EXTINCTION AND 2V: A SIMPLE STEREOSCOPIC SOLUTION OF GENERAL APPLICATION

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Abstract

A new stereographic method of 2V determination, usable with extinction data obtained via any type of rotation apparatus, is presented. It is based on very simple geometrical considerations and, provided the orientations of the three principal axes of the indicatrix, however obtained, are known, employs a minimum of extinction data—the extinction angle associated with a single carefully selected general wave-normal. The sensitivity of this single extinction measurement, in terms of 2V determination by this method, varies considerably, and depends on both the orientation of the wave-normal and the value of 2V. This is illustrated by two contoured stereograms.

Introduction

There have recently been presented several purely graphical methods for the determination of 2V on the basis of varying numbers of extinction measurements plotted in stereographic projection (Wilcox, 1960; Tocher, 1962, 1964a; Joel, 1964). These are all connected, explicitly or implicitly, with coplanar extinction curves (Joel and Tocher, 1964) in so far as all the wave-normals involved are coplanar and at right angles to a single axis, Po, about which the crystal is rotated. Moreover, in order that the necessary great circles containing either the wave-normals (Tocher, 1962, 1964a; Joel, 1964) or the axis of rotation (Wilcox, 1960, 1961) may be constructed or utilised with the least manipulation of the net, plotting has been done with Po plotted in the center or on the primitive circle respectively. Most of these methods are, of course, adaptable for use with conical extinction curves (Joel and Tocher, 1964), but the siting of the associated wave-normals in general positions in the stereogram, even when Po is plotted centrally or on the primitive circle, makes the graphical constructions tedious and time-consuming, particularly with the methods of Tocher (1962, 1964a) and Joel (1964).

In addition to the above graphical methods, others involving coplanar and/or conical extinction curves and no curves (Joel, 1963) have been more recently advanced (Tocher, 1964b; Villarroel and Joel, 1965): these make deliberate use of the special case where the wave-normal locus contains an optic axis but, in each case, the number of experimental observations required after location of the optic axial plane may be considerable.

The graphical method now presented, although based on more general considerations, has the advantage of fairly rapid application: after the indicatrix axes and the optic axial plane have been located, the comparatively short graphical construction required is based on only one
extinction measurement and can be applied with equal facility for general wave-normals in all parts of the stereogram.

**The Basis of the Method**

Consider any wave-normal, $N$ (Fig. 1), in a general position with respect to the principal planes, $\beta B_1$, $\beta B_2$, and $B_1 B_2$, of a biaxial indicatrix. The associated wave-front, $F$, containing vibrations $P$, $P'$, will cut the $\beta B_1$ and $\beta B_2$ planes in points $S$ and $T$ respectively. It will also cut the circular sections, $\beta a_1$ and $\beta a_2$, at the points $a_1$ and $a_2$, one in the acute, the other in the obtuse angle between $S$ and $T$.

![Fig. 1. Stereogram, center C, illustrating both the geometrical basis of the method and the final determination of the optic axes, $A_1$, $A_2$, and associated circular sections, $\beta a_1$, $\beta a_2$, by interpolation.](image)

Since, by the geometry of the indicatrix, the circular sections are symmetrically disposed about $\beta$ to the planes $\beta B_1$ and $\beta B_2$, i.e. since angles $a_1 \beta S$ and $S \beta a_2$ are equal, then, in the wave-front, $S$ must lie between $a_1$ and $a_2$. Moreover, $P$ must bisect the angle $a_1 a_2$. Thus, since both $P$ and $S$ must lie between $a_1$ and $a_2$, no circular section can cut the wave-front between $P$ and $S$. But the circular sections must cut the wave-front at points on either side of and equidistant from vibration $P$. Therefore, over an angular distance of $PS$ on either side of $P$, i.e. between points $S$ and $S_1$ where $SP = PS$, no circular section can cut the wave-front. Similar considerations apply to points $T$ and $T_1$, equidistant from and on either

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$^1$ $B_1$ and $B_2$, the two bisectrices, are not differentiated as $\alpha$ and $\gamma$ or vice versa in this paper, for the value of $2V$ about both bisectrices is determinable without a knowledge of the associated refractive indices. These, if required for sign determination, must be determined by an entirely different investigation.
side of vibration $P'$. It may be noted here that these considerations are confined to two octants$^2$ of the indicatrix diametrically opposed across $\beta$ and, in consequence, are concerned with restricting the circular section traversing these octants to within an angle defined by planes $\beta S_1$ and $\beta T_1$: this circular section can cut the wave-front only between points $S_1$ and $T_1$.

Attention may now be directed towards the triangles polar to $\Delta S \beta S_1$, $\Delta S_1 \beta T_1$, and $\Delta T_1 \beta T$. These, $\Delta B_2 N s_1$, $\Delta s_1 N t_1$, and $\Delta t_1 N B_1$ respectively, are defined by adjacent parts, $B_2 s_1$, $s_1 t_1$, and $t_1 B_1$, of the optic axial plane, on the one hand, and by the great circles, passing through $N$, whose poles are $S$, $S_1$, $T_1$ and $T$, on the other. Since, of the three angles $S \beta S_1$, $S_1 \beta T_1$, and $T_1 \beta T$, the circular section under consideration can lie only in angle $S_1 T_1$ it is clear that of the three parts of the optic axial plane, $B_2 s_1$, $s_1 t_1$, and $t_1 B_1$, in the associated polar triangles, the corresponding optic axis can lie only between $s_1$ and $t_1$.

The second optic axis can therefore lie only between $s_2$ and $t_2$, where these are symmetrically disposed to $s_1$ and $t_1$ respectively across both bisectrices; and the associated circular section can cut the wave-front only between $S_2$ and $T_2$, where these are the points of intersection of the great circles polar to $s_2$ and $t_2$ respectively with the wave-front.

**Practical Application**

As a preliminary to the application of this graphical method, it is essential that the orientations of the principal axes and planes of the indicatrix be known as accurately as possible. These are determinable in a variety of ways depending upon the apparatus employed: by coplanar extinction curves, using the simple or modified (Villarroel and Joel, 1965) spindle stage (Joel and Garaycochea, 1957; Tocher, 1962; Garaycochea and Wittke, 1964) or the universal stage (Joel and Muir, 1958a); or by conical extinction curves using the inclined spindle stage or universal stage (Joel and Tocher, 1964). Orthoscopic methods with the universal stage are, in general, insufficiently accurate in this connection and should only be used for approximation purposes prior to the exact location of the principal axes by extinction methods.

On the basis of the extinction angle associated with any general wave-normal, the construction may now proceed. If using the simple spindle stage, the wave-normal will have to lie in the coplanar wave-normal locus used for determining the indicatrix; but with more versatile apparatus like the modified spindle stage, the inclined spindle stage, or the universal stage, it may be chosen in any suitable orientation within the

$^2$ The octants in question are, of course, those outlined by the three principal planes, $\beta B_1$, $\beta B_2$ and $B_1 B_2$, of the indicatrix.
range of the instrument regardless of the wave-normal locus or loci used in determining the indicatrix.

Wave-normal N, wave-front F, and vibrations P and P' having been plotted (Figs. 1, 2), points S and T can be identified and points S₁ and T₁ located. Next, points s₁ and t₁ are plotted where the great circles polar to S₁ and T₁ respectively cross the optic axial plane (although the great circles themselves need not be plotted it is worth checking that both pass through N). These in turn give rise, respectively, to s₂ and t₂, symmetrically disposed to the former across both bisectrices. Finally, points S₂ and T₂, where the great circles polar to s₂ and t₂ respectively cut the wave-front, are located. This ends the first stage of the determination and from here the final location of the optic axes and circular sections may be carried out by one of two methods: by interpolation or by progression.

By interpolation. Those portions of the optic axial plane, s₁t₁ and s₂t₂, within which the optic axes must lie, are of equal angular width, but, in general, the corresponding portions of the wave-front, S₁T₁ and S₂T₂ respectively, within which the latter must be cut by the circular sections, are of unequal angular width. This means that, for trial positions of the optic axes symmetrically disposed about the bisectrices in the arcs s₁t₁ and s₂t₂, the corresponding trial positions a₁ and a₂ (the points where the circular sections cut the wave-front in arcs S₁T₁ and S₂T₂ respectively) will, in general, be asymmetrically disposed about vibrations P and P'. Only one such trial pair of a₁ and a₂ points will have the necessary symmetrical disposition to vibrations P and P'. These are the true a₁ and a₂ positions, and the corresponding trial positions within the arcs s₁t₁ and s₂t₂ in the optic axial plane are the positions of the optic axes A₁ and A₂ respectively. Figure 1 shows several such trial positions for A₁, A₂ and a₁, a₂; the finally determined true positions are, in each case, indicated.

By progression. At the end of the first stage it was shown that, of the two circular sections, one must cut the wave-front between S₁ and T₁, the other between S₂ and T₂. However, although these two arcs, S₁T₁ and S₂T₂, subtend equal angles at β, they are not, within the wave-front, symmetrically disposed to the vibrations P and P'. In fact, the portions of the wave-front free of intersection by circular sections, originally shown to be the arcs S₁P₁ and T₁P₁, are now extended to embrace the arcs S₁P₂ and T₁P₂. But whereas the original arcs were determined on the premise that they must be bisected by P and P' respectively, this is no longer the case: S₂P > PS₁ and T₂P' > P'T₁. Thus, to restore the condition of the original premise, arcs PS₁ and P'T₁ must be extended to points S₁' and T₁' respectively (Fig. 2) such that S₂P = PS₁' and T₂P' = P'T₁'.
by a circular section from $S_1T_1$ to $S_1'T_1'$ now renders invalid the arc $s_1t_1$ within which an optic axis may fall. This must now be reduced to arc $s_1't_1'$ by the great circles polar to $S_1'$ and $T_1'$. Next follows a reduction of arc $s_2t_2$ to $s_2't_2'$ so that $s_1't_1'$ and $s_2't_2'$ are equal and symmetrically disposed about both bisectrices. And this, of course, necessitates a corresponding reduction in arc $S_2T_2$ to $S_2'T_2'$ by the great circles polar to $s_2'$ and $t_2'$.

At this point, the end of the second cycle of the progressive construction, the portions of the wave-front free of intersection by circular sections are $S_1'PS_2'$ and $T_1'P'T_2'$, neither of which is bisected by the relevant vibration. A third cycle and, if necessary, subsequent cycles will reduce still further the arcs within which the circular sections may cut the wave-front and those within which the optic axes may lie. It is clear, in fact, that as the process is continued, the several arcs progressively decrease in size and converge on the points $a_1$, $a_2$ and the optic axes $A_1$, $A_2$ respectively.

In the more sensitive cases (see below) two complete cycles are sufficient to limit each optic axis to within a $1^\circ$ arc of the optic axial plane (Fig. 2) and to effect virtual coincidence of points $S_2'$ and $T_2'$ on the wave-front—this latter coincidence serves, of course, to define point $a_2$ where one circular section cuts the wave-front. Point $a_1$ is now readily determinable between $S_1'$ and $T_1'$: it is symmetrically disposed with respect to $a_2$ about both $P$ and $P'$. The optic axes are now determinable with a high degree of accuracy as the poles of planes $\beta a_1$ and $\beta a_2$. The accuracy of the final determination is, of course, dependent upon many
factors not the least of which is plotting accuracy. However, given a good net of at least 20 cms diameter, plotting of points and planes can normally be done to within $\frac{1}{2}^\circ$ of their correct positions. Of course, even this minimum tolerance can, with practice, with greater care, or with the use of a larger net be improved upon.

**Sensitivity**

The sensitivity of this method, as of all other methods of 2V determination, varies with conditions. It depends upon two factors: the angle 2V, and the orientation of the chosen wave-normal N with respect to the optic axes and the principal planes of the indicatrix.

![Partial stereograms, center C, depicting one octant of the indicatrix in the cases where 2V=20° (Fig. 3) and 2V=80° (Fig. 4). The contours show the variation in value of the sensitivity ratio, $S_1T_1/S_2T_2$, for general wave-normals.](image)

However, while the sensitivity of previously advanced methods has had, perforce, to be described purely qualitatively, that of this method can be fairly simply presented in a more quantitative manner. A simple numerical indicator of the sensitivity in the case of any chosen wave-normal can be obtained by comparing the angular lengths of the two arcs, $S_1T_1$ and $S_2T_2$, at the end of the first complete graphical cycle: the greater the ratio $S_1T_1/S_2T_2$, the greater the sensitivity for the wave-normal concerned.

In each of Figs. 3 and 4, one octant of the indicatrix has been contoured to show how the value of the ratio $S_1T_1/S_2T_2$ varies for different wave-normals: Figure 3 is representative of small 2V (20°); Fig. 4 of large 2V (80°). In all cases the sensitivity decreases towards the minimum ($S_1T_1/S_2T_2=1$) as the wave-normal approaches the $\beta B_1$ and $\beta B_2$ planes, and towards the maximum ($S_1T_1/S_2T_2=\infty$) as the wave-normal approaches
an optic axis. For both of these special cases (see below) the problem is insoluble by this method. It is of interest and of great value, however, that, for most positions of N within the $\beta B_1 B_2$ triangle, the sensitivity of the method reaches its maximum when $2V = 90^\circ$—the case where, for the direct determination of $2V$, the universal stage is, in normal random thin section work, least often usable and, even when usable, is at its least accurate (Johnston, 1953; Wyllie, 1959; Munro, 1963). Exceptions to this general rule, of course, exist for wave-normals near the optic axial plane: here, the sensitivity normally shows an increase as an optic axis is approached, regardless of the value of $2V$.

**Special Cases**

*N on the $\beta B_1$ or $\beta B_2$ plane*. In this case, vibration P coincides with point S on the same principal plane as and $90^\circ$ from N, while vibration P' and point T both coincide with the bisectrix polar to that plane. Since the method is based on the non-coincidence of P and P' with S and T respectively, it can not provide a solution in this case.

*N on the optic axial plane*. Here, the wave-front passes through $\beta$; in general, vibration P and both of the points S and T coincide with $\beta$; and vibration P' falls on the optic axial plane $90^\circ$ from N. As above, due to the coincidence of P with S and T, the method can provide no solution.

**Refinements**

Using this method alone, the most obvious refinement consists of ensuring that a wave-normal of high sensitivity is used in the final determination of $2V$. Since the sensitivity of a general wave-normal in any given position depends upon the value of $2V$, an approximation to the value of $2V$ should first be made, using any convenient general wave-normal of reasonable sensitivity. Only then can an idea of the sensitivity distribution of the available wave-normals be obtained and a suitable choice made for the final determination. Comparison of Figs. 3 and 4 shows, for example, that curves of low sensitivity, up to about 2.0, are more or less coincident for all values of $2V$, and that only thereafter does the effect of $2V$ become very apparent.

However, although the theoretical sensitivity approaches infinity as the wave-normal approaches an optic axis, it must, at the same time, be remembered that close to an optic axis there is also a rapid decrease in the sharpness of the extinction position: a suitable balance must therefore be struck between these two phenomena in order to achieve simultaneously high sensitivity and an acceptable degree of sharpness in the extinction position. If $2V$ is large, there is little difficulty for, even as far
as 25° in any direction from an optic axis, the sensitivity ratio has a value as high as 4.0 (Fig. 4). If 2V is small, however, the sensitivity distribution is radically different: comparable sensitivity (4.0) with acceptable sharpness in extinction (N about 20° from an optic axis) is best achieved within about 10° of the optic axial plane and on the obtuse bisectrix side of an optic axis, where the sensitivity gradient is low (Fig. 3).

With the universal stage, a very wide choice of wave-normals is normally available and a satisfactory solution can usually be obtained. With the various types of spindle stage, the choice of wave-normals is much more restricted, but reference to Figs. 3 and 4 will normally permit one to make a good final choice from those available.

**Other Uses**

Although this method is obviously at its most useful when neither optic axis is directly determinable, it is clear that it may be used with profit in other circumstances. It may, for example, be used in those cases where an optic axis, although directly observable with the universal stage, is near the limit of tilt. It may also be used to supplement the Biot-Fresnel construction advocated for use by Joel and Muir (1958b) in the case where one optic axis is directly observable with accuracy: the two methods employ the same data.

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**References**


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