THE GOLDSCHMIDT TWO-CIRCLE METHOD. CALCULATIONS IN THE MONOCLINIC SYSTEM, ILLUSTRATED BY MONAZITE FROM WEYMOUTH, MASS.

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Determination of Elements from Coördinate Angles

Figure 34 is the gnomonic projection of a few forms of a monoclinic crystal (monazite) constructed from the angles of the *Winkeltabellen*. If it is compared with the orthorhombic projection (p. 159) it will be seen to be similar to it, with the notable

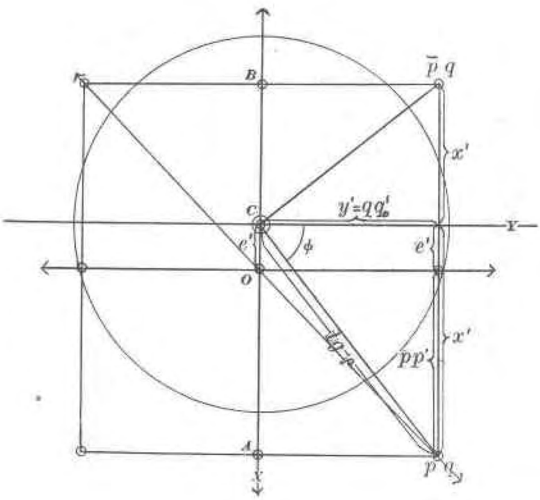

Fig. 34.

exception that the projection center (C) is not a node of the network marked out by the face-poles. This has its center at O, the projection of the basal pinacoid, which lies on the X coördinate but at a distance from C depending on the angle (β) between the *a* and *c* axes. Nevertheless the calculation proceeds as in the orthorhombic system with reference to C, for in the two-circle measurement the angles are measured, no matter what the system of the crystal, from the face or hypothetical plane normal to the prism zone. The angle φ of any face (p q) of the crystal of figure

34 is then measured from the Y coördinate at C. The angle ρ is the angle between the face-pole and the point C and tan ρ represents the distance of the face-pole from C. We can therefore use the same relation as before:

$$x' = \sin \varphi \tan \rho. \qquad y' = \cos \varphi \tan \rho.$$

Before proceeding to deduce the formulas connecting angles and elements, however, it is necessary to draw attention to a further relation illustrated in figure 35. This is a vertical section thru the projection of figure 34 along the X coördinate, showing the sphere of projection with center at S. The plotting of the face p q by its angular coördinates φ and tan ρ in figure 34 is based on the assumption that the height of the plane of projection above the center of the sphere is equal to the radius (h = 1). In the Goldschmidt system the symbols are based upon three elements, p_0 q_0 r_0, of which p_0 and q_0, normal to the two vertical pinacoids, are measured in the horizontal plane of the projection; r_0, normal to the basal pinacoid, is always taken as unity whatever its direction. In the rectangular systems r_0 coincides in direction with h. In this case however r_0 deviates from h, depending upon the value of μ (supplement of β). Now if we wish r_0 to be unity, the plane of projection must be lowered by a certain amount as shown (for monazite where $\mu = 76° 20'$)

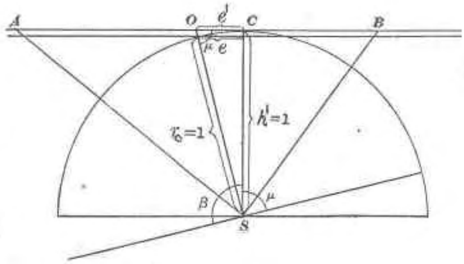

Fig. 35.

in figure 35. Evidently all units measured in the upper plane of projection will be larger than in the parallel lower plane; they will however be in the same relative proportion, and the integral multipliers, p and q, will be the same in both cases. The true elements p_0 q_0 ($r_0 = 1$) can be derived from those calculated from the coördinate angles by multiplying the latter by sin μ as shown below. All values measured in the upper plane ($h' = 1$) will be indicated by primed letters, in the lower plane by the same letters unprimed.

From figure 34 we have for the face p q:

$$x' = \sin \varphi \tan \rho = pp_0' + e'. \qquad y' = \cos \varphi \tan \rho = qq_0'.$$

For the face $\overline{p\ q}$:

$$x' = \sin \bar{\varphi} \tan \rho = pp_0' - e'. \qquad y' = \cos \bar{\varphi} \tan \rho = qq_0'.$$

From figure 33 we have:

$$e' = \cot \mu = \tan \rho \text{ of face } (001). \qquad r_0' = \frac{1}{\sin \mu}.$$

The values of x' and y' are calculated from the angles for each face. The x' values give a series of equations which may be solved either for p_0' or e', it being assumed that p and q have been determined graphically by projection. e' may be known from direct measurement of μ, the angle between the pinacoids 100 and 001. q_0' is obtained directly by averaging the values of y'.

Prisms.—The direction-line of a prism $\frac{p}{q} \infty$ is a line thru O and the face p q. Its position is defined by the equation

$$\tan \varphi = \frac{pp_0'}{qq_0'}.$$

Prism angles give therefore the ratio only of p_0' to q_0', as in the orthorhombic system.

Transformation of Elements to the Plane where $r_0 = 1$

From figure 35 we have the following relations:

$$\frac{h}{r_0(=1)} = \sin \mu \qquad \text{therefore} \qquad h = \sin \mu;$$

$$\frac{e'}{e} = \frac{h'}{h} = \frac{1}{\sin \mu} \qquad \text{therefore} \qquad e = e' \sin \mu.$$

The same proportion will evidently hold for any other units measured in the two planes and we have therefore:

$$p_0 = p_0' \sin \mu. \qquad q_0 = q_0' \sin \mu.$$

Transformation of Polar to Linear Axes

$$\mu = 180° - \beta.$$

$$p_0 = \frac{c}{a} \qquad q_0 = c \sin \beta = c \sin \mu. \qquad r_0 = 1.$$

$$e = \cos \beta. \qquad h = \sin \beta.$$

$$a = \frac{q_0}{p_0 \sin \mu} = \frac{q_0'}{p_0' \sin \mu}, \qquad c = \frac{q_0}{\sin \mu} = q_0', \qquad \cot \beta = e'.$$

Calculation of Angles from Elements

For p q and 0q, figure 34:

$$\tan \varphi = \frac{x}{y} = \frac{pp_0 \pm e}{qq_0}.$$

$$\tan \rho = \frac{x}{h \cdot \sin \varphi} = \frac{pp_0 \pm e}{h \cdot \sin \varphi} = \frac{y}{h \cdot \cos \varphi} = \frac{qq_0}{h \cdot \cos \varphi}.$$

For p0:

$$\tan \varphi = \infty. \qquad \varphi = 90°. \qquad \tan \rho = \frac{x}{h} = \frac{pp_0 \pm e}{h}.$$

For prisms:

$$\tan \varphi = \frac{pp_0}{qq_0}. \qquad \tan \rho = \infty \cdot \rho = 90°.$$

Forms for the calculation of these relations may be found in *Winkeltabellen*, p. 19a and 19b.

An illustration of the discussion of a monoclinic mineral on the above lines is given by Goldschmidt in an article on lorandite.[1] A very full discussion in English is to be found in the paper by A. S. Eakle on the crystal form of colemanite.[2]

To illustrate the use of the formulas we may discuss the gnomonic projection of inyoite given by Rogers in this journal.[3] Measuring directly from this projection in terms of the accompanying scale No. 4, which gives decimal parts of unity, we obtain:

$$e' = Zc = 0.46, \qquad p_0' = cd = 0.775, \qquad q_0' = ce = 0.63.$$

[1] *Z. Kryst. Min.* **30**, 291, 1898.

[2] *Bull. Dept. Geol. Univ. Cal.*, **3**, 42, 1902.

[3] *Am. Min.*, **4**, 137, 1919.

From the transformation formulas:

$$\cot \beta = e'. \qquad \beta = 65° \, 18'.$$

$$c = q_0' = 0.63. \qquad a = \frac{q_0'}{p_0' \sin \mu} = 0.894.$$

Projection of a Monoclinic Crystal on Clinopinacoid, 010

Certain monoclinic minerals (epidote, azurite, etc.) often show prismatic development parallel to the *b* axis (orthodome zone). In such cases it is much the easiest way to measure the crystal with this zone as prism zone (normal to V circle on the goniometer). It can then be projected in this position, the symbols determined graphically, and the elements calculated. Transformation of symbols and elements to normal position follows. It seems desirable to illustrate this procedure, as the case arises not infrequently.

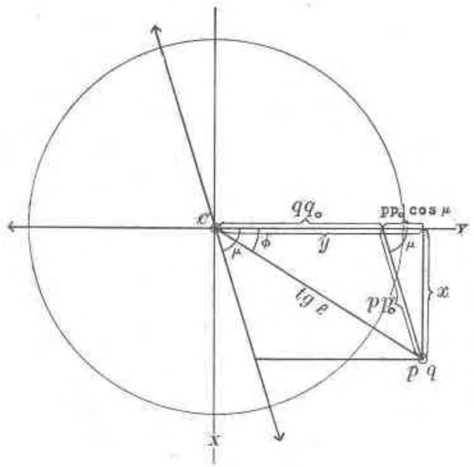

Fig. 36.

In figure 36 is shown part of such a projection. The Y coördinate is the direction-line of the orthopinacoid, C the face-pole of the clinopinacoid.

For a face p q for which φ and ρ are known we have the following relations:

$$x = \sin \varphi \tan \rho = pp_0 \sin \mu,$$

$$y = \cos \varphi \tan \rho = qq_0 + pp_0 \cos \mu.$$

If μ is known by measurement these equations give values for p_0 and q_0, the elements for this position.

Calculation of Angles

$$\cot \varphi = \frac{y}{x} = \frac{qq_0 + pp_0 \cos \mu}{pp_0 \sin \mu} = \frac{qq_0}{pp_0 \sin \mu} + \cot \mu,$$

$$\tan \rho = \frac{x}{\sin \varphi} = \frac{pp_0 \sin \mu}{\sin \varphi} = \frac{y}{\cos \varphi} = \frac{qq_0 + pp_0 \cos \mu}{\cos \varphi}.$$

While the equations are not of the best form for logarithmic calculation, still they lead to the desired result in a direct manner.

Transformation of Elements and Symbols to Normal Position

Let I be the normal position with elements $p_0\ q_0\ r_0(=1)$.

Let II be the 010 position with elements $p_0''\ q_0''\ r_0''(=1)$. Then

$$p_0 = \frac{q_0''}{p_0''}. \qquad q_0 = \frac{1}{p_0''}.$$

$$p_0'' = \frac{1}{q_0}. \qquad q_0'' = \frac{p_0}{q_0}.$$

$$\text{Symbol for any face (pq) (I)} = \frac{1}{q}\frac{p}{q} \text{ (II)}.$$

$$\text{Symbol for any face pq (II)} = \frac{q}{p}\frac{1}{p} \text{ (I)}.$$

Illustration of Projection on 010

As an illustration of this procedure the projection and drawing of a crystal of monazite is given in figure 37. The crystal was attached to the matrix in such a manner that it could only be measured in position (II). The faces were poor, so that elements were not calculated. The forms with symbols in both positions are as follows:

	II	I	Miller		II	I	Miller
a.....	0∞	$\infty 0$	100	e......	10	01	011
b.....	0	0∞	010	r......	1	1	111
c.....	$\infty 0$	0	001	v......	$\bar{1}1$	$\bar{1}1$	$\bar{1}11$
m....	01	∞	110	s......	$\frac{1}{2}$	12	121
w....	∞	10	101	o......	$-\frac{1}{2}$	$\bar{1}2$	$\bar{1}21$

The elements determined graphically are:

$$p_0'' = \frac{5.55}{5} = 1.11, \qquad q_0'' = \frac{5.275}{5} = 1.055,$$

$$p_0 = \frac{q_0''}{p_0''} = .950. \qquad \text{Calculated} \quad .955.$$

$$q_0 = \frac{1}{p_0''} = .901. \qquad \text{Calculated} \quad .899.$$

$$\mu = 76° \, 15'. \qquad \text{Calculated} \quad 76° \, 20'.$$

Coördinate Angles in Winkeltabellen for Abnormal Position

In the *Winkeltabellen*, pp. 7 and 8, the subject is treated of the relations of forms, symbols and coördinate angles for the projections of a given crystal on the three pinacoids. It will be seen there that the coördinate angles for all three positions are related thru certain additional angles, ξ_0 and η_0, ξ and η, contained in the angle tables, the significance of which is explained in a diagram on page 5. In the case of the monazite crystal of figure 36 we have a projection on b. For this position:

$$\varphi'' = 90° - \xi_0, \qquad \rho'' = 90° - \eta.$$

Turning to the angle-table for monazite, we find these values and can compare the measured and calculated angles as follows:

	Calculated				Measured	
	ξ_0	η	$\varphi''=90°-\xi_0$	$\rho''=90°-\eta$	φ	ρ
m 110	90° 00′	43° 17′	0° 00′	46° 43′	0° 00′	46° 00′
e 011	13 40	41 58	76 20	48 02	74 33	48 49
w 101	50 48	0 00	39 12	90 00	38 06	90 00
r 111	50 48	30 20	39 12	59 40	38 00	60 40
o $\bar{1}21$	−36 29	56 06	−53 31	33 54	−52 36	33 20

The crystal was poor and measured on the matrix under disadvantageous conditions; the agreement of measured and calculated angles is therefore poor, but sufficed for identification of the mineral.

Drawing of Crystals in Abnormal Position

The drawings of this crystal are added because they illustrate the method of securing a perspective drawing in correct position

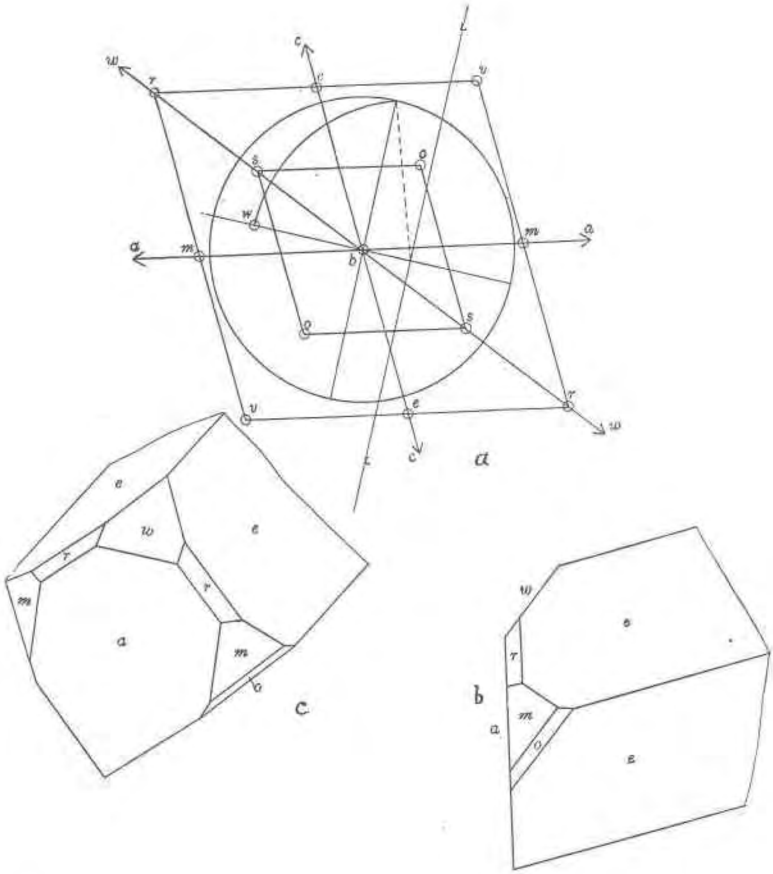

Fig. 37.

directly from this abnormal projection. The case offers splendid demonstration of the flexibility of the Goldschmidt method of drawing.

Figure 37b is an orthographic projection of the crystal on (010) made from the gnomonic projection, 37a, exactly as described for the ordinary basal projection on a previous page (Cryst. Draw., p. 91). Figure 37c is a perspective projection of the crystal in normal position (the edge a to m should be vertical) and is obtained from 37b by the aid of the guide-line, LL and angle-point, W, shown in the drawing. The position of these aids to construction is that which they assume if we think of them in normal position and rotated with the crystal to the new position. The two drawings are related by lines normal to LL and the construction follows exactly the rules given in the paper on crystal drawing, page 93.

The crystal of monazite here figured is interesting as marking a new locality for this mineral. The single crystal found occured in a cavity of a quartz vein in Weymouth, Mass. The vein is lenticular, following the bedding of the Cambrian slate of the region. The quartz crystals are large and fine, clear, with the usual forms. The monazite crystal, about 3 mm. in diameter, is clear and of dark amber color. The writer is indebted to Mr. T. H. Clark for the loan of the specimen and for the knowledge of the locality.

LISTS OF THE MONOCLINIC MINERALS INCLUDED IN GOLDSCHMIDT'S WINKELTABELLEN. Edgar T. Wherry. *Washington, D. C.*—The minerals are arranged as in the preceding list in increasing order of axis *a*. The approximate value of the monoclinic angle μ is also given; in most cases this is identical with β as given by Dana, but it is the complement of β as given by some other authors.

Monoclinic Minerals

	a	c	μ	Page
Euclasite (Euklas)	0.32	0.33	80°	135
Autunite (Kalkuranite)	0.35	0.35	90	194
Heulandite	0.40	0.86	89	177
Claudetite	0.40	0.34	86	97
Brewsterite	0.40	0.84	86	79
Ganophyllite	0.41	1.83	87	155
Kroehnkite	0.45	0.44	73	203
Wapplerite	0.46	0.27	85	362
Copiapite	0.48	0.98	72	102
Mordenite	0.50	1.07	89	244
Epistilbite	0.51	0.58	56	131
Amphibole	0.55	0.29	75	37
Kaolinite	0.57	1.60	83	196
Chlorite group	0.58	2.26	90	400
Freieslebenite	0.59	0.93	88	151
Allactite	0.61	0.33	84	33
Pharmacolite	0.61	0.36	83	263
Brushite	0.62	0.34	85	81
Homilite	0.62	1.28	89	179
Gadolinite	0.63	1.32	89	153
Hydroherderite	0.63	0.64	90	174
Datolite	0.63	0.63	90	110
Lautarite	0.63	0.64	74	215
Sapphirinite	0.65	0.93	80	310
Botryogenite	0.65	0.60	62	75
Fiedlerite	0.66	0.89	77	146
Hyalophanite	0.66	0.55	64	143
Orthoclase	0.66	0.56	64	143
Quenstedtite	0.67	0.66	78	291
Gypsum	0.69	0.41	81	167
Harmotomite	0.70	1.23	55	170
Phillipsite	0.70	1.23	56	264
Cuspidinite	0.72	1.94	90	105
Realgar	0.72	0.49	66	292
Chalcomenite	0.72	0.98	89	92
Picromerite	0.74	0.50	75	266
Vivianite	0.75	0.70	76	359
Erythrite (Kobaltblüthe)	0.75	0.70	75	199
Titanite	0.75	0.85	60	344
Cyanochroite	0.76	0.50	74	107
Durangite	0.77	0.82	65	121
Colemanite	0.78	0.54	70	100
Malachite	0.78	0.40	89	228
Symplesite	0.78	0.68	73	336
Wolframite	0.83	0.87	90	366
Azurite (Kupferlasur)	0.85	0.88	88	207
Whewellite	0.87	1.37	73	363
Leadhillite	0.87	1.11	90	217
Kieserite	0.91	1.77	89	198
Atelestite	0.93	1.51	71	57
Wagnerite, Kjerulfinite	0.96	0.75	72	361
Crocoite (Rothbleierz)	0.96	0.92	77	297