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The error associated with point-counting

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Abstract

Modal analysis by point-counting is discussed as a stochastic sampling process dictated by the influences of grid distance and grain transition probabilities. Four sampling processes are recognized (Bernoulli, Markov-Bernoulli, Markov-Markov, and Null-Markov) and the probability mass functions appropriate to each described. The counting normal distribution is found to be appropriate to sampling processes in which a Markovian influence occurs. Published methods of error estimation, based on the binomial distribution, are applicable only to the Bernoulli sampling process. A method of point-counting for unbanded rocks, in which data are collected as k sets of size N and the analytical error estimated in terms of the sample standard deviation, is recommended.

Introduction

Many systems of igneous rock nomenclature, such as those of Nockolds (1954), Chayes (1957), and Streckeisen (1967), use an estimate of the percentages of a rock's constituent minerals (the "mode") as a classificatory criterion. It is now common practice to determine the mode by a point-counting method, in which the identity of the mineral underlying each of a series of equally-spaced points on a grid is determined and tabulated. Thus, for the ith mineral, found at x_i points out of a total of N points counted, the best estimate of its percentage in a rock is $\hat{p}_i = 100x_i/N$. Of critical importance to the petrographer is the error associated with this estimate, and that is the subject of this paper.

One approach to estimating these errors for minerals in granitic rocks was presented in the classic work of Chayes (1956). In this approach, the total error (analytical error, *Ea*) is the sum of two essentially independent errors: that due to the counting process (counting error, *Ec*), and that due to the size of the sample taken from the rock (sampling error, *Es*). Chayes hypothesized that the counting error could be approximated adequately in terms of the binomial distribution, even though he pointed out that the Bernoulli criteria may not be satisfied during point-counting (Chayes, *op. cit.*, p. 91). The sample error represents that part of the analytical error not accounted for by the counting error. Bayly (1960, 1965) developed empirical relationships to estimate sample variance (square of *Es*) from grain size and sample area for some granitic rocks.

In this paper, we will approach the problem of estimating point-counting errors by an examination of the conditions that exist during a modal analysis, and then by a discussion of the appropriate stochastic sampling processes. Finally, we will recommend a method of point-counting for unbanded rocks that allows an unambiguous estimate of these errors.

The point count

The type of sampling process carried out during a point count depends on two unrelated factors, only one of which the operator can control. The mechanics of point-counting are such that the operator may select the distance between successive points on a grid (the grid distance). With respect to the rock's "grain size," one of two conditions exists: that in which the grid distance exceeds the grain size, or that in which grid distance is less than grain size. The second factor is the nature of the rock under investigation. Of the many rock properties that may affect a point count, such as texture, perhaps the most important is the distribution of the constituent crystals. Again, two possibilities are apparent: crystals are stochastically independent, or the composition of any crystal is related to the compositions of adjacent crystals. Discussions of point-counting have assumed the former

(Van der Plas and Tobi, 1965; Agterberg, 1974) without any supporting evidence. Contrary to this assumption, both Vistelius and his coworkers and Whitten and Dacy (1975) have shown that grain transitions along linear traverses in several granitic rocks possess a Markov property, *i.e.* the compositions of adjacent crystals have some degree of interdependence. As both factors interact in the modal analysis of a rock, four possible situations exist. They are:

(1) grid distance[1] > grain size, adjacent crystals independent;

(2) grid distance < grain size, adjacent crystals independent;

(3) grid distance < grain size, adjacent crystals dependent; and

(4) grid distance > grain size, adjacent crystals dependent.

Some appropriate distributions

Bernoulli process. The Bernoulli sampling process is a process in which (i) every observation in a set of N observations must be of one of two kinds, one of which is termed a success; (ii) the probability that a given observation will be a success is a fixed quantity, π; and (iii) all observations are stochastically independent (Hogg and Craig, 1970, p. 87). Such a process can be described statistically by the well-known binomial probability mass function.

Situation (1), in which grid distance exceeds grain size and adjacent grains are independent, represents a Bernoulli sampling process. A random sampling method is used commonly to fulfill the third Bernoulli condition, but grid sampling may achieve the same result in some cases. The third Bernoulli condition merely requires that no interdependence exists between the results of successive observations. If the grid distance exceeds grain size and the grains are independent, such a grid will meet this condition. Homogeneity of the rock ensures that the second Bernoulli condition is satisfied. Therefore, any sample cut from the rock for modal analysis will have the same composition as the rock itself, and the analytical error of any mineral will be precisely the error determined by the moments of the binomial distribution: *i.e.* no "sample error" (in the sense of Chayes) will exist. Thus, the analytical error of the ith mineral, expressed as a percentage, can be estimated by

$$\hat{E}a = 100\sqrt{(\hat{\pi}_i \cdot \hat{q}_i)/N},$$

where $\hat{\pi}_i = x_i/N$, $\hat{q}_i = (1 - \hat{\pi}_i)$, x_i is the number of points located on the ith mineral, and N is the total number of points counted. The chart prepared by Van der Plas and Tobi (*op. cit.*) provides a convenient method of estimating this error. Although we have discussed this situation in some detail, we suspect that it rarely occurs during modal analysis.

A variation of this process would occur if counts are made on a series of samples whose sizes are such that each sample is effectively homogeneous, but the rock is heterogeneous on a larger scale. In this event, the quantity π_i would vary from sample to sample. The mean ($\hat{\pi}^*_i$), averaged over all samples, would be the quantity of interest, and the results of the counts for the individual samples would have a mixed binomial distribution. The analytical error for $\hat{\pi}^*_i$ would then be greater than that calculated from the binomial distribution. The characterization of the point-counting process given by Chayes (1956) implies that $x_i = N\hat{\pi}^*_i$ should have a mixed binomial distribution and that the decomposition of analytical variance into counting and sample variances is a special case of analysis of variance.

A second variation of the Bernoulli process occurs if each sample area is increased to the point that sample-to-sample variation in π_i is small, but inhomogeneity exists within each sample. If π_i has a beta distribution throughout the rock for example, the results of counting would have a beta-binomial distribution, and the analytical error of $\hat{\pi}^*_i$ would be greater than that determined from the binomial distribution (Kendall and Stuart, 1969). The characterization developed by Bayly (1960, 1965) implies that the distribution of x_i should be transformed from a mixed binomial to a compound binomial as sample area, or grid distance, increases.

Markov-Bernoulli process. This sampling process occurs when situation (2) exists (grid distance < grain size, adjacent grains independent). During point-counting two types of transitions are involved: first, the transition from one point to the next on the same grain; and second, the transition across a grain boundary from one crystal to a crystal of the same mineral or to a crystal of a different mineral. We define a Markov-Bernoulli process as one in which the first transition is Markovian (*i.e.* has a non-zero probability) and the second is Bernoullian; this implies that the compositions of adjacent crystals are

[1] In the discussions that follow, the categorization of grid distance relative to grain size implies that grid distance is constant throughout an analysis and that the distance between successive traverses is equal to the grid distance.

Table 1. Summary of the results of a simulated Markov-Bernoulli sampling process

θ	k	$\bar{x}$	s^2	F
0.00	-	(4.00)*	(2.40)	(1.00)
0.10	1000	4.02	2.81	1.17
0.20	1000	3.98	3.74	1.56
0.30	500	4.15	4.83	1.79
0.40	500	4.07	5.05	2.10
0.50	500	4.00	5.96	2.48
0.60	500	3.91	7.81	3.25
1.00	-	(4.00)	(24.00)	(10.00)

θ = probability that two successive observations selected at random will fall on the same grain
k = number of sets with N=10 counted
$\bar{x}$ = mean of all counts in k sets with same θ value
s^2= sample variance of x
$F = s^2/N\pi(1-\pi)$ where $\pi = 0.4$
* values in parentheses calculated from the known exact distribution involved.

stochastically independent, but that consecutive observations might be made on the same crystal.

We have found that the exact probability distribution of the random variable, x_i, for a sampling process such as described here can be written as a series expansion, and have also investigated the distribution by Monte Carlo methods. A Monte Carlo experiment that we performed illustrates the characteristics of the process, irrespective of how distantly it may seem to relate to current point-counting practice. In the Monte Carlo experiment, the probability that a randomly selected point will fall on a mineral (π) was fixed at 0.40, and the probability that two successive observations selected at random will fall on the same crystal (θ) was varied from 0.00 to 0.60. For each θ value, k sets of counts with $N = 10$ were made. The value of k was either 500 or 1000, depending on the magnitude of θ. For each θ, the statistic $F = s^2/N\pi(1 - \pi)$, the ratio of the observed variance to that predicted by the binomial, Bin (N, π), distribution, was calculated. The results are given in Table 1. It can be seen that F and θ increase sympathetically and that the observed variance always exceeds that predicted by the binomial distribution when θ is greater than zero. The small value of N used in this experiment was for computational simplicity. Such a small value does not imply a small count length, for the much larger quantity kN actually represents the count length. The series expansion for the distribution of x_i is untidy, but reveals that the variance must be greater than $N\pi(1 - \pi)$ and is a monotonically increasing function of θ.

In practical terms, this experiment indicates that under the point-counting condition described here the analytical error should always exceed that predicted by the binomial distribution, but the difference should decrease as grid distance is increased (*i.e.* as the probability θ is decreased).

Markov-Markov process. In situation (3), where grid distance is less than grain size and adjacent crystals are dependent, the sampling process includes the two types of transitions discussed in the previous section. In this case, however, both transitions are Markovian. We will leave a discussion of an appropriate distribution until later, but it is important to note that the correlation of mineralogies of adjacent crystals may be either positive or negative. If the correlation is negative, this may result in an analytical error of less than that predicted for the appropriate π under the binomial distribution.

The subject of this discussion is restricted to the precision of modal analysis. We note in passing that the correlations induced by banding in inhomogeneous rocks can be investigated in terms of these models by multivariate autocorrelation methods. The problems of grid layout and analysis of the data are too complex to be considered here. Bartlett (1975) discusses some of the available techniques of sampling and analysis of spatial data.

Null-Markov process. Under the conditions in which grid distance is greater than grain size and the adjacent grains are dependent, only one type of transition occurs. This transition is Markovian in nature, and the influence of positive or negative crystal correlations will be strongly reflected in the deviation of the analytical error from that predicted by the binomial model.

The Markovian influence and the counting normal distribution

Theoretical discussion. In three of four processes described, the Markovian influence leads to statistics the distributions of which are very cumbersome to calculate, even if a simplified first-order Markov process is assumed. In real cases, the situation may be more complex. For example, points on a grid where grid distance is greater than grain size might not fall on contiguous grains. Further, if an observation is not of the ith mineral, it may be any of several different minerals, and the transition probabilities surely vary from mineral pair to mineral pair. In addition, rocks are not strictly equigranular, and even if they were a section would not cut each crystal to expose an equal area; hence it is possible that either θ (as defined previously) will vary from place to place within the sample or the number of grains between successive observations will vary. Thus, the real sampling

process may well be represented by a "(compound M-th order Markov)–(mixed N-th order Markov) process." Out of the foregoing discussion, three points become clear. (1) There is a central tendancy for point-count data, most simply expressed in terms of π_i, the proportion of the ith mineral in the rock. (2) There exists a clear possibility that observations may be correlated, and that this correlation may be positive or negative. (3) There is a great deal of disorder, of several kinds.

We have been accustomed to equating disorder with entropy in physical systems; exactly the same concept is applicable to information (Shannon, 1948). From this concept, maximum entropy probability distributions have been developed by Kagan *et al.* (1973), Kemp (1975), Brockman (1976), and others. The concept underlying these distributions is best expressed by Good's Principle of Maximum Entropy:

> "Let x be a random variable whose distribution is subject to some set of constraints. Then entertain the null hypothesis that the distribution is one of maximum entropy, subject to those constraints"

(Good, 1963, p. 912). With respect to count-type data, there are two apparent restraints: the average composition of the rock constrains the expectation of a count x_i of N points to be $E(x_i) = \pi_i N$, where π_i is the proportion of the ith mineral in the rock; and the various types of correlations in a given situation govern the variance of x_i.

Brockman (1976, p. 25–28) shows that under these constraints x has the distribution that he calls the counting normal distribution. This distribution has the form

$$P(x) = \beta e^{-(x-\alpha)^2/2\gamma},$$

where

$$\beta^{-1} = \sum_{j=0}^{N} e^{-(j-\alpha)^2/2\gamma},$$

and α and γ are selected so that $E(x) = \pi N$ and $\mathrm{Var}(x) = \sigma^2$. The moments of this distribution must be found by direct evaluation from the definitions. Alpha and γ must be found iteratively, but are of little real interest as it can be shown that the random variable $\hat{p} = 100x/N$ (which is what interests the petrographer) has a distribution that approaches the normal distribution very closely for moderate values of N, provided $E(x)$ is not too near 0 or N and that σ^2/N^2 is not too large. Then, if points are counted in sets of size N, and k such sets are counted,

$$\hat{\pi} = \bar{x}/N,$$

and to quote the results in percentage present, let $\hat{p} = 100\hat{\pi}$. Thus $E(\hat{p}) = 100\hat{\pi}$ and

$$\hat{\mathrm{Var}}(\hat{p}) = \{10^4/(k^2N^2)\}\{\Sigma(x^2) - (\Sigma x)^2/k\}$$

In practical applications we replace $\hat{\mathrm{Var}}(\hat{p})$ by

$$s^2p = k\,\hat{\mathrm{Var}}(\hat{p})/(k-1) = \{10^4/[(k-1)N^2k]\}\{\Sigma(x^2) - (\Sigma x)^2/k\}$$

Table 2. Point-count data for two granitic rocks

Texas Pink Granite (N=16, k=37)					
	$\hat{p}$	s^2	σ_b^2	F	X^2
Quartz	26.52	1.97	3.29	0.599	21.564
Kfeldspar	47.30	2.48	4.21	0.589	21.204
Plagioclase	20.10	1.82	2.71	0.672	24.192
Biotite	6.08	0.45	0.96	0.466	16.776

Oban River Leucoadamellite (N=16, k=36)					
	$\hat{p}$	s^2	σ_b^2	F	X^2
Quartz	33.68	2.91	3.88	0.750	26.250
Kfeldspar	36.98	2.59	4.05	0.639	22.365
Plagioclase	22.57	1.88	3.03	0.620	21.700
Biotite	6.77	0.47	1.10	0.427	14.945

$\hat{p}$ = estimated percentage of mineral present
s^2 = sample variance of $\hat{p}$
σ_b^2 = variance of $\hat{p}$ estimated from the binomial distribution
$F = s^2/\sigma_b^2$
$X^2 = (k-1)F$

An experiment. In an attempt to test the applicability of the counting normal distribution, point-count data were collected on two nonfoliated, coarse-grained granitic rocks: the Texas Pink Granite (TPG) and the Oban River Leucoadamellite (ORLA). In each case, a grid with grid distance just greater than the largest crystal was used and either 37 (TPG) or 36 (ORLA) sets of counts with $N = 16$ were made on each rock, as described in the section "A recommended method of modal analysis." The data are given in Table 2.

It is obvious that the conditions of data collection represent either a Bernoulli or a Null-Markov sampling process. A test for the "fitness" of observed variances to the binomial model is the well known F test. Thus, a variance estimate, s^2, computed from k

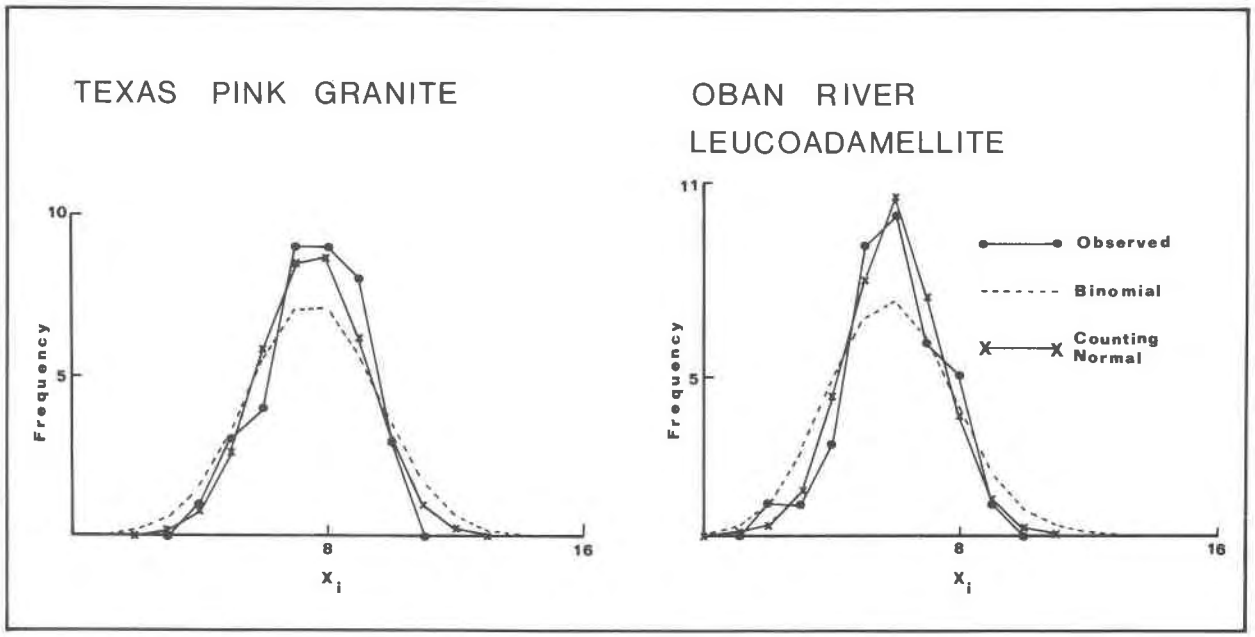

Fig. 1. Comparison of the observed variation in counts for alkali feldspar (x_i) with those predicted from the binomial and counting normal distributions for two granitic rocks. $N = 16$ for both cases, and $k = 37$ and 36 for the Texas Pink Granite and the Oban River Leucoadamellite respectively.

sets of measurements can be compared with the binomial variance, σ_b^2, by

$$F = s^2/\sigma_b^2 \sim F(k-1, \infty).$$

A more powerful test can be constructed by combining the individual F statistics. It is well known that

$$X^2 = (k-1)F(k-1, \infty) \sim \chi^2(k-1).$$

In addition, if the variances of m different minerals in the k sets are estimated, then approximately

$$\{(m-1)/m\} \sum_{i=1}^{m} X^2 \sim \chi^2(m-1)(k-1),$$

a form of the additivity theorem for chi-squares (Hogg and Craig, 1970, p. 159), modified to allow for non-independence. For large values of $(k-1)(m-1)$,

$$Z = \sqrt{2\chi^2} - \sqrt{2(k-1)(m-1) - 3} \sim N(0, 1)$$

approximately (Fisher and Yates, 1948, p. 33). To test the possibility that the data represent a simple Bernoulli process, we tested the null hypothesis $Ho: E(s^2) = \sigma_b^2$ against the alternative $Ha: E(s^2) \neq \sigma_b^2$. Using the sum of X^2 from the variance data (Table 2) and computing the Z scores led to a Z of -3.398 and -3.079 for the Texas Pink Granite and the Oban River Leucoadamellite, respectively. Thus, for each rock the null hypothesis can be rejected at a confidence level of 0.99. Brockman (1976, p. 84–107) has discussed procedures for testing the fitness of the counting normal.

A non-quantitative test of the applicability of the counting normal distribution to these data was made by determining the predicted number of points expected to fall on each mineral in a count of $N = 16$ for k sets, using the binomial and counting normal distributions and comparing them graphically with the counts obtained during point-counting. In all cases the predicted results from the counting normal were closer to those obtained by the counting process than those from the binomial. Figure 1 shows the results for alkali feldspar in each rock.

A recommended method of modal analysis

Although our characterizations of sampling processes are theoretical in nature and, hence, potentially applicable to all types of rocks, we will restrict our discussion of the practical aspects of point-counting to the modal analysis of unbanded rocks.[2]

It is common practice in point-counting to count N points (usually 1000 or more) as one set. From these data, the percentages of constituent minerals are cal-

[2] The interested reader is referred to Chayes (1956, Ch. 2) for a discussion of the problems associated with modal analysis of banded rocks.

culated and errors allocated to the results, using either the graph of Van der Plas and Tobi (1965) or the relationship deduced by Bayly (1965). As we have pointed out previously, the graph of Van der Plas and Tobi is applicable only to those cases in which the point count represents a simple Bernoulli sampling process. Even if the operator selects a grid distance greater than the diameter of the largest grain in the sample, he has no assurance that the observations are stochastically independent or that π is constant throughout the sample. Thus, the "analytical error" so calculated may be a very poor approximation.

Bayly's method of estimating analytical error results from empirical observations on a few granitic rocks. There is no reason to believe that his expression for sample variance is applicable to any rock other than those used in his experiment. In addition, the expression was developed from data on several rocks in which the calculated sample variance was negative (Bayly, 1960, Table 1; Bayly, 1965, Table 2), a meaningless statistical entity. He noted this anomaly (Bayly, 1965, p. 209) and pointed out that the binomial distribution may overestimate counting variance; nevertheless, he continued to use the binomial distribution to estimate this component in his data analysis.

As an indication of the approximate nature of error estimations based on the two approaches described above, the observed errors for the Texas Pink Granite and the Oban River Leucoadamellite (Table 2) have been compared with those predicted from the work of Bayly (1965) and Van der Plas and Tobi (1965). The error for each mineral is quoted as twice the standard error (which gives a confidence interval of $\hat{p} \pm 2s$ with a level of confidence of 0.95) in Table 3. An examination of the table reveals that the observed errors are less than the errors predicted by both methods in all cases.

The counting normal distribution appears to best describe the observed distribution of minerals in sampling situations that possess the Markov property. Although the two rocks used in the experiment give analytical errors less than those predicted by the binomial distribution (Tables 2 and 3), we do not wish to imply that any sampling process that includes a Markovian component (*i.e.* situations 2 through 4) will behave in a similar manner. The counting normal distribution allows for variances of less than or greater than those estimated by the binomial (Brockman, 1976). No prediction as to the efficiency of a sampling process can be made prior to the actual modal analysis, unless tests for grain transition probabilities are made using the grid distance of the proposed analysis. It should be further emphasized that collection of data as one large set (as is commonly the case) precludes any realistic estimate of analytical error whatsoever.

Table 3. Some estimations of analytical error for two granitic rocks

Texas Pink Granite				
	$\hat{p}$	2s	2σ	2sa
Quartz	26.52	2.81	3.63	3.97
Kfeldspar	47.30	3.15	4.10	8.61
Plagioclase	20.10	2.70	3.29	3.89
Biotite	6.08	1.34	1.96	2.08

Oban River Leucoadamellite				
	$\hat{p}$	2s	2σ	2sa
Quartz	33.68	3.41	3.94	4.06
Kfeldspar	36.98	3.22	4.05	4.22
Plagioclase	22.57	2.74	3.48	3.59
Biotite	6.77	1.37	2.09	2.12

$\hat{p}$ = percentage of mineral present
2s= twice sample standard deviation (observed - from Table 2)
2σ= twice standard error as determined from binomial distribution (after Van der Plas and Tobi, 1965)
2sa = twice analytical error as determined by the method of Bayly (1965)

This discussion leads to the inevitable conclusion that analytical errors in point-counting cannot be estimated realistically from established formulae, for a variety of reasons. Chief among these is the sampling process used. Although the sampling process used in an individual point count may be recognized, the moments of some appropriate distributions (such as the counting normal) are difficult to calculate. It is probably more common that the operator does not know what type of crystal transitions exist in the rock; thus, he is unaware of the sampling process being used. To circumvent these problems, we recommend that point counting of unbanded rocks be carried out in the following fashion.

(a) Determine the largest diameter of all crystals in the thin section or rock slab to be analyzed, and fix grid distance so as to slightly exceed this value. The grid distance is to be kept constant throughout the analysis and is used to set both the distances between successive points along a traverse and the distance between successive traverses.

(b) Collect the data as a series of k sets, each set containing a total of N points. Each set is collected on a square grid of $\sqrt{N}$ traverses, each traverse contain-

ing $\sqrt{N}$ points. Thus, to count approximately 1000 points, 30 sets of $N = 36$ would be appropriate. If analyzing a thin section, locate the position of the first set by using a random number table to get x,y coordinates for the counting stage. For the analysis of a rock slab, locate the position of the first set by placing a transparent overlay grid on the surface with eyes closed. The positions of all other sets are located such that (i) no piece of the sample is covered by more than one set, and (ii) the outline of the grid for each set is at least two grain diameters away from the outlines of all other sets.

(c) Estimate the percentage of the ith mineral by

$$\hat{p}_i = 100 \left\{ \sum_{j=1}^{k} (x_j)/kN \right\},$$

where x_j is the number of points falling on the ith mineral in set j.

(d) Determine the variance of the ith mineral, expressed as a percentage, from

$$s^2\hat{p}_i = \{10^4/[(k-1)N^2k]\}\{\Sigma (x^2) - (\Sigma x)^2/k\}$$

(e) Determine the confidence interval for each mineral at an approximate 95 percent level of confidence as $\hat{p}_i \pm 2\sqrt{s^2\hat{p}_i}$.

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