

PRECISION AND COMPONENTS OF VARIANCE IN THE
MODAL ANALYSIS OF A COARSE-GRAINED
AUGEN GNEISS

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ABSTRACT

Large sodium cobaltinitrite-stained slabs of hand specimens are magnified under a binocular microscope and counted on a large mechanical stage. The method is recommended for rocks that are so coarse grained as to require a very large number of thin sections per specimen for a moderately precise estimate of their mode, but not so coarse grained as to make field identification of minerals a reliable procedure.

The method is applied to a quartz-mangeritic augen gneiss, and the variability between the specimens is broken down into components of variance: (1) The operator plus counting component of variance is not significantly different from binomial expectation. (2) The within-specimen component of variance is used to predict the number of slabs required per specimen to obtain a mode of given precision. (3) The between-specimen component of variance is unknown, but a rough estimate of its maximum size is possible and is useful for designing sampling plans in similar rocks.

INTRODUCTION

In the course of a survey of a geologically unknown region in the Grenville Province of the Canadian Shield, 100 miles north of Montreal, an intricately deformed complex of meta-igneous rocks was mapped (Schryver, 1966a, b, 1968). In subsequent work it was desired to obtain knowledge of the mineralogical composition and the degree of compositional homogeneity of the most abundant rock type of the complex, *e.g.*, for determination of which specimens can be considered "typical," and chemical analysis of which specimens can be considered of more than local value only. This most abundant rock type, a coarse-grained augen gneiss of quartz-mangeritic (or, quartz-pyroxene-monzonitic) composition, forms a mappable unit underlying an area of approximately 120 square miles.

The gneiss is green to greyish green, and weathers light grey or buff. The essential minerals are potassium feldspar, plagioclase, quartz, orthopyroxene and clinopyroxene. The accessory minerals are hornblende, iron-titanium oxides, apatite and zircon, in order of abundance. Biotite and garnet are rare.

Slightly to strongly perthitic *orthoclase* and *plagioclase* (An₂₅₋₃₀) occur in roughly equal amounts, in grains ranging in size from a fraction of a millimeter to 5 cm. The large grains are subhedral crystals, the finer ones are, at least in part, still recognizable as products of granulation and recrystallization of larger crystals and they commonly form lenticular aggregates. Every textural gradation between the megacrysts and the lenticular aggregates has been observed. Under the microscope, the distinction between megacrysts and fine-grained aggregates is less clear than in the field: both consist of irregular intergrowths of perthite and plagioclase. The few megacrysts that retain their identity under

the microscope are invariably perthites. Both megacrysts and lenticular aggregates are referred to as 'augen'.

Ortho- and clinopyroxene occur in small (*e.g.* 0.5 mm) irregular, subhedral to anhedral grains. They may occur as separate grains, but commonly they form fine lamellar intergrowths consisting of roughly equal amounts of ortho- and clinopyroxene. Commonly, quartz, pyroxenes, hornblende, magnetite and the other accessories occur together in thin discontinuous stringers curving around the feldspar augen.

The rock is invariably well lineated, and commonly foliated, both by preferred form orientation of the feldspar augen and the quartz-mafic stringers.

Point counting of rock surfaces is practically the only method to determine quantitatively the mineralogical composition of most rocks (however, see Erdosh, 1967). It is also the cheapest method to test compositional homogeneity in terms of mineral abundances, and hence to approximate homogeneity in terms of some of the major elements.

THE PROBLEM

The concept of IC (Identity Change) number was introduced by Chayes (1956, Chap. 9) to predict sample size (number of thin sections to be analyzed) as a function of the precision required. It is clear from his investigation that the original concept of IC number was meant to apply only to roughly equigranular rocks in which three mineral components of approximately equal abundance account for the bulk of the rock. The present author, however, was faced with the problem of determining the modes of metamorphic rocks that are (1) inequigranular to strongly porphyritic/porphyroblastic, (2) made up of at least four mineral components of unequal abundances, (3) strongly lineated and foliated, and, to complicate matters, (4) without a definite break in grain size between 'megacrysts' and 'matrix'.

Neither the model studies of Solomon (1963), nor the empirical investigation of Shaw and Harrison (1955) suggest any solution to the problem. Thus, from the onset it was clear that the precision of the modal analyses could not be predicted from any directly measurable parameter. An attempt was made in the field to determine modes by point counting of entire outcrops with the help of a fishing net having rectangular meshes of 1 square inch. However, by staining potassium feldspar on slabs and thin sections of a few specimens, it soon became clear that the two most abundant minerals, potassium feldspar and plagioclase, could not be distinguished satisfactorily in the field, and that the three minerals pyroxene, amphibole and magnetite could only be distinguished from each other when they occur in uncommonly large grains. Also, the absence of a break in grain size between megacrysts and matrix made it impossible to estimate the abundance of megacrysts by the macroscopic method.

THE METHOD

The next best method, and the one finally adopted, seems to be the point-counting of slabs of hand specimens stained with sodium cobaltinitrite. In order to estimate both the modal composition and the precision of the estimate, two slabs of approximately 40 cm² each were cut from each specimen. The size of the slabs was determined by the fact that from most specimens it was possible to cut areas of that size, and not larger. The instruments used for counting include a binocular microscope with zoom lens (magnifying 7× to 30×, giving a lower limit of resolution of approximately 0.1 mm in routine counts), a specially constructed large mechanical stage (similar to the one described by Smithson, 1963), and a Swift point counter connected to the stage.

SPECULATIONS ABOUT ACCURACY

In reflected light, minerals display properties other than those normally used to identify them with a petrographic microscope. But identification very quickly becomes routine if, particularly in the beginning, frequent recourse is made to a polarizing microscope and immersion oils to substantiate a particular identification. Due partly to the nature of the illumination and partly to the limited enlargement, it continued to be necessary to count some minerals as a single component. This was the case, for instance, for orthopyroxene, clinopyroxene and hornblende. Furthermore, grains of diameters less than 0.1 mm cannot be identified, thus the abundance of any mineral that occurs in a significant amount in grains of so small a diameter will be estimated too low. A phenomenon similar to the Holmes effect (*e.g.* Chayes, 1956, Chap. 11) occurs in slab analysis. When dark minerals such as pyroxene or magnetite are embedded in water-clear quartz, it is difficult to see which mineral lies at the surface of the slab, and consequently the abundance of the dark minerals might be overestimated.

The best method to substantiate or discard some of these speculations on the accuracy of slab analysis lies in comparison of the results from the slabs with those from a number of thin sections cut from the same surface area. Possibly, the instrumental method recently proposed by Erdosh (1967) also might allow comparison.

THE COUNTING PLUS OPERATOR COMPONENT OF VARIANCE

Chayes (1956, Chaps. 4 and 5) has shown that the counting plus operator component of variance is smaller than, or at most equal to, binomial expectation if a systematic sample of points is counted over a thin section. In order to test whether this holds for the present method of analysis, duplicate counts were made of 18 slabs, the replicates being properly randomly distributed among the routine counts of many more

slabs. The results are given in Table 1, where:

$$\bar{X} = \sum_{j=1}^{18} (X_{j1} + X_{j2})/36,$$

where X_{jr} = estimate of abundance of a mineral component from analysis r of slab j . Each X_{jr} is based on a count of approximately 930 points (counting grid 2×2 mm).

$$S_0^2 = \sum_{j=1}^{18} (X_{j1} - X_{j2})^2/36.$$

$\sigma_0^2 = \bar{X}(100 - \bar{X})/930$: variance estimate of a binomial distribution.

$\chi^2 = (18 - 1)S_0^2/\sigma_0^2$, a statistic distributed as chi-square under the hypothesis that S_0^2 can be considered as an estimate of the population variance σ_0^2 .

H_0 = null hypothesis that $S_0^2 = \sigma_0^2$, H_a = alternative hypothesis that $S_0^2 \neq \sigma_0^2$.

The critical region of the test, *i.e.* the region of rejection of H_0 is given by $\chi^2 < \chi_{1-\alpha/2, 17}^2$, and $\chi^2 > \chi_{\alpha/2, 17}^2$, where α = the size of the critical region and has been selected at 0.05.

TABLE 1. COMPARISON OF SAMPLE VARIANCE AND BINOMIAL VARIANCE

Mineral	\bar{X}	S_0^2	σ_0^2	χ^2	Outcome of test
K-feldspar	41	1.41	2.60	9.22	Accept H_0
Plagioclase	37	1.88	2.51	12.73	Accept H_0
Quartz	11	0.98	1.05	15.87	Accept H_0
Mafic ^a	10	0.72	0.97	12.62	Accept H_0
Ore ^b	1.7	0.194	0.180	18.32	Accept H_0

^a Mainly ortho- and clinopyroxene, minor amphibole.

^b Mainly iron-titanium oxides.

A similar test, but now with the alternative hypothesis that $S_0^2 < \sigma_0^2$, shows that S_0^2 cannot be considered as being significantly smaller than σ_0^2 .

Therefore, the counting plus operator component of variance is not significantly different from the expected variance of a binomial distribution. A corollary of the outcome of the test is that the counting process is under control.

PRECISION AND THE WITHIN-SPECIMEN COMPONENT OF VARIANCE

The within-specimen component of variance (σ_1^2) was estimated for each major mineral by duplicate analysis of a number of specimens. At

first the author treated each specimen separately, and estimated σ_1^2 from single pairs of slabs. It soon became clear that, in accordance with statistical theory, the variances so estimated fluctuate very greatly. Rocks of presumably very similar composition and fabric gave very different estimates of variance. Thus it became necessary to group the specimens and calculate a pooled within-specimen component of variance.

The magnitude of σ_1^2 depends on a number of properties of the rock, the most important of which are: (a) modal composition: σ_1^2 changes with changing abundance, (b) grain size: σ_1^2 increases with increasing grain size, (c) the actual frequency distribution of the abundance in the rock: see Appendix, (d) vectorial fabric properties. Some remarks can be made about the last point: σ_1^2 is different for different orientations of the surface counted with respect to the foliation and lineation. It is clear that estimates of minimum variance are obtained from planes cut at right angles to the foliation. Furthermore, Chayes (1956, Chap. 2) has shown that, under certain restrictions, one of the edges of the counting grid should make an angle of between 30° and 50° with the trace of the foliation to get best (minimum bias) results. In lineated rocks, the smallest diameter of the grains commonly lies in the plane at right angles to the lineation. Therefore, one should not simply select any counting surface at right angles to the foliation, but the unique surface at right angles to the lineation (assuming the lineation lies in the foliation plane). Despite this the author has cut most of his specimens in two directions, one at right angles to the lineation, and one parallel to the lineation but at right angles to the foliation plane. In this way the estimates of pairs of slabs can be considered as independent, even for relatively small specimens and large grains. The results of the analysis of 59 specimens (118 slabs) of mangeritic augen gneiss are given in Table 2, where:

$$\bar{X} = \sum_{i=1}^{59} (X_{i1} + X_{i2}) / 118,$$

where X_{ij} = estimate of abundance of a mineral component in slab j of specimen i . Each X_{ij} is based on a count of approximately 500 points over an area of 40 cm^2 (counting grid $4 \times 2 \text{ mm}$).

$$S_a^2 = \sum_{i=1}^{59} (X_{i1} - X_{i2})^2 / 118.$$

$$\hat{\sigma}_1^2 \geq S_a^2 - 2\sigma_0^2 \quad (\text{See Appendix}). \quad (01)$$

The 95 percent-confidence intervals are calculated from

$$t_{\alpha/2} \cdot 59 \cdot S_a, \quad (02)$$

where $t_{\alpha/2} = \alpha/2$ -percentage point of the t -distribution, and is selected at 0.05. Thus, these are 95 percent-confidence intervals for the abundances of mineral components in a specimen of which one slab of 40 cm² has been counted under the conditions specified above.

TABLE 2. STATISTICS CONCERNING THE VARIABILITY IN HAND SPECIMENS OF MANGERITIC AUGEN GNEISS

Mineral	\bar{X}	S_a^2	95%-confidence interval for X_{ij} (\pm)	$\hat{\sigma}_1^2 \geq$
K-feldspar	40	11.79	6.9	6.59
Plagioclase	37	8.14	5.7	3.12
Quartz	11	4.56	4.3	2.46
Mafic	10	3.64	3.8	1.70

X_{ij} = estimate of abundance of a mineral in slab j of specimen i .

From the statistics calculated it is possible to predict the precision of a mode based on n slabs per specimen from the usual formula for the confidence interval of a mean:

$$t_{\alpha/2,k} \cdot S_a / \sqrt{n} \tag{03}$$

In the case of the mangeritic augen gneiss where two slabs of each of $k = 59$ specimens have been counted, the actual 95 percent-confidence intervals are $1/\sqrt{2}$ smaller than those given in Table 2.

If the width of a $100(1-\alpha)$ percent-confidence interval is indicated by $2H$,

$$2H = 2 \cdot t_{\alpha/2,k} \cdot S_a / \sqrt{n}. \tag{04}$$

Considering n as the unknown, and specifying the magnitude of H , (04) can be rearranged into

$$n = (S_a \cdot t_{\alpha/2,k} / H)^2, \tag{05}$$

where n = sample size required (number of slabs to be cut from a hand specimen) to obtain a $100(1-\alpha)$ percent-confidence interval equal to $2H$. Note that k = number of degrees of freedom of S_a^2 .

For instance, if it is decided that the width of the 95 percent-confidence intervals should be similar to those in Chayes' work on granites, as tabulated in his Figure 12 (Chayes 1956, p. 82), then

$$H^2 \leq 0.02\bar{X}^2, \tag{06}$$

and equation (05) becomes the inequality:

$$n \geq S_a^2 \cdot (t_{0.025,k})^2 / 0.02\bar{X}^2. \tag{07}$$

Keeping the count length constant at 500 points per 40 cm², and taking $k = 59$,

$$n \geq (\hat{\sigma}_1^2 + 2\sigma_0^2) \cdot (t_{0.025, 59})^2 / 0.02\bar{X}^2. \quad (08)$$

For potassium feldspar and plagioclase, $n = 2$ satisfies this condition: for quartz and mafic minerals, however, at least 8 slabs should be counted.

By doubling the count length, and thus reducing the counting component of variance by a factor of 2,

$$n \geq (\hat{\sigma}_1^2 + \sigma_0^2) \cdot (t_{0.025, 59})^2 / 0.02\bar{X}^2 \quad (09)$$

For potassium feldspar and plagioclase $n = 1$ would just suffice: for quartz and mafic minerals at least 6 slabs should be counted.

Therefore, in the particular case of the mangeritic augen gneiss, 1000 points should be counted, distributed over either one or two 40 cm² areas, if inequality (06) should hold. For quartz and mafic minerals either 8×500 or 6×1000 points should be counted. Now, the time required to count 2000 additional points (approximately 1 hour) is more than the time required to cut and stain two additional slabs (approximately 1/2 hour, if done in large batches). Thus, the choice between the alternatives is obvious, if a sufficiently large "hand" specimen is available.

THE BETWEEN-SPECIMENS COMPONENT OF VARIANCE

If it is known, or one is willing to assume that specimens are selected at random from a given type of rock, it is possible to estimate the between-specimen component of variance (σ_2^2), and construct confidence limits around the mean composition of that rock *type*. Also, the relative magnitudes of σ_0^2 , σ_1^2 , and σ_2^2 will indicate which level of sampling is responsible for most of the variability in the modes, and, therefore, at which level of sampling one should concentrate to reduce that variability.

In the problem encountered by the author, and probably by anyone sampling a geologically unknown and relatively poorly exposed area, this part of the problem is wrought with many difficulties. During field work, specimens were collected wherever a fresh piece of rock could be hammered off an outcrop, and observed outcrops are distributed on regularly spaced traverse lines throughout the rock body of interest. It is clear that the sample so obtained cannot be accepted as a probability sample (Cochran, *et al.*, 1954). This becomes even more obvious if one considers that (a) contact zones with other map units were studied in detail, and a disproportionately large number of specimens is from such zones, (b) commonly, when the field identification of the rock

type was in doubt, a specimen was collected, and (c) from large homogeneous-looking areas of exposure, only one, and rarely two specimens were taken. This is a very common procedure in reconnaissance mapping.

The author believes that a very rough indication of the *maximum* size of the between-specimen component of variance can still be obtained from such a sample, even if it were only to determine orders of magnitude of the variance ratios for the design of a subsequent, truly random-sampling plan. If it is accepted that the maximum size of σ_2^2 can be approximated, then rough estimates of the maximum size of confidence intervals for the average abundances of mineral components can be calculated for the rock type.

The results of the analysis of the 59 specimens of mangeritic augen gneiss are given in Table 3, where:

$$\bar{X} = \sum_{i=1}^{59} \bar{X}_i / 59,$$

where \bar{X}_i = estimate of mean abundance of a mineral component in specimen i . Each \bar{X}_i is based on a count of approximately 1000 points over an area of 2×40 cm².

$$S_b^2 = \sum_{i=1}^{59} (\bar{X} - \bar{X}_i)^2 / 59.$$

$$\hat{\sigma}_2^2 \leq S_b^2 - S_a^2 / 2 \text{ (See Appendix)} \tag{10}$$

The 95 percent-confidence intervals are calculated from

$$t_{\alpha/2, /59-1} \cdot S_b / \sqrt{59}, \tag{11}$$

where $\alpha=0.05$. These are the maximum sizes of the 95 percent-confidence intervals of mineral components in the mangeritic augen gneiss.

TABLE 3. STATISTICS CONCERNING THE VARIABILITY OF THE MANGERITIC AUGEN GNEISS

Mineral	\bar{X}	S_b^2	Order of magnitude of maximum size of	
			95%-confidence interval for \bar{X} (\pm)	$\hat{\sigma}_2^2$
K-feldspar	40	39.28	1.6	33
Plagioclase	37	27.53	1.4	23
Quartz	11	19.65	1.2	17
Mafic	10	14.01	1.0	12

APPENDIX

If m independent analyses (counts) are made of each of n randomly cut rock slabs from each of k randomly collected specimens, and if the assumptions underlying the normal analysis of variance are fulfilled (Cochran, 1947), it is well known that the following table for the analysis of variance can be written:

TABLE 4. ANALYSIS-OF-VARIANCE TABLE AND EXPECTED VALUES OF VARIANCES

Source of variation	Degrees of freedom	Mean square is an estimate of	Variance is an estimate of
Between specimens	$k-1$	$\sigma_0^2 + m\sigma_1^2 + nm\sigma_2^2$	$\sigma_0^2/mn + \sigma_1^2/n + \sigma_2^2$
Within specimens	$k(n-1)$	$\sigma_0^2 + m\sigma_1^2$	$\sigma_0^2/m + \sigma_1^2$
Between analyses	$kn(m-1)$	σ_0^2	σ_0^2

We have called σ_0^2 the counting plus operator component of variance, σ_1^2 the within-specimen component of variance, and σ_2^2 the between-specimen component of variance.

(1) Since, from previous theoretical considerations and experimental work it is known that σ_0^2 is, at most, equal to binomial expectation, the present experiment was not designed to estimate σ_0^2 , and throughout most of the analysis $m=1$. Nevertheless, the duplicate analyses of a few slabs allowed testing of the hypothesis that the estimator of σ_0^2 (*i.e.* S_0^2) is, in fact, not significantly different from binomial expectation. For ease of reference the symbol σ_0^2 has been used to indicate the expected size of the counting variance for a count length of 930 points per slab. Since the count length throughout most of the analysis was 500 points per slab, the counting variance is approximately twice as large as σ_0^2 , and has been written as $2\sigma_0^2$.

(2) S_a^2 is an estimator of $\sigma_0^2/m + \sigma_1^2$. Now, $m=1$, and the counting component of variance = $2\sigma_0^2$, therefore:

$$\hat{\sigma}_1^2 > S_a^2 - 2\sigma_0^2 \quad (01)$$

how much larger depends on the difference between the true size of σ_0^2 and the value from binomial expectation.

(3) S_b^2 is an estimator of $\sigma_0^2/nm + \sigma_1^2/n + \sigma_2^2$, and is thus equal to $S_a^2/n + \sigma_2^2$, where $n=2$. Therefore:

$$\hat{\sigma}_2^2 = S_b^2 - S_a^2/2 \quad (10)$$

Why $\hat{\sigma}_2^2$ should be regarded as a very rough estimate of maximum size has been discussed.

The abundance of the ore minerals has not been taken further than the first step of the statistical treatment (estimation of σ_0^2), mainly because the abundance is very unlikely to have a normal, or even symmetrical frequency distribution (e.g., Agterberg, 1961). The effect of truncation of the distribution at 0 percent is not negligible, unlike those of the major minerals. For the major minerals, the actual percentage points of the t -distribution are not exactly those for $\alpha=0.05$, but the discrepancy is probably very small. For the minor minerals, a lognormal, or rather an asymmetric beta distribution (Cramer, 1946) is a better approximation than the normal one. However, from an inspection of Table 1, it is clear that many more than 500 points per slab should be counted to give an estimate of the abundance of the ore minerals of a relative precision similar to that of the major minerals. This has not been done in the present analyses, and it seemed futile to take the very poor estimates of the ore minerals through a series of complicated calculations. It is noted here that the ore minerals form the only mineral component for which an increase in count length alone would materially improve the estimate of abundance.

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