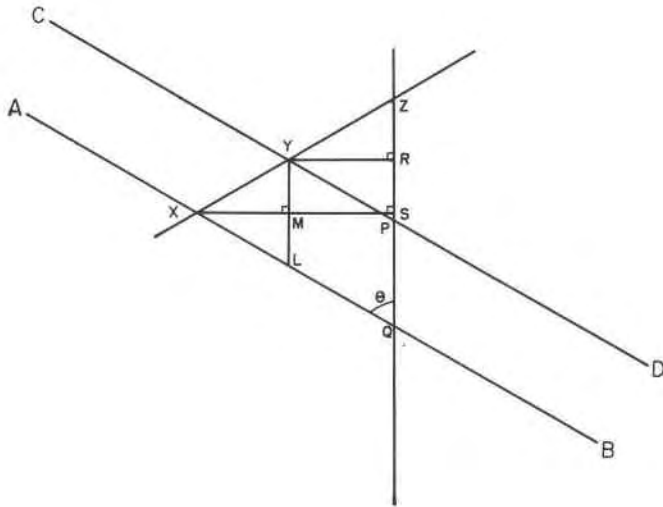


FIG. 5. Geometric proof for Fig. 2. Given: $AB \parallel CD$; PQ equals some constant. Then: $ZQ = ZP + PQ$. (2) $PQ = YL$. (3) $YL/MX = ZQ/XS$. (4) $ZQ = (YL)(XS)/MX = (PQ)(XS)/(XS - YR)$. Thus, for fixed values of XS , YR and PQ , ZQ is independent of the slope angle $90^\circ - \theta$.

CONSTRUCTION AND PROOFS

N_D Correction Chart

The geometric solution is given by the equation:

$$ZQ = (XS)(PQ)/XS - YR$$

Fig. 5

The similarity between the geometric construction and Fig. 1 is readily evident. PQ equals $N_D(2) - N_D(1)$ (0.005 in the following calculations). XS and YR are equivalent to the linear distances on the Hartmann Net between λ_1^* and λ_2^* respectively. ZQ is the correction factor $N_D(M) - N_D(L)$. The relationship between XS , YR and λ_1^* , λ_2^* is given by Cauchy's equation. The numerical values shown in Table 1 will con-

TABLE 1. CONVERSION TABLE FOR DETERMINING WAVE LENGTHS EQUIVALENT TO LINEAR DISTANCES IN GEOMETRIC FORMULAE, MEASURED FROM N_D (589 $m\mu$.)

$m\mu$	656	650	640	630	620	610	600	590	589
cm	-5.00	-4.66	-4.04	-3.36	-2.60	-1.79	-0.95	-0.08	0.00
$m\mu$	580	570	560	550	540	530	520	510	
cm	0.86	1.83	2.87	3.95	5.11	6.32	7.60	8.96	
$m\mu$	500	490	486	480	470				
cm	10.41	11.95	12.93	13.59	15.32				

struct a chart approximately 20 cm.×20 cm., and may be scaled to construct a chart of any size. Solving the equation for various values of ZQ (1, 2, 3, etc.) and plotting gives Fig. 2.

Dispersion Correction Chart

The equation for the solution of the dispersion correction is:

$$AB = FD - (CE)(DC)/(YP) \tag{Fig. 6}$$

In Table 1 DC equals 18 cm., and values for YP may be obtained by subtraction. For example, YP for 0.510 mμ and 0.610 mμ is 8.96 - (-1.79) = 10.75 cm. Geometric construction is the same as that of the first chart except that the wave lengths need not be scaled to the Hartmann Net, and may be plotted linearly. Actual plotting will give curved lines for correction factor, but these may be drawn as straight lines (Fig. 4) with a small loss in accuracy.

ERRORS

Although the error involved in one determination is a complex function, experience has shown that the accuracy of the chart is well within the experimental error of the method when using white light and the colored Becke line. It is generally less than 0.0004 for Fig. 2 and less than 0.001 for Fig. 4.

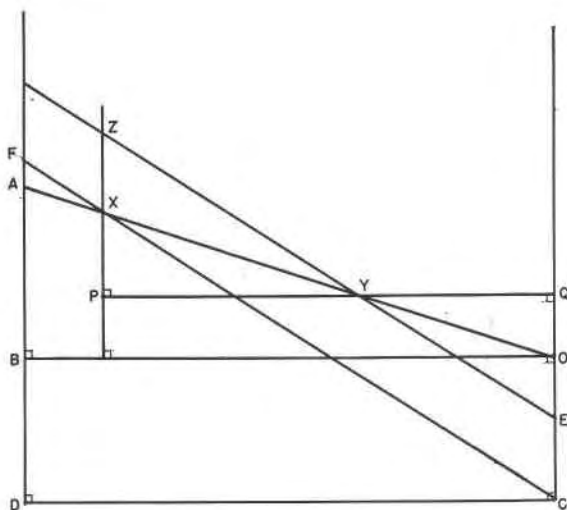


FIG. 6. Geometric proof for Fig. 4. Given: ZE||FC (Note FD is equivalent to (N_F-N_C) of liquids; AB is equivalent to (N_F-N_C) of mineral) Points X and Y represent wave lengths of agreement. (1) ZP=XP+EC. (2) ZP/PY=FD/DC. (3) ZP=(PY) (FD)/DC. (4) XP=(PY) (FD)/(DC)-(EC). (5). XP/YP=AB/DC. (6) AB=(XP) (DC)/YP. (7) AB=FD-(CE) (DC)/YP by substitution for (XP).

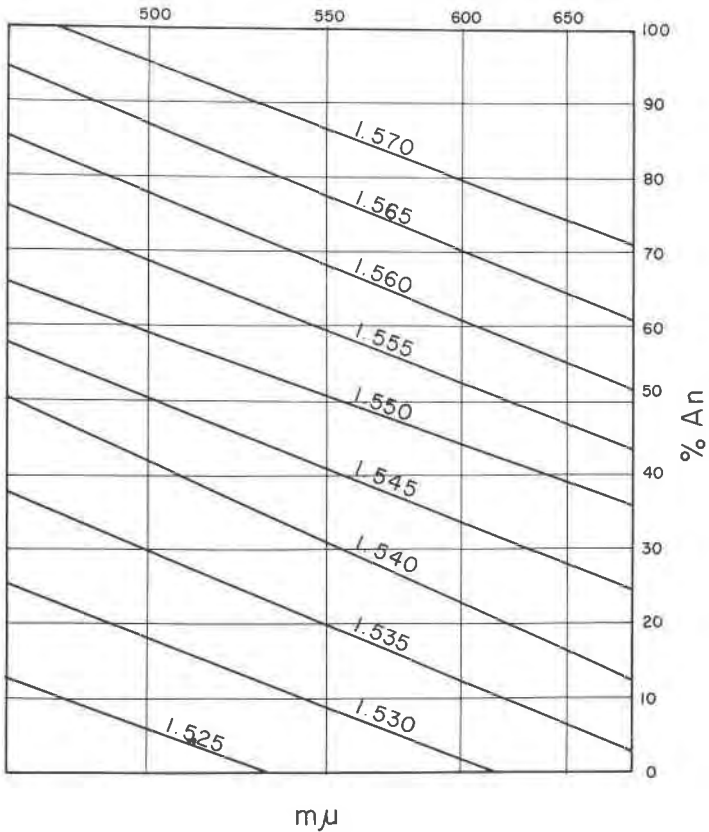


FIG. 7. Plagioclase composition diagrams. The slanting lines represent various possible liquids.

PLAGIOCLASE DIAGRAM

Tsuboi's classic study of the relation of index to composition in feldspars is well known (Tsuboi, 1923, 1934). However, Tsuboi's original diagram was a rather complicated affair involving several steps. A simplification adapted to high dispersion liquids is shown in Fig. 7. After determining the wave length of agreement for a particular liquid, one reads up to the appropriate liquid line and across to the composition. Figure 7 is for N_x' only, but others (N_x , N_z') are just as simply constructed.

CONSTRUCTION

Using Tsuboi's original chart, determine the composition for as many wave lengths per liquid as deemed necessary. Using composition and

wave length of agreement as abscissa and ordinate as desired, plot the points for each liquid on a new graph and connect them with smooth curved lines (the charts shown were plotted using only two points per liquid and have a probable calculated error in composition of less than 2%). It will be noted that the wave lengths are scaled as in a Hartmann Net. This is necessary only if one wishes to have the liquid lines plot as nearly straight lines. Figure 7 was based on University of Wisconsin liquids.

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