

Mineral assemblages in stereographic projection

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Abstract

Stereographic projection of mineral assemblages in four-component systems can yield clearer illustration of compatibility than more conventional diagrams, while still permitting study of possible reactions between assemblages.

Graphical representation of mineral assemblages in terms of four components is commonly based on a compositional tetrahedron within which four-phase assemblages are portrayed by internal tetrahedra. In order to show the chemographic relations more clearly, one may project all four-phase assemblages with one common phase from the composition of that phase onto a convenient surface. For example, Thompson (1957) simplified the relationships between muscovite-bearing assemblages, as shown in an AKFM tetrahedron, by projection from the composition of muscovite onto the extended AFM plane. This works well if the composition of the common mineral lies on an edge of the tetrahedron but is less convenient if it lies on a face or within the tetrahedron. Moreover, if the tie line from the common mineral to some phase does not intersect the plane of projection, the composition of that phase will project to infinity.

However, there is no necessity for the surface of projection to be flat: the prime requirement is for a projection in which assemblages do not overlap.

Stereographic projection is well-suited to problems of this kind. No points have to be projected to infinity. If the composition of the common phase is placed at the center of the reference sphere, all tie lines from that composition will project to the surface of the sphere without overlap. In the case of a common mineral whose composition lies on a face of the tetrahedron, taking that face as the equatorial plane results in all tie lines from the common mineral projecting onto one hemisphere. For assemblages containing the common mineral, three-phase assemblages appear as segments of great circles and four-phase assemblages as spherical triangles (respectively equivalent to tie lines and "three-phase" triangles in a projection such as the AFM diagram).

To illustrate the use of stereographic projection, a set of assemblages with diopside in common is considered. (These assemblages are simplified after those in zoned skarn from Albion Basin, Utah, which will be the subject of a separate paper.) The assemblages with diopside are:

quartz–anorthite–tremolite
quartz–anorthite–grossular
quartz–grossular–calcite
grossular–calcite–corundum
corundum–calcite–spinel
calcite–spinel–forsterite.

With P , T , $\mu(\text{H}_2\text{O})$, and $\mu(\text{CO}_2)$ externally controlled, these assemblages can be depicted in the Al_2O_3 – CaO – MgO – SiO_2 tetrahedron of Figure 1a.

The same assemblages are more clearly shown in stereographic projection. Although such a projection could be based on the tetrahedron of Figure 1a, judicious choice of components permits simplification. In order to have diopside at the center of the reference sphere and corundum projecting to the center of the stereogram, one chooses a tetrahedron based on Al_2O_3 – CaO – MgO – Si_2O_4 , as in Figure 1b. Thus in the resultant stereogram all ties to corundum are radial, periclase–spinel–corundum–anorthite lie on a diameter, and of course, all alumina-free phases project to the circumference. The stereogram of the above assemblages is shown as Figure 2, on which the compositions of some other common minerals in calc-silicate rocks are also plotted.

It is immediately obvious that the four-phase assemblages are all compatible: something that is not readily apparent from Figure 1.

Not only are projections useful for portrayal of mineral assemblages, but also for deduction of possible reactions. This is true for stereographic projection as well as for the more conventional projections.

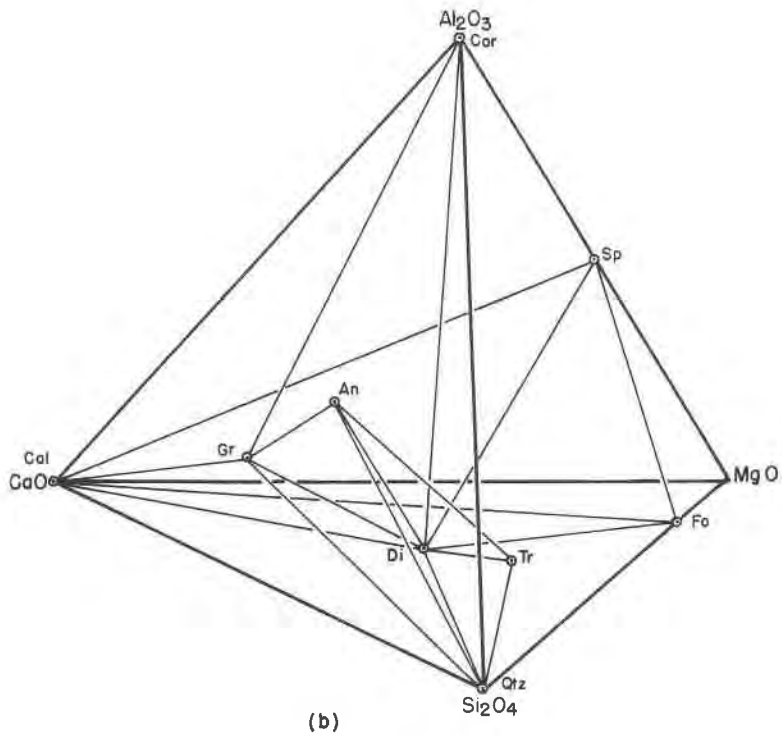
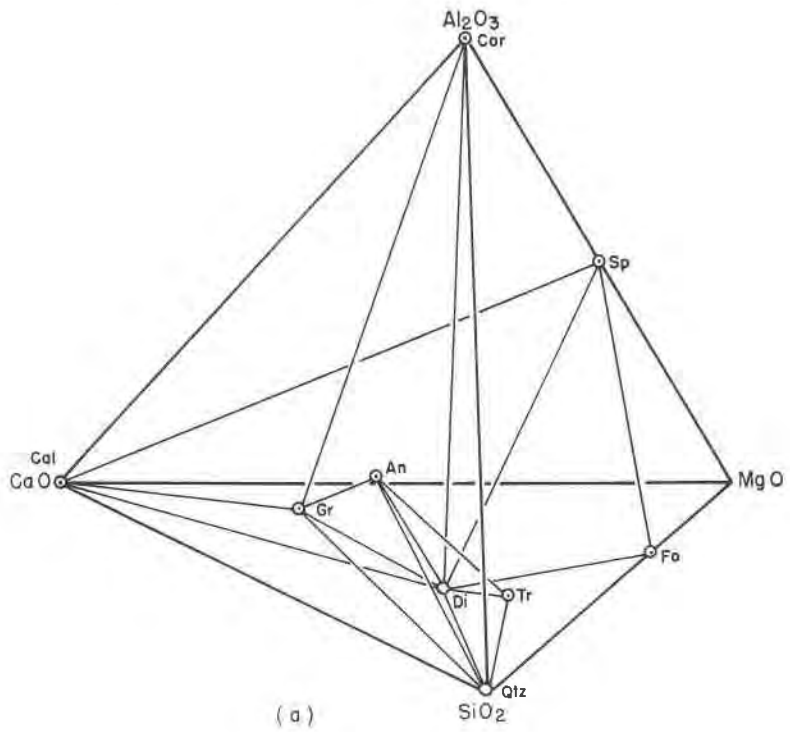


FIG. 1. Compositional tetrahedra illustrating the four-phase assemblages with diopside (see text), (a) based on $\text{Al}_2\text{O}_3\text{-CaO-MgO-SiO}_2$, (b) based on $\text{Al}_2\text{O}_3\text{-CaO-MgO-Si}_2\text{O}_4$. Abbreviations as for Fig. 2.

As examples, one can consider the following reactions involving (a) crossed tie lines (nonterminal), (b) an internal phase (terminal), (c) collinearity, and (d) degeneracy:

- (a) $4 \text{ grossular} + 3 \text{ tremolite} = 3 \text{ anorthite} + \text{corundum} + 15 \text{ diopside} + 3 \text{ H}_2\text{O}$ (a non-terminal reaction limiting the occurrence of grossular and tremolite in diopside-bearing assemblages)
- (b) $6 \text{ grossular} = 16 \text{ diopside} + 8 \text{ H}_2\text{O} = 3 \text{ idocrase} + 2 \text{ tremolite} + 7 \text{ quartz}$ (a terminal reaction limiting the occurrence of grossular in diopside-bearing assemblages)
- (c) $\text{anorthite} + \text{spinel} = \text{corundum} + \text{diopside}$ (a collinearity in projection)

(d) $\text{grossular} + 2 \text{ CO}_2 = \text{anorthite} + 2 \text{ calcite} + \text{quartz}$ (a degenerate reaction, not involving MgO).

The main disadvantage of using stereographic projection for analytical as opposed to illustrative purposes is that tie lines are arcs of great circles rather than straight lines. Subtle differences in curvature and collinearity of points on a curve are not easily perceived by eye: a stereonet is virtually essential.

In summary, stereographic projection of mineral assemblages can yield clearer illustration of compatibility than may otherwise be possible, and, although less readily, still permits study of possible reactions as in more conventional projections.

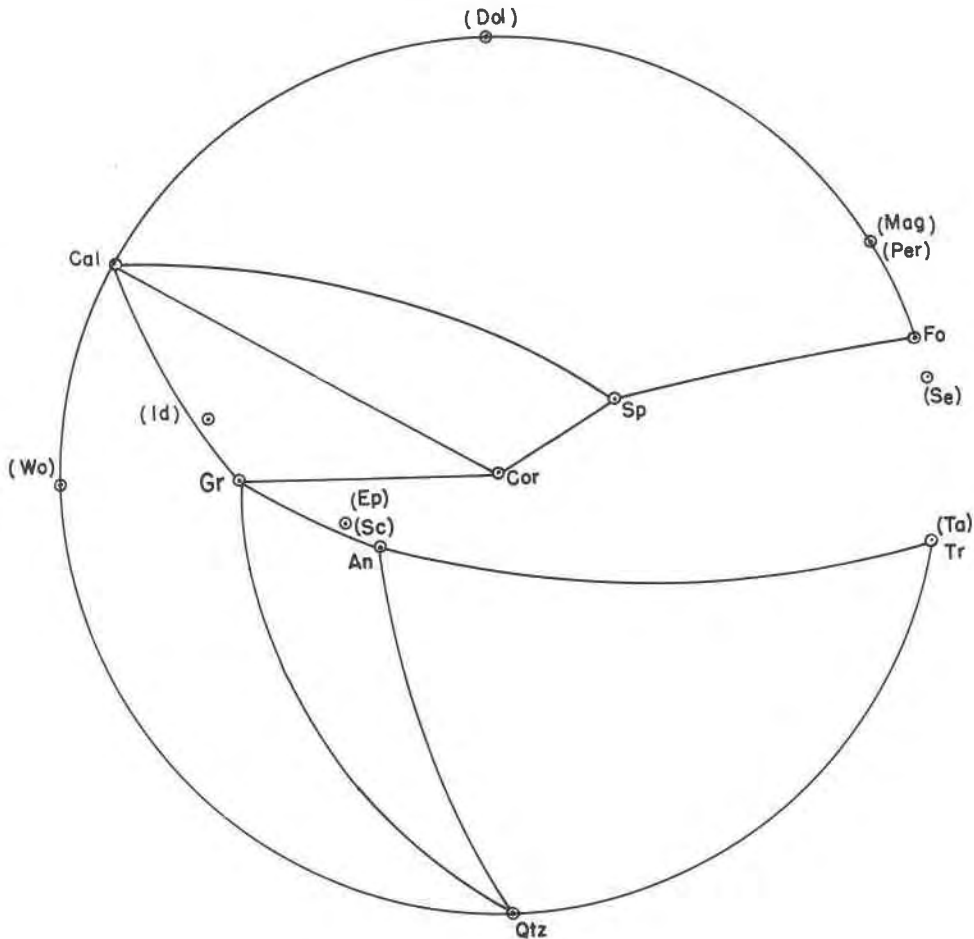


FIG. 2. Stereographic projection of the four-phase assemblages with diopside, based on the tetrahedron of Fig. 1b. Diopside lies at the center of the reference sphere, corundum at the north pole, and the basal face of the tetrahedron lies in the equatorial plane. Additional minerals are given in parentheses. An = anorthite, Cal = calcite, Cor = corundum, Di = diopside, Dol = dolomite, Ep = epidote, Fo = forsterite, Gr = grossular, Id = idocrase, Mag = magnesite, Per = periclase, Qtz = quartz, Sc = scapolite, Se = serpentine, Sp = spinel, Ta = talc, Tr = tremolite, Wo = wollastonite.

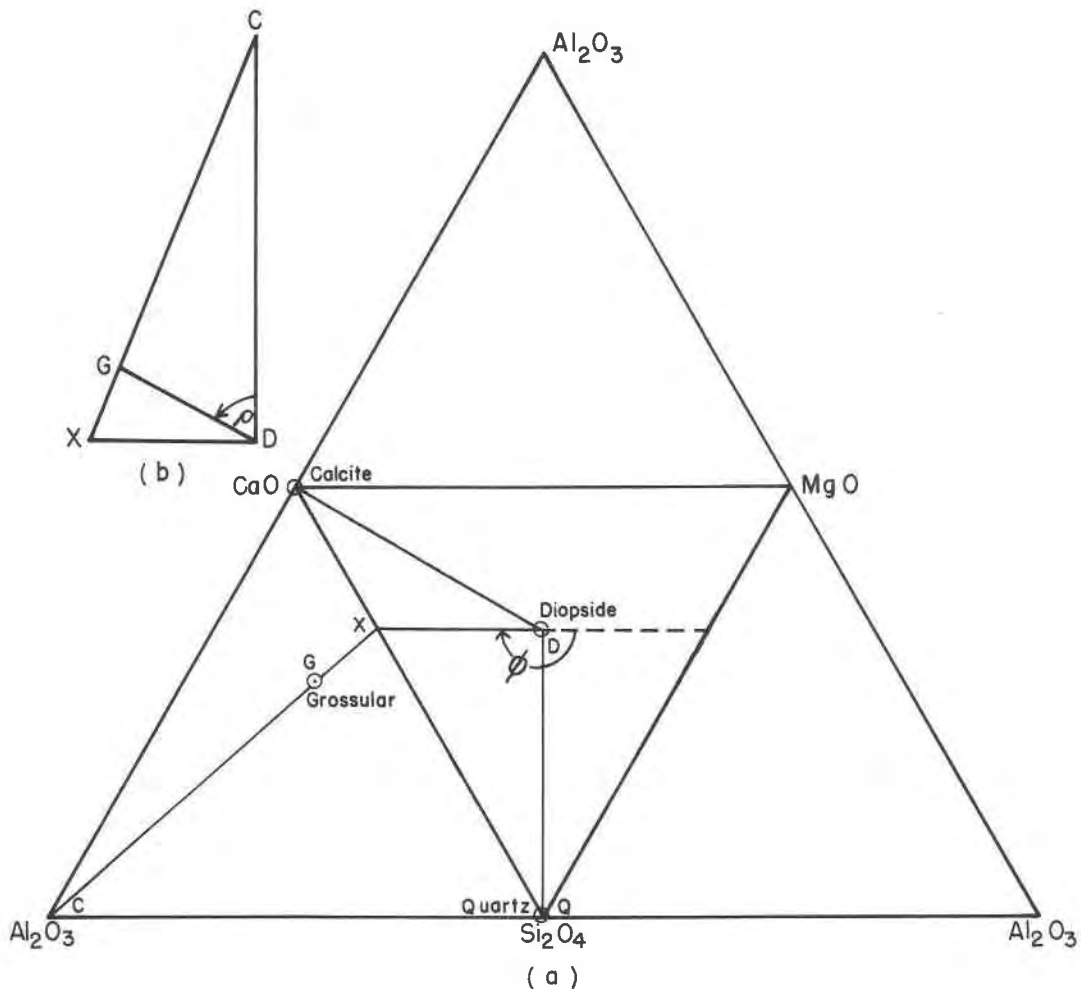


FIG. 3. Illustration of constructions for plotting mineral points (see appendix).

Appendix

Location of mineral points in stereographic projection

The Al_2O_3 - CaO - MgO - Si_2O_4 tetrahedron is shown in exploded form as Figure 3a. To describe the orientation of the tie line between diopside and the mineral in question we may use two angles, one being the azimuth of the tie line and the other the inclination from the vertical. Employing the convention of crystallography, these are ϕ and ρ respectively (e.g. Bloss, 1971).

In the present case, quartz lies at $\phi = 90^\circ$, $\rho = 90^\circ$, calcite at $\phi = -120^\circ$, $\rho = 90^\circ$, and so on. To determine the location, of say, grossular, consider its composition written as $3(CaO) (Al_2O_3) 1.5 (Si_2O_4)$, recalculated in terms of mole fractions as 0.545 (CaO), 0.182 (Al_2O_3), 0.273 (Si_2O_4) and plotted in Figure 3a as shown. To obtain ϕ , project the composition from

Al_2O_3 onto the base of the tetrahedron, by extending CG to X , whence $\phi = 90^\circ + \angle QDX = 180^\circ$. To determine ρ , draw the triangle XDC where DC is the altitude of tetrahedron (and $\angle XDC = 90^\circ$) as shown in Figure 3b. Then $\rho = \angle GDC = 61.5^\circ$. Of course, values of ϕ and ρ may be determined graphically, or calculated from the same construction.

In the general case, consider Figure 3 with Si_2O_4 , CaO , MgO , and Al_2O_3 replaced by a general set of components A , B , C , and D respectively, the place of diopside being taken by a mineral ABC . Retain the definitions of ϕ and ρ . To calculate ϕ and ρ for the tie-line between the mineral ABC and another mineral in the system $ABCD$, one can proceed as follows:

(1) Recalculate the composition of the mineral in terms of mole fractions A , B , C , and D . Let them be X_A , X_B , X_C , and X_D .

(2) In order to project this composition onto the

base of the tetrahedron, calculate

$$a \equiv \frac{X_A}{X_A + X_B + X_C} \quad b \equiv \frac{X_B}{X_A + X_B + X_C}$$

(The equivalent expression for "c" is not necessary because $a+b+c = 1$.)

(3) Note that $\sin 60^\circ = 0.866$, $\cos 60^\circ = 0.500$, the altitude of the tetrahedron is 0.943, and for the mineral ABC, $a=b=0.333$.

Then, let

$$\tan \theta \equiv \frac{0.866 (a - 0.333)}{(0.333 - b) - 0.5 (a - 0.333)}$$

If

$$b < 0.333, \quad \phi = \theta$$

$$b > 0.333, \quad \phi = 180^\circ + \theta$$

$$b = 0.333, \quad \phi = 120^\circ (a > 0.333)$$

$$\text{or } -60^\circ (a < 0.333)$$

$$(4) \quad \tan \rho = \frac{(a - 0.333) (1 - X_D)}{0.943 \sin \phi X_D} \\ = \frac{(0.333 - b) (1 - X_D)}{0.943 \sin (120^\circ - \phi) X_D}$$

(If $a = 0.333$, the second expression is used.)

e.g. for grossular, $X_D = 0.182$, $a = 0.333$, $b = 0.667$

$\therefore \tan \theta = 0$, $\theta = 0$, $\phi = 180^\circ$

and $\tan \rho = 1.840$, $\rho = 61.5^\circ$

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References

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