The powdered mineral was tested with phenolphthalein but gave only a faint coloration. The gently ignited material gave a decided red color and also upon leaching a slight precipitate with ammonium oxalate. As will be seen from the analyses the water content is constant. The CO_2 is apparently an integral part of the mineral molecule. That CO_3 can replace SiO_3 is shown by the experiments of Lemberg,[1] who synthesized both carbonate and silicate-cancrinite. The ratio of the bases to the silica is very large. The only formula which will satisfy all the bases is an orthosilicate one in which the bases are present in some complex grouping. The mineral appears to be most nearly related to sodalite.

THE GOLDSCHMIDT TWO-CIRCLE METHOD. INTRODUCTION TO THE TRICLINIC SYSTEM

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The Gnomonic Projection

Figures 38 and 39 show the gnomonic projections of the upper and lower ends respectively of the anorthite crystal discussed in the following paper by Professor Parsons, assuming that the same faces would be developed on the upper end as were found by measurement on the lower end. The two figures are similar, the differences being such as to be very puzzling unless the relations of the two are clearly in mind. The one may be obtained from the other by pricking the face-poles thru on to the back of the paper, and then turning the paper over in such a way that the right-and-left coördinate maintains its direction but with ends interchanged. Figure 38 is that which would be obtained by plotting the angles of the respective forms as given in the *Winkeltabellen*.[2] In figure 39 the forms have the same ρ values and the same φ values but measured in the opposite sense; the latter is also true of the angle ν; and the y and q

[1] *Z. Deut. Geol. Ges.*, **1885**, 662.

[2] In the project thus made the direction-lines of faces in the prism zone would center at S, the projection center, and would not pass thru nodes of the network of face-poles of the terminal faces. By transferring the direction-lines by parallels thru O, the pole of the base, they become, as shown in the figure, diagonals of the network; and their zonal relations and symbols then become evident.

coördinates are measured in the reverse direction. Thus pq of figure 38 is equivalent to $p\bar{q}$ of figure 39. The latter is similar to Parsons' figure 43, in which the forms are indicated by the face numbers used in the measurements; the horizontal coördinate of 38 is the direction-line of the two faces 15 and 18 of figure 43.

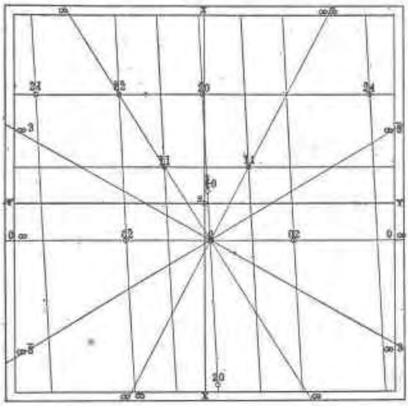

Fig. 38

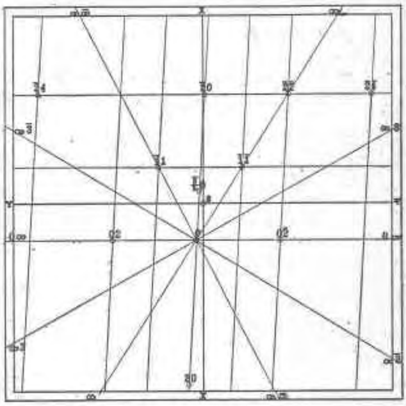

Fig. 39

Gnomonic projections of anorthite crystal in two complementary positions. Palache, p. 185.

Calculation of Elements from Coördinate Angles

The elements to be determined are p_0, q_0, r_0 (= 1), the polar elements; λ, μ, ν, the angles which they include; φ_0 and ρ_0, δ and d, or x_0 and y_0, respectively the angles, polar coördinates or rectangular coördinates of the basal pinacoid. The relations are similar to those described in the monoclinic system, the gnomonic projection made from measured angles being in a plane at h = 1. The values of the elements for $r_0 = 1$ are derived from projection elements determined in this plane by multiplying by cos ρ_0 as shown below.

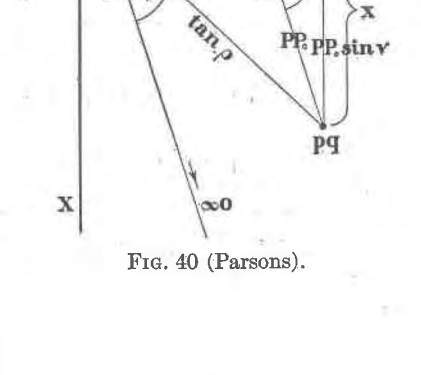

Fig. 40 (Parsons).

From figure 40 in which $r_0 = 1$ we derive the relations:
(1) $x = x_0 + pp_0 \sin \nu$ (2) $y = y_0 + qq_0 + pp_0 \cos \nu$
If $h = 1$ the similar relations hold:
(3) $x' = x_0' + pp_0' \sin \nu$ (4) $y' = y_0' + qq_0' + pp_0' \cos \nu$
(5) $x' = \sin \varphi \tan \rho$ (6) $y' = \cos \varphi \tan \rho$

Since φ and ρ for each face are known by measurement we can calculate x' and y' from equations 5 and 6; see table 3, page 192 of Parsons' paper for illustration. p and q are also known from the graphical determination; see figures 39 and 43.

We have therefore to calculate the five independent quantities: x_0' and $p_0' \sin \nu$ from equation 3 (Parsons, table 8); y_0', q_0' and $p_0' \cos \nu$ from equation 4 (Parsons, table 9); an independent value of ν from the prisms (Parsons, table 10). These elements may be termed the *projection elements*. They are identical with those which may be determined graphically from the projection, this serving as a check on the calculations.

Preceding all these calculations, however, is the determination of the value v_0, which is the mean value of the vertical-circle reading for the face 0∞ derived from all the measurements. This calculation, much more laborious in the triclinic system than in any of the others, where there is a symmetrical distribution of the faces about the vertical axis, is explained at length in Parsons' paper and illustrated in his tables 4, 5, 6 and 7.

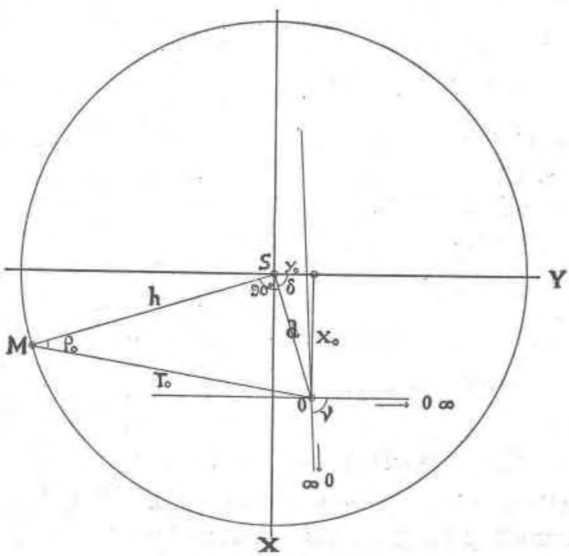

FIG. 41 (Palache).

Calculation of Polar Elements from Projection Elements

Figure 41 shows with greater detail the central portion of figure 40. In it O is the projection of the base with rectilinear coördinates x_0 and y_0, and polar coördinates δ and d. M is the angle-point of the line SO and the triangle SMO represents the relations at the crystal center of h, the normal to the projection plane, and r_0, the normal to the base.

The projection-elements first calculated, p_0', q_0', x_0', and y_0', are measured in a plane where h is unity. The polar elements are to be measured in a parallel plane cutting r_0 at unity which means that each will be shorter in the proportion of $h : r_0$. For example $p_0 = p_0' \cdot \frac{h}{r_0}$

From figure 41 it is evident that $\frac{h}{r_0} = \cos \rho_0$; but $\tan \rho_0 = \frac{x_0'}{\sin \delta}$ and $\tan \delta = \frac{x_0'}{y_0'}$. We can therefore calculate δ and ρ_0 and thus find $\cos \rho_0$.

Therefore $p_0 = p_0' \cos \rho_0$ $\qquad x_0 = x_0' \cos \rho_0$

$q_0 = q_0' \cos \rho_0$ $\qquad y_0 = y_0' \cos \rho_0$

It is also evident from the same figure that the following relations hold:

$$\tan \rho_0 = \frac{d}{h} = d' \ (h = 1) \qquad d' = \sqrt{(x_0')^2 + (y_0')^2}$$

The remaining polar elements are the angles λ, μ, and ν. Of these ν, the angle between the pinacoids $\infty 0$ and 0∞, has already been determined. λ, the angle between 0∞ and 0, and μ, the angle between $\infty 0$ and 0, are calculated by the following equations, the derivation of which is not shown in the figures:

$$\cos \lambda = \frac{y_0}{r_0} = y_0 \text{ for } r_0 = 1.$$

$$\cos \mu = y_0 \cos \nu + x_0 \sin \nu.$$

Transformation of the Elements

1. *Relations of polar and linear axes.* The fundamental equation relating the polar axes (normal to the three pinacoids)

and the linear axes (parallel to edges of the pinacoidal body) is as follows:

$$a : b : c :: \frac{\sin \alpha}{p_0} : \frac{\sin \beta}{q_0} : \frac{\sin \gamma}{r_0} :: \frac{\sin \lambda}{p_0} : \frac{\sin \mu}{q_0} : \frac{\sin \nu}{r_0}$$

Substitution in this ratio of the known values (for any system, remembering that sin 90° = 1) gives a triple ratio which, solved in two operations, yields the desired transformation. In the case of the triclinic system b and r_0 are taken as unity. To find a and c, given p_0 and q_0,

$$a : 1 : c :: \frac{\sin \lambda}{p_0} : \frac{\sin \mu}{q_0} : \frac{\sin \nu}{1}.$$

$$a : 1 = \frac{\sin \lambda}{p_0} : \frac{\sin \mu}{q_0} \qquad a = \frac{q_0 \sin \lambda}{p_0 \sin \mu}$$

$$1 : c = \frac{\sin \mu}{q_0} : \frac{\sin \nu}{1} \qquad c = \frac{q_0 \sin \nu}{\sin \mu}$$

Similarly, given a and c, to find p_0 and q_0, we use the middle member of the fundamental ratio and obtain:

$$p_0 = \frac{c \sin \alpha}{a \sin \gamma} \qquad q_0 = \frac{c \sin \beta}{\sin \gamma}$$

2. *Relations of the polar and linear axial angles.* If the three angles λ, μ, and ν be regarded as the sides of a spherical triangle, then the supplements of the three angles α, β, and γ are equal to the opposite angles of the triangle. The reverse is also true, the relation being reciprocal. The calculation of either set from the other resolves itself therefore into the calculation of the angles of an oblique spherical triangle whose sides are given.

Given λ, μ, ν and let $\sigma = \frac{\lambda + \mu + \nu}{2}$

$$\sin \frac{\alpha}{2} = \sqrt{\frac{\sin \sigma \sin (\sigma - \lambda)}{\sin \mu \sin \nu}}$$

$$\sin \frac{\beta}{2} = \sqrt{\frac{\sin \sigma \sin (\sigma - \mu)}{\sin \nu \sin \lambda}}$$

$$\sin \frac{\gamma}{2} = \sqrt{\frac{\sin \sigma \sin (\sigma - \nu)}{\sin \lambda \sin \mu}}$$

For the reverse case the formulas are identical but λ, μ, and ν are substituted for α, β, and γ respectively thruout and *vice versa*.

These fundamental relations are deduced and proved by Goldschmidt in *Index der Krystallformen*, pages 5–9. They form the foundation of his whole system of crystallographic discussion, and it is hoped that they may some day be adequately presented to American readers.

Calculation of Angles from Elements

The following relations may be derived from the diagram of figure 40.

$$\tan \varphi = \frac{x}{y} = \frac{x_0 + pp_0 \sin \nu}{y_0 + qq_0 + pp_0 \cos \nu} \qquad \tan \rho = \frac{x}{h \sin \varphi} = \frac{y}{h \cos \varphi}$$

$$\text{For a prism } \infty \frac{q}{p}, \ \tan \varphi = \frac{pp_0 \sin \nu}{qq_0 + pp_0 \cos \nu}; \ \rho = 90°$$

Forms for the most rapid carrying out of the somewhat laborious computations, with adequate controls, will be found in *Winkeltabellen*, pages 19b and 20.

CALCULATION IN THE TRICLINIC SYSTEM, ILLUSTRATED BY ANORTHITE.

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The methods involved in the complex problem of measuring and calculating the axial ratios of a triclinic crystal are illustrated by the following measurements and calculations of a crystal of anorthite from Vesuvius, made by the writer in the laboratory of Professor Victor Goldschmidt in 1909.

The crystal was slightly elongated but there was no cleavage apparent to guide in orienting it, so that the zone with the longest edges was assumed to be the prism zone, and the crystal was adjusted on the goniometer with this zone parallel to the axis of the vertical circle. Readings were obtained from 19 faces as shown in Table 1.

From these readings a gnomonic projection was made (Fig. 42), from which it is at once evident that this crystal is a simple