

The crystal structure of CaGeO₃ perovskite and the crystal chemistry of the GdFeO₃-type perovskites

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Abstract

A single-crystal X-ray study indicates that the perovskite form of CaGeO₃ is orthorhombic (*Pbnm*), and isotypic with GdFeO₃, although it was previously reported as a cubic form; the cell dimensions are $a = 5.2607(6)\text{\AA}$, $b = 5.2688(10)$, $c = 7.4452(15)$ and $V = 206.36(6)\text{\AA}^3$ ($Z = 4$; $D_{\text{calc}} = 5.17\text{ g/cm}^3$). The crystal structure is close to that of SmAlO₃. Although the unit cell of CaGeO₃ is pseudocubic, the structure itself is very distorted through the tilting and distortion of polyhedra. The oxygen polyhedra are less tilted and less distorted than those of other GdFeO₃-type perovskites. The structural deformation of the GdFeO₃-type perovskite is determined primarily by the size-ratio of two kinds of cation occupying A and B sites. Some structural characteristics such as O(2)–O(2)–O(2) and A–O(1)–B angles and bond-length distortions exhibit systematic relationships as a function of the observed tolerance factor which is newly defined here. A strong correlation between the Goldschmidt tolerance factor and the observed tolerance factor has made possible some predictions for GdFeO₃-type perovskites.

Introduction

The perovskite structure, with general formula ABO₃, consists of a framework of B octahedra that share corners with each other and triangular faces with cuboctahedra containing A cations in twelve coordination. For compositions in which their constituent atoms are not of the ideal relative size, many distorted types of perovskite may replace the ideal structure. Some metasilicates and metagermanates are known to crystallize as perovskites in cubic (SrGeO₃; above 50 kbar), hexagonal (BaGeO₃; above 95 kbar), and orthorhombic phases (MgSiO₃; above 300 kbar) (Shimizu *et al.*, 1970; Liu, 1976a; Yagi *et al.*, 1978; Ito and Matsui, 1978). The high-pressure transformation of a metasilicate to a perovskite form is geophysically important for interpreting seismic wave velocities in the Earth's lower mantle. Also, germanates are useful as structural analogs of common silicate minerals at high pressures because new dense phases frequently exist at much lower pressures than the corresponding isotypic silicates.

CaGeO₃ is a typical example of such germanates and is known to transform from the wollastonite through a garnet-like structure to the perovskite structure at more than 65 kbar and 900°C (Susaki and Akimoto, private comm.). CaGeO₃ has previously been indexed as cubic perovskite with $a = 3.723\text{\AA}$ (Ringwood and Major, 1967), although Prewitt and Sleight (1969) reported a doubling of the unit cell ($a = 7.448\text{\AA}$). In order to confirm the cell dimension and the space group of CaGeO₃ perovskite, we

examined a single crystal using X-ray diffraction techniques, to refine its crystal structure, and to make a systematic study of crystallographic correlations among the GdFeO₃-type perovskites. A preliminary communication on this result has been reported (Sasaki *et al.*, 1981).

Experimental

Sample

A polycrystalline specimen of CaGeO₃ perovskite was synthesized by hot-pressing CaGeO₃ wollastonite powder in squeezer solid-media apparatus for two hours at $P = 100\text{ kbar}$ and at $T = 1000^\circ\text{C}$ (see details in Liebermann *et al.*, 1977). After sintering at elevated pressure and temperature, the run was slowly cooled (20–60 minutes) to room temperature after which the pressure was released. Examination of the recovered specimen using a polarizing microscope and X-ray powder diffraction analyses confirmed that it was a single phase with the perovskite structure. A single crystal of parallelepiped shape and dimensions, $0.14 \times 0.10 \times 0.07\text{ mm}$ was extracted from the polycrystalline aggregate and prepared for single-crystal X-ray diffraction study.

Space group determination

The unit cell and space group of CaGeO₃ perovskite were determined with the aid of precession and Weissenberg photographs and intensity data collected with a Picker four-circle diffractometer. The space group deter-

mination was more difficult than expected, because the unit cell is pseudocubic, dimensionally and in intensity distribution for strong reflections. The cell dimensions determined with the four-circle diffractometer (graphite monochromatized $\text{MoK}\alpha_1$; $\lambda = 0.70926\text{\AA}$) are as follows:

$$\begin{aligned} a &= 5.2607(6)\text{\AA} \\ b &= 5.2688(10) \\ c &= 7.4452(15) \\ V &= 206.36(6)\text{\AA}^3 \\ Z &= 4 \text{ (formula units/cell)} \\ D_{\text{calc}} &= 5.17 \text{ g/cm}^3 \end{aligned}$$

The cell dimensions have the relation $a \cong b \cong \sqrt{2} a_p$ and $c \cong 2a_p$ (a_p : a pseudocubic subcell parameter), maintaining the characteristic of GdFeO_3 -type perovskites. In order to emphasize the difference from cubic perovskite, the precession photograph for $hk1$ is shown in Figure 1. If this crystal were cubic with $a = 3.7\text{\AA}$, no diffraction spots would appear on photographs for $l = 2n + 1$ on the orthorhombic cell. The intensity distribution indicates orthorhombic symmetry. The systematic absences are: no conditions on hkl ; $k = 2n$ on $0kl$; $h + l = 2n$ on $h0l$; no conditions on $hk0$; ($h = 2n$) on $h00$; ($k = 2n$) on $0k0$; ($l = 2n$) on $00l$. The possible space group is either D_{2h}^{16} - $Pbnm$ or C_{2v}^2 - $Pbn2_1$. On the basis of no evidence for noncentro-

symmetry and the results of least-squares refinement of the structure, the space group is most likely $Pbnm$. Although some weak reflections apparently violating the above determination could be detected, they were reflections caused by a small twinned part of the crystal, as will be described in the following section.

Twining

Preliminary investigation using the four-circle diffractometer and long-exposure precession photographs showed the presence of two types of reflections inconsistent with the GdFeO_3 -type perovskite of space group $Pbnm$. The first type is of reflections on the reciprocal points at half-integral coordinates, such as $1/2\ 5/2\ 0$, $1/2\ 5/2\ 2$, $1/2\ 3/2\ 3$, $1/2\ 1/2\ 4$, $1/2\ 5/2\ 4$, $3/2\ 3/2\ 6$, and $1/2\ 5/2\ 6$. Another is of the 031 and 013 reflections that apparently violate the extinction rule for the b -glide plane. All of the above reflections, however, can be clearly explained by the existence of twinning related by the following transformation matrices:

$$\begin{pmatrix} a^{*'} \\ b^{*'} \\ c^{*'} \end{pmatrix} = \begin{pmatrix} 1/2 & -1/2 & 1 \\ 1/2 & -1/2 & -1 \\ 1/2 & 1/2 & 0 \end{pmatrix} \begin{pmatrix} a^* \\ b^* \\ c^* \end{pmatrix}$$

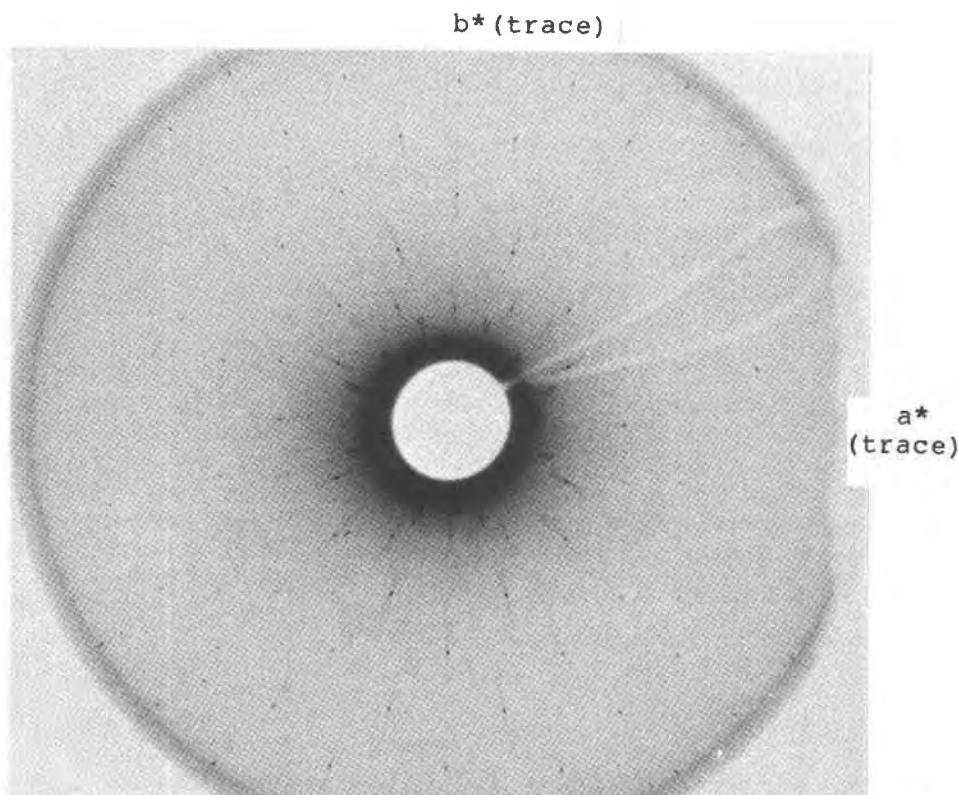


Fig. 1. Precession photograph of the $hk1$ net for CaGeO_3 perovskite, taken with $\mu = 25^\circ$ (non-filtered $\text{MoK}\alpha$ radiation; 35 kV and 15 mA for 336 hours). If this perovskite were cubic with $a = 3.7\text{\AA}$, no diffraction spots would appear for this net. The symmetry mmm (orthorhombic) is required for Laue group.

Table 1. Fractional atomic coordinates and thermal parameters ($\times 10^4$) of CaGeO₃ perovskite obtained in least-squares refinements: (1) after and (2) before the correction of intensities diffracted by the twinned crystal. The β_{ij} 's are defined by $\exp[-(h^2\beta_{11} + k^2\beta_{22} + l^2\beta_{33} + 2hk\beta_{12} + 2hl\beta_{13} + 2kl\beta_{23})]$.

(1) After correction:				
	Ca	Ge	0(1)	0(2)
x	-0.0051(3)	0	0.0606(9)	0.7157(6)
y	0.0283(2)	1/2	0.4916(8)	0.2830(6)
z	1/4	0	1/4	0.0320(5)
β_{11}	46(2)	21(1)	36(7)	49(5)
β_{22}	66(2)	25(1)	66(8)	48(5)
β_{33}	30(1)	10(1)	15(4)	24(2)
β_{12}	-9(2)	0(1)	4(6)	-16(5)
β_{13}	0	-0(1)	0	1(3)
β_{23}	0	-0(.4)	0	17(3)
B(equiv)	0.64(2)	0.24(1)	0.49(8)	0.53(5)
Ref. (meas)	689			
(used)	412			
R	0.035			
R(wt)	0.037			
(2) Before correction:				
	Ca	Ge	0(1)	0(2)
x	-0.0051(5)	0	0.0587(16)	0.7159(10)
y	0.0273(3)	1/2	0.4911(17)	0.2829(9)
z	1/4	0	1/4	0.0321(9)
β_{11}	46(4)	24(3)	42(17)	56(13)
β_{22}	68(4)	31(3)	118(23)	49(12)
β_{33}	24(4)	11(3)	22(17)	36(11)
β_{12}	-6(7)	-0(3)	2(17)	-20(11)
β_{13}	0	0(2)	0	11(10)
β_{23}	0	0(1)	0	1(10)
B(equiv)	0.60(6)	0.29(4)	0.75(27)	0.64(17)
Ref. (meas)	689			
(used)	343*			
R	0.035			
R(wt)	0.052			

*The reflections greater than 4 σ were used in refinement.

$$\begin{pmatrix} h' \\ k' \\ l' \end{pmatrix} = \begin{pmatrix} 1/2 & -1/2 & 1/2 \\ 1/2 & -1/2 & -1/2 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} h \\ k \\ l \end{pmatrix}$$

There is no doubling of spots or peaks for any reflections that overlap. Since the reflections at half integral reciprocal points do not overlap each other, the comparison in intensities of such reflections gave roughly the volume ratio of 1:0.14(1).

Data collection and structure refinement

The ω -2 θ scan technique of the four-circle diffractometer was used to collect at room temperature the intensity data up to $2\theta = 80^\circ$ in the bisecting mode. Each reflection was scanned at the appropriate speed with a maximum measurement time of four minutes; the scan width on 2θ ($^\circ$) was $2.0 + 0.7 \tan\theta$. A standard reflection used was stable within $\pm 1.5\%$ of the integrated intensity during data collection. Of a total of 689 reflections measured, the reflections less than three times the calculated standard deviation were omitted in refinements. A set of intensities was corrected for Lorentz and polarization factors and a prismatic absorption correction was applied using the program ACACA (Wuensch and Prewitt, 1965). The linear

Table 2. Interatomic distances (\AA) and angles ($^\circ$) in the Ge octahedron. Standard deviations are in parentheses. Square brackets, [], show the value calculated for the ideal cubic perovskite with the same polyhedral volumes.

Ge i - 0(1) i	x 2	1.889(.9)
- 0(2) v	x 2	1.898(3)
- 0(2) viii	x 2	1.889(3)
Mean <Ge - O>		1.892 [1.861]
Bond-length distortion, Δ_B		0.005
0(1) i - 0(2) v	x 2	2.684(5)
- 0(2) iv	x 2	2.665(5)
- 0(2) i	x 2	2.671(5)
- 0(2) viii	x 2	2.677(5)
0(2) v - 0(2) viii	x 2	2.659(4)
- 0(2) iv	x 2	2.696(4)
Mean <O - O>		2.675 [2.632]
0(1) i - Ge i	- 0(2) viii	90.3(2)
0(1) i -	- 0(2) v	90.3(2)
0(2) v -	- 0(2) viii	89.2(1)
0(2) i -	- 0(2) viii	90.8(1)

absorption coefficient is 177.1 cm^{-1} and the transmission factors for this crystal varied from 0.19 to 0.33.

All parameters were refined simultaneously using the full-matrix least-squares program, RADY, which is a modified version of RADIEL (Coppens *et al.*, 1979). The

Table 3. Interatomic distances (\AA) and angles ($^\circ$) with standard deviation in parentheses for cuboctahedron. Square bracket, [], shows the value calculated for the ideal cubic perovskite with the same polyhedral volume.

Ca i - 0(1) i	x 1	2.346(5)	
- 0(1) iv	x 1	2.465(5)	
- 0(1) iiii	x 1	2.929(5)	
- 0(1) v	x 1	2.849(5)	
- 0(2) i'	x 2	2.352(4)	
- 0(2) iv'	x 2	2.568(4)	
- 0(2) v	x 2	2.597(4)	
- 0(2) viii	x 2	3.069(4)	
Mean <M - O>		2.647 [2.632]	
Bond-length distortion, Δ_A		9.40	
0(1) i	- 0(1) v	x 2	3.303(7)
0(1) v	- 0(1) iiii	x 2	4.197(7)
- 0(2) vii	x 4	2.684(5)	
- 0(2) ii'	x 4	2.665(5)	
0(1) iv	- 0(2) iiii'	x 4	2.671(5)
- 0(2) vi	x 4	2.677(5)	
0(2) i'	- 0(2) ii'	x 2	3.246(5)
0(2) viii	- 0(2) viii	x 2	4.199(6)
0(2) i'	- 0(2) iv'	x 4	2.659(4)
0(2) iiii'	- 0(2) vi	x 4	2.696(4)
0(2) i'	- 0(2) iiii'	x 4	4.196(5)
- 0(2) vii	x 4	4.571(5)	
- 0(2) v	x 2	3.256(4)	
0(2) iv'	- 0(2) viii	x 2	4.251(4)
0(1) i	- Ca i - 0(1) iv	x 1	86.67(16)
0(1) iv	- 0(1) iiii	x 1	101.83(15)
0(1) iiii	- 0(1) v	x 1	93.18(13)
0(1) v	- 0(1) i	x 1	78.31(15)
0(2) v	- 0(2) vi	x 2	107.89(11)
0(2) iiii'	- 0(2) iv'	x 2	78.41(12)
0(2) i'	- 0(2) v	x 2	82.13(11)
0(2) iv'	- 0(2) viii	x 2	97.51(10)
0(2) viii	- 0(2) viii	x 2	86.34(10)
0(2) i'	- 0(2) ii'	x 2	87.27(13)
0(2) v	- 0(2) viii	x 2	55.23(9)
0(2) viii	- 0(2) i'	x 2	57.85(10)
0(2) i'	- 0(2) iv'	x 2	65.26(11)
0(2) iv'	- 0(2) v	x 2	62.92(10)
0(1) i	- 0(2) vii	x 2	57.42(9)
0(1) v	- 0(2) vii	x 2	53.79(8)
0(1) v	- 0(2) ii'	x 2	60.77(10)
0(1) iiii	- 0(2) ii'	x 2	59.58(10)
0(1) iiii	- 0(2) iiii'	x 2	57.56(10)
0(1) iv	- 0(2) iiii'	x 2	64.06(11)
0(1) iv	- 0(2) vi	x 2	63.80(9)
0(1) i	- 0(2) vi	x 2	65.54(9)

Table 4. The metal-metal distances (Å) and Ge—O—Ge and O(2)—O(2)—O(2) angles (°), with standard deviations in parentheses.

Ge i - Ge i11	3.7227(4)
- Ge ii	3.7226(8)
Ca i - Ca i11	3.7606(19)
- Ca v	3.6853(19)
- Ca ii	3.7349(8)
Ca i - Ge i	3.248(1)
- Ge i11	3.349(1)
- Ge v	3.204(1)
- Ge vi1	3.105(1)
Ge i - O(1) i	160.4(3)
- O(2) viii - Ge i11	159.0(2)
Ca i - O(1) i	99.6(1)
Ca i11 - O(1) i	90.0(1)
Ca i - O(2) i'	97.3(1)
- O(2) iv'	90.6(1)
- O(2) iv'	86.7(1)
- O(2) v - Ge vi1	86.1(1)
- O(2) v - Ge i	91.2(1)
- O(2) viii - Ge i	77.9(1)
- O(2) viii - Ge i11	81.1(1)
- O(2) i' - Ge i11	103.8(1)
O(2) i - O(2) viii - O(2) i'	154.72(18)
O(2) viii - O(2) i' - O(2) iv'	105.10(14)
O(2) i' - O(2) i1' - O(2) vi1	100.18(14)

residuals of the function $\sum w_i(|F_{\text{obs}}| - |F_{\text{calc}}|)^2_i$ were minimized with w_i kept as unity. Atomic scattering factor tables and anomalous dispersion coefficients were taken from *International Tables for X-Ray Crystallography, Vol. IV* (1974), Fukamachi (1971), and Tokonami (1964) and a secondary extinction correction was applied following Becker and Coppens (1974).

Results and discussion

Crystal structure of CaGeO₃

The final positional and thermal parameters obtained are listed in Table 1¹, in which we include data both before and after correction for the intensity of the twinned crystal. As seen from the table, the shift of atomic parameters due to the twinned crystal is relatively small. The interatomic distances and angles are also given in Tables 2, 3, and 4. Figure 2 shows a projection of the CaGeO₃ structure on (001) along with labels for the atoms related by symmetry operations. The atomic coordinates for this perovskite structure are similar to those of SmAlO₃ (Marezio *et al.*, 1972).

The cell distortion factor, d , is useful for estimating the departure from an ideal cubic model (for which $d = 0$):

$$d = \{(a/\sqrt{2} - a_p)^2 + (b/\sqrt{2} - a_p)^2 + (c/2 - a_p)^2\}/3a_p^2 \times 10^4,$$

where $a_p = (a/\sqrt{2} + b/\sqrt{2} + c/2)/3$. The value for

CaGeO₃ ($d = 0.004$) is close to that for SmAlO₃ (0.003) and is much smaller than those for ScAlO₃ (5.64), GdTlO₃ (6.00), and GdFeO₃ (4.08). Although the cell for CaGeO₃ is dimensionally close to being cubic, the polyhedra are considerably distorted as in characteristic orthorhombic perovskites. The most prominent distortion from the ideal cubic perovskite is in the tilting of the polyhedra (Fig. 2). The O(2)ⁱ-O(2)^{viii}-O(2)^{i'} angle, which shows the degree of the tilting in the xy plane, is 154.7(2)° in contrast to 180° for a cubic perovskite. The value of O(2)^{i'}-O(2)^{ii'}-O(2)^{vii} angle, corresponding to the tilting on the xz plane, is 100.2(1)° as opposed to 90° for a cubic perovskite. The polyhedra themselves have the following characteristics: (1) The ratio of $(\langle M-O \rangle - \langle M-O \rangle_{\text{cubic}})/\langle M-O \rangle$ shows that the oxygens around the Ge atom are more extended from the ideal cubic positions (0.016) than those around Ca (0.006), where $\langle M-O \rangle_{\text{cubic}}$ was estimated by constraining a cubic cell to have the same volume as the actual unit cell (see Table 2). (2) When we define the bond-length distortion, Δ , as $1/n \sum \{(r_i - r)/r\}^2 \times 10^3$, it is clear that the Ca cuboctahedron ($\Delta_A = 9.4$) is distorted, whereas the Ge octahedron is close to a regular one ($\Delta_B = 0.005$). The SmAlO₃ perovskite has similar values of the bond-length distortion: $\Delta_A = 8.7$ and $\Delta_B = 0.001$. The distortions, however, are less than for those of most GdFeO₃-type perovskites such as MgSiO₃ ($\Delta_A = 27.31$, $\Delta_B = 0.226$), ScAlO₃ ($\Delta_A = 37.50$, $\Delta_B = 0.034$) and GdFeO₃ ($\Delta_A = 28.13$, $\Delta_B = 0.032$). (3) The O—Ca—O angles, which in the ideal cubic perovskite are all 90°, vary from 107.9(1)° to 53.79(8)° (see Table 3).

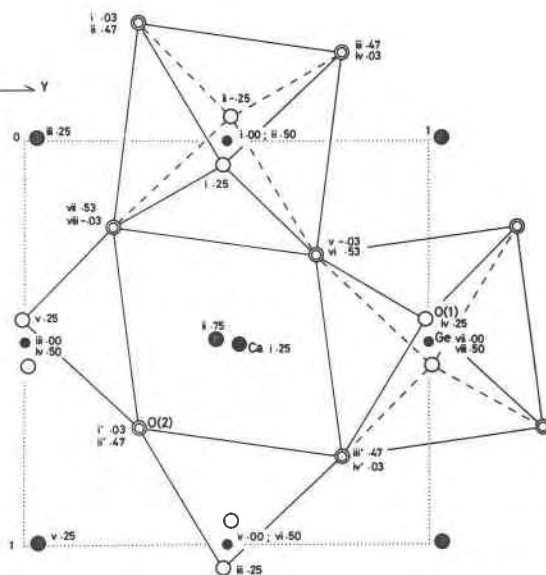


Fig. 2. Structure of CaGeO₃ perovskite projected on (001); Ca sites ruled, Ge sites solid, O(1) sites open, O(2) sites doubled. Each of the atoms is numbered with lower-case Roman numerals. Most atoms for which $0.5 < z < 1.0$ have been omitted for clarity.

¹To receive a copy of structure factor tables and Table 5, order document AM-83-233 from the Business Office, Mineralogical Society of America, 2000 Florida Ave. NW, Washington, D. C. 20009. Please remit \$1.00 in advance for microfiche.

Table 6. Comparison of tolerance factors and some structural characteristics such as B-O(1)-A and O(2)-O(2)-O(2) angles and bond-length distortions among orthorhombic GdFeO₃-type perovskites, ABO₃, based on single-crystal X-ray structure refinements.

	$\langle B-O \rangle$ (Å)	$\langle A-O \rangle$ (Å)	t_{obs}	Δ_B	Δ_A	B-O(1)-A 1-1-1 (°)	O(2)-O(2)-O(2) 1-viii-1' (°)	O(2)-O(2)-O(2) 1'-11'-vii (°)	t_{IR}	Reference
SeMgO ₃	2.150	2.808	0.924	0.374	58.49	112.41	119.52	114.88	0.883	1
ScAlO ₃	1.901	2.607	0.970	0.034	37.50	105.07(5)	133.60(4)	109.13(4)	0.830	2
YAlO ₃	1.911	2.655	0.982	0.017	20.34	102.55(31)	145.89(68)	103.86(53)	0.884	3
SmAlO ₃	1.899	2.657	0.989	0.001	8.66	100.18(13)	155.41(21)	100.66(16)	0.906	4
MgSiO ₃ *	1.790	2.476	0.978	0.226	27.31	103.8(15)	139.9(16)	107.7(15)	0.900	5
MgSiO ₃ *	1.791	2.474	0.977	0.002	28.27	103.3(13)	138.1(16)	108.2(16)	0.900	6
CaTiO ₃	1.926	2.712	0.996	0.001	5.62	95.6	161.3	98.4	0.889	7
YTiO ₃	2.039	2.800	0.971	0.177	37.69	105.67(15)	134.65(18)	107.82(15)	0.826	8
LaTiO ₃	2.018	2.816	0.987	0.003	11.69	100.97(74)	151.62(57)	101.62(47)	0.874	8
NdTiO ₃	2.022	2.803	0.980	0.025	21.73	103.47(12)	143.72(14)	104.49(12)	0.857	8
SmTiO ₃	2.038	2.814	0.976	0.074	28.36	104.62(20)	139.65(24)	106.17(20)	0.847	8
GdTiO ₃	2.039	2.811	0.975	0.170	32.48	105.04(12)	137.52(14)	106.66(12)	0.838	8
InCrO ₃	1.987	2.708	0.964	0.096	38.40	105.18(16)	131.96(22)	110.39(19)	0.814	9
SeMnO ₃	2.235	2.894	0.916	0.424	64.51	112.76	117.16	115.86		1
LaMnO ₃ #	2.018	2.818	0.987	3.75	16.44	101.15(16)	143.79(31)	101.89(26)	0.952	10
PrMnO ₃ #	2.037	2.823	0.980	5.55	27.51	100.91(16)	133.45(29)	103.61(26)	0.871	11
YFeO ₃	2.015	2.771	0.972	0.081	34.26	104.67(29)	135.32(33)	108.09(31)	0.836	12
PrFeO ₃	2.008	2.791	0.983	0.008	17.67	102.25(11)	146.48(17)	103.75(13)	0.873	13
NdFeO ₃	2.011	2.790	0.981	0.007	20.24	102.89(11)	144.71(14)	104.53(12)	0.868	13
SmFeO ₃	2.013	2.785	0.978	0.040	24.83	103.48(9)	141.46(11)	105.54(9)	0.857	13
EuFeO ₃	2.013	2.783	0.978	0.038	26.85	103.64(11)	140.56(14)	105.82(12)	0.853	13
GdFeO ₃	2.012	2.781	0.977	0.032	28.13	103.87(16)	140.19(15)	105.83(13)	0.848	13
TbFeO ₃	2.012	2.775	0.975	0.047	30.17	104.03(9)	138.42(11)	106.79(9)	0.844	13
DyFeO ₃	2.012	2.773	0.975	0.059	32.02	104.15(12)	137.22(14)	107.10(12)	0.839	13
HoFeO ₃	2.010	2.768	0.974	0.054	33.36	104.36(13)	136.55(17)	107.44(14)	0.835	13
ErFeO ₃	2.011	2.765	0.972	0.038	34.85	104.83(12)	135.72(14)	107.83(12)	0.831	13
TmFeO ₃	2.011	2.763	0.972	0.029	35.87	104.67(13)	135.07(17)	108.26(15)	0.828	13
YbFeO ₃	2.011	2.759	0.970	0.029	37.61	104.69(17)	134.00(20)	108.61(16)	0.825	13
LuFeO ₃	2.010	2.755	0.969	0.030	38.62	105.01(15)	133.01(18)	109.30(15)	0.822	13
SeCoO ₃	2.148	2.805	0.923	0.688	58.30	112.39	121.00	114.15		1
TeCoO ₃	2.159	2.855	0.935	1.88	50.24	112.28	128.87	109.48		1
SeNiO ₃	2.113	2.768	0.926	0.285	55.72	111.74	121.59	113.64		1
SeCuO ₃	2.177	2.839	0.922	13.5	60.26	113.12	130.81	111.54		1
SeZnO ₃	2.160	2.815	0.922	0.775	59.14	111.99	120.63	114.51		1
CaGeO ₃	1.892	2.647	0.989	0.005	9.40	99.57(14)	154.72(18)	100.18(14)	0.923	14
SrZrO ₃ #	2.091	2.919	0.987	0.006	10.65	101.18(31)?	152.92(35)	101.12(27)	0.887	15
BaCeO ₃ #	2.241	3.128	0.987	0.044	10.57	101.13(62)	151.10(77)	102.97(76)	0.878	16
BaPrO ₃ #	2.223	3.111	0.990	0.001	9.63	100.99(97)	154.6(12)	101.8(10)	0.886	16
NaTaO ₃ #	1.978	2.765	0.988	0.001	8.23	99.51(6)	155.81(8)	99.43(6)	0.894	17

$\langle B-O \rangle$: Mean atomic distances for B site (VI coordinate).

$\langle A-O \rangle$: Mean atomic distances for A site (XII coordinate).

t_{obs} : Observed tolerance factor: $t = \langle A-O \rangle / \sqrt{2} \langle B-O \rangle$.

t_{IR} : Tolerance factor calculated from the Shannon & Prewitt's ionic radii (VI and XII coordinates for B and A site, respectively).

Δ_B : Bond-length distortion for B-site (VI coordinate).

Δ_A : Bond-length distortion for A-site (XII coordinate).

*: X-ray powder diffraction study.

#: Neutron powder diffraction study.

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using the polynomial approximation of $y = \sum a_j (1-t_{\text{obs}})^j \times 10^3$ ($j = 0, 1, 2, \dots, n$; where $n = 2$ or 3) (see "A Fit" in this table). (4) Also, these curves can be extended to fit over the full range as shown in "C Fit" of Table 7.

The distortions of polyhedra in GdFeO₃-type perovskites

It is useful to plot distortions of polyhedra as well as tilts versus t_{obs} . Figures 6 and 7 show the correlation of bond-length distortions Δ_A and Δ_B versus t_{obs} for the A and B sites, respectively. Figure 6 shows that bond-length distortions for the larger A site have a systematic relationship with t_{obs} : (1) As the value of t_{obs} decreases, the distortions increase. (2) The trend can be approximated as a linear function within each of the groups, I and II. (3) The equations for distortions versus t_{obs} allow an interpolation within a group but does not permit an extrapolation. (4) The fit of the polynomial approximation, shown by the form of $R_k = \sum \{ |Y(\text{obs}) - Y(\text{calc})| / |Y(\text{obs})| \}_k$, is not as good for distortions of polyhedra as it is for tilting (see Table 7). Although a comparison of bond-length distortion for the B site indicates similar characteristics to that of Δ_A , there is no strong systematic relationship (Fig. 7).

Application for germanates and silicates

The systematic relationships observed in Figures 3 to 7 suggest that it might be possible to predict the deformation of perovskites containing various ions. For this purpose, we must be able to estimate the value of t_{obs} from known quantities such as ionic radii. In Figure 8, we

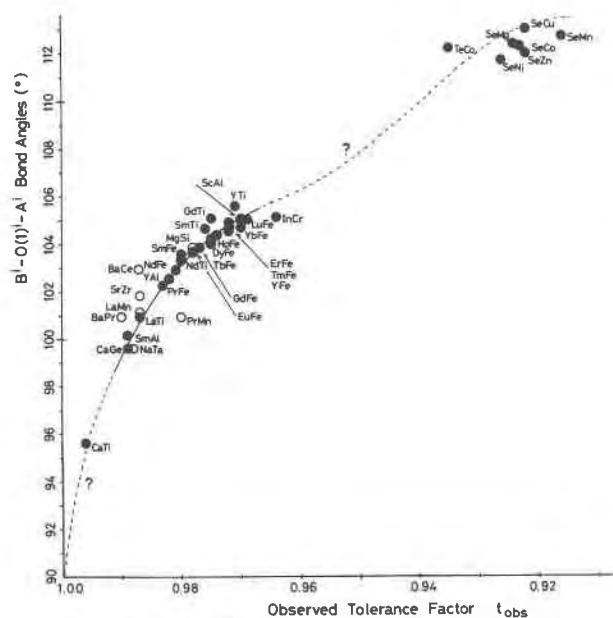


Fig. 5. $B^i-O(1)^j-A^i$ angle vs. t_{obs} for the GdFeO₃-type perovskite. Samples plotted are the same as those in Fig. 3. The line is based on a polynomial equation (C Fit) shown in Table 7.

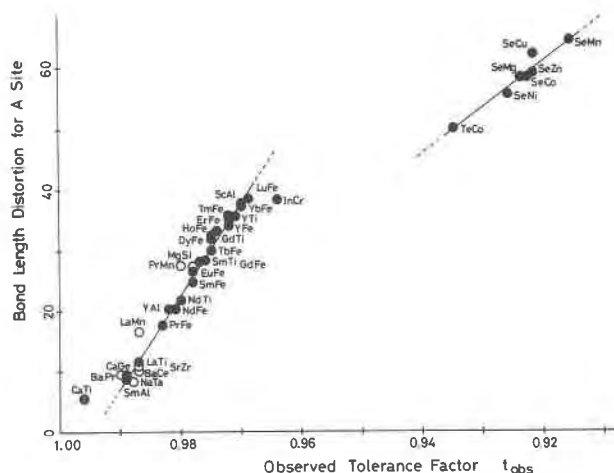


Fig. 6. Bond-length distortion for A site vs. t_{obs} for the GdFeO₃-type perovskites. Samples plotted are the same as those in Fig. 3.

plot the Goldschmidt tolerance factor, $t_{\text{IR}} = (r_A + r_O) / \sqrt{2(r_B + r_O)}$ versus t_{obs} for the perovskite crystals shown in Table 6, where r_A , r_B and $r_O (= 1.4 \text{ \AA})$ are the empirical ionic radii of the respective ions in the compound ABO₃ (Shannon and Prewitt, 1969; Shannon, 1976). Radii data for coordination numbers eight (A site) and six (B site) were used because values for twelve coordination for

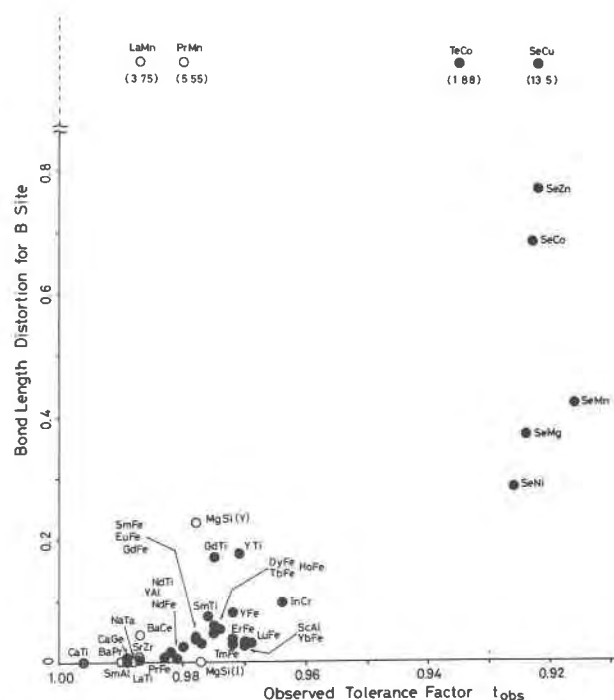


Fig. 7. Bond-length distortion for B site vs. t_{obs} for the GdFeO₃-type perovskites. Samples plotted are the same as those in Fig. 3. For MgSiO₃, Y and I represent values after Yagi *et al.* (1978) and Ito and Matsui (1978), respectively.

Table 7. The polynomial equations relating some bond angles and bond-length distortion versus t_{obs} using the mean squares method with least-squares calculations. The form is given by: $Y = \sum a_j(1-t_{\text{obs}})^j$ ($j = 0, 1, 2, \dots, n$, where $n = 3, 4$, or 5).

	a_0	a_1	a_2	a_3	a_4	R_k	Reference
A Fit (Group I) ($0.96 < t_{\text{obs}} \leq 0.99$)							
$O(2)^i-O(2)^{viii}-O(2)^{i'}$	167.353	-0.88694	-0.031787	0.80562×10^{-3}		0.009	Fig. 3
$O(2)^{i'}-O(2)^{ii'}-O(2)^{vii}$	94.153	0.62812	-0.004793			0.002	Fig. 4
$B^i-O(1)^i-A^i$	93.703	0.67376	-0.009870			0.002	Fig. 5
Δ_A	15.340	-2.41560	0.206090	-0.33625×10^{-2}		0.024	Fig. 6
B Fit (Group I + Cubic) ($0.96 < t_{\text{obs}} \leq 1.00$)							
$O(2)^i-O(2)^{viii}-O(2)^{i'}$	180.001	-3.44519	0.149365	-0.45824×10^{-2}	0.57285×10^{-4}	0.008	Fig. 3
$O(2)^{i'}-O(2)^{ii'}-O(2)^{vii}$	89.998	1.50629	-0.069647	0.20036×10^{-2}	-0.22033×10^{-4}	0.002	Fig. 4
$B^i-O(1)^i-A^i$	90.019	1.20693	-0.033576	0.33067×10^{-3}		0.002	Fig. 5
Δ_A	-0.0015	0.68943	-0.013916	0.31847×10^{-2}	-0.69645×10^{-4}	0.023	Fig. 6
C Fit (Groups I and II + Cubic) ($0.91 < t_{\text{obs}} \leq 1.00$)							
$O(2)^i-O(2)^{viii}-O(2)^{i'}$	179.956	-2.70904	0.048539	-0.29879×10^{-3}		0.025	Fig. 3
$O(2)^{i'}-O(2)^{ii'}-O(2)^{vii}$	90.018	1.12058	-0.020962	0.13474×10^{-3}		0.004	Fig. 4
$B^i-O(1)^i-A^i$	90.034	1.25746	-0.040800	0.61616×10^{-3}	-0.32303×10^{-5}	0.003	Fig. 5
Δ_A	-0.2526	0.37869	0.066647	-0.15607×10^{-2}	0.98440×10^{-5}	0.036	Fig. 6

t_{obs} : The observed tolerance factor

$R_k = \frac{\sum \{|Y(\text{obs}) - Y(\text{calc})| / |Y(\text{obs})|\}}{\sum \{|Y(\text{obs}) - Y(\text{calc})| / |Y(\text{obs})|\}}$; For the calculation of $O(2)^i-O(2)^{viii}-O(2)^{i'}$, $Y = 180 - Y$.

many ions are lacking. It is very significant that the GdFeO₃-type perovskites with the same B ion can be characterized as a linear function in the $t_{\text{IR}}-t_{\text{obs}}$ diagram, and that each function for the different B ions has almost the same slope. These characteristics make the estimation of t_{obs} possible for structures containing arbitrary kinds of ions.

Let us consider several germanates and silicates because of their geophysical importance. Average gradients of the $t_{\text{obs}}-t_{\text{IR}}$ lines of titanates, aluminates, and orthoferites were assumed for germanates and silicates making use of the data for CaGeO₃ (this study) and MgSiO₃ (Yagi *et al.*, 1978). The estimated values of t_{obs} , which we shall call t'_{obs} , can be derived from the assumption that:

$$t'_{\text{obs}} = 0.274 t_{\text{IR}} + 0.736 \quad (1)$$

for germanates, and

$$t'_{\text{obs}} = 0.274 t_{\text{IR}} + 0.731 \quad (2)$$

for silicates (see Fig. 8).

SrGeO₃ is known to crystallize as a cubic perovskite (Shimizu *et al.*, 1970). The t'_{obs} value of 1.003 (SrGeO₃) estimated from $t_{\text{IR}} = 0.975$ is consistent with the observation that SrGeO₃ is a cubic perovskite. The value of $t'_{\text{obs}} = 1.019$ for hexagonal BaGeO₃ (Shimizu *et al.*, 1970) may show that the orthorhombic GdFeO₃-type perovskites can be separated crystallographically from hexagonal ones. On the other hand, for the silicate CaSiO₃, which was reported to transform into a cubic perovskite at $P = 160$ kbar and $T = 25^\circ\text{C}$ (Liu and Ringwood, 1975), the value of $t'_{\text{obs}} = 1.002$ ($t_{\text{IR}} = 0.99$) is close to 1.0, in accord with the formation of cubic perovskite.

A cubic CdGeO₃ perovskite with $a = 3.7\text{\AA}$ has been

reported to exist at 130 kbar and 900°C (Ringwood and Major, 1967). The value of $t'_{\text{obs}} = 0.987$ ($t_{\text{IR}} = 0.916$), determined from Equation (1), is close to those of CaGeO₃ and SmAlO₃. This means that CdGeO₃ should crystallize in space group *Pbnm*, but with more distorted polyhedra than is found for CaGeO₃. Because CaGeO₃ was previously reported incorrectly as cubic perovskite, a more detailed study of the symmetry for CdGeO₃ is required. If CdGeO₃ is orthorhombic, the t'_{obs} value predicts a crystal structure similar to that of LaTiO₃: $O(2)^i-O(2)^{viii}-O(2)^{i'} = 152^\circ$, $O(2)^{i'}-O(2)^{ii'}-O(2)^{vii} = 102^\circ$, $\text{Ge}^i-\text{O}(1)^i-\text{Cd}^i = 101^\circ$, and $\Delta_{A(=\text{Cd})} = 11$. MnGeO₃ quenched from 250 kbar and 1400–1800°C has been found to be orthorhombic (Liu, 1976b). The value of $t'_{\text{obs}} = 0.973$ ($t_{\text{IR}} = 0.865$) supports this report and predicts the space group *Pbnm* (or *Pbn2₁*). Liu (1977) reported MgGeO₃ as an orthorhombic perovskite, having space group *Pmmm* or *P222*. Examination of the $t_{\text{obs}}-t_{\text{IR}}$ diagram shows that MgGeO₃ is within the stability of the GdFeO₃-type perovskite ($t'_{\text{obs}} = 0.966$; $t_{\text{IR}} = 0.839$) and has similar characteristics to InCrO₃. However, because the report for MgGeO₃ was based on powder data of five mixed phases, it is also recommended that the space group be reexamined.

It should be emphasized that the data are consistent with MgSiO₃ (Yagi *et al.*, 1978; Ito and Matsui, 1978) being a GdFeO₃-type perovskite (see Figs. 3 to 7). In contrast, Madon, Bell, Mao, and Poirier (1980) assigned tetragonal symmetry to MgSiO₃ ($a = b = 9.3$, $c = 13.1\text{\AA}$), based on lattice parameters measured using electron diffraction patterns. Their observations, however, are not sufficient to confirm the symmetry and their conclusion may result from other factors such as twinning or pseudo-symmetry. It should be noted that an experiment using

only electron microscopy is not sufficient to determine the symmetry.

We may use the observed relationship to predict values of t'_{obs} for perovskites of some germanate and silicate compounds which have not yet been synthesized. For this purpose, the values of t'_{obs} associated with various cations were calculated for germanates and silicates from t_{IR} and are shown in Table 8. The compounds for which the t'_{obs} lies within the range for group I would probably crystallize as orthorhombic perovskites and have the tilting and distortion of oxygen polyhedra as estimated from Table 7: these are FeGeO_3 ($t'_{\text{obs}} = 0.969$), CoGeO_3 (0.967), and ZnGeO_3 (0.967) for germanates, and MnSiO_3 ($t'_{\text{obs}} = 0.985$), FeSiO_3 (0.981), CoSiO_3 (0.979), and ZnSiO_3 (0.978) for silicates. On the other hand, BaSiO_3 ($t'_{\text{obs}} = 1.035$) and SrSiO_3 (1.017) should not occur as the GdFeO_3 -type perovskite. However, the above discussion illustrates only one geometrical aspect concerning crys-

Table 8. Estimated values of t_{obs} for some germanates and silicates.

	t_{IR}	t'_{obs}		t_{IR}	t'_{obs}
Germanates			Silicates		
MgGeO_3	0.839	0.966	CaSiO_3	0.990	1.002
MnGeO_3	0.865	0.973	MnSiO_3	0.927	0.985
FeGeO_3	0.850	0.969	FeSiO_3	0.911	0.981
CoGeO_3	0.843	0.967	CoSiO_3	0.904	0.979
ZnGeO_3	0.843	0.967	ZnSiO_3	0.904	0.979
SrGeO_3	0.975	1.003	SrSiO_3	1.045	1.017
CdGeO_3	0.916	0.987	CdSiO_3	0.982	0.986
BaGeO_3	1.033	1.019	BaSiO_3	1.108	1.035

tallization of orthorhombic perovskites; often other factors such as crystal-field effects determine what phases are stable. For example, although FeSiO_3 is one of the possible orthorhombic perovskites (see Table 8), it is thought to be unstable with respect to mixed oxides (Yagi et al., 1978).

Summary

CaGeO_3 perovskite is orthorhombic ($Pbnm$) and isostructural with GdFeO_3 . The oxygen polyhedra are less tilted and less distorted than those of most GdFeO_3 -type perovskites. Comparison between structural characteristics and t_{obs} has made possible useful predictions for the existence of previously-unknown GdFeO_3 -type perovskites. The following predictions are obtained for germanate and silicates: (1) CdGeO_3 should crystallize as an orthorhombic perovskite rather than a cubic one; (2) the perovskite form of MgSiO_3 probably has the GdFeO_3 -type structure; (3) the space group, $Pmmm$ (or $P222$), reported for a MgGeO_3 perovskite should be reexamined with the possibility of it being $Pbnm$; (4) in the geometrical view of crystals, germanate and silicate perovskites containing Mg, Mn, Co, Fe, and Zn ions in the large A site should have the GdFeO_3 -type structure.

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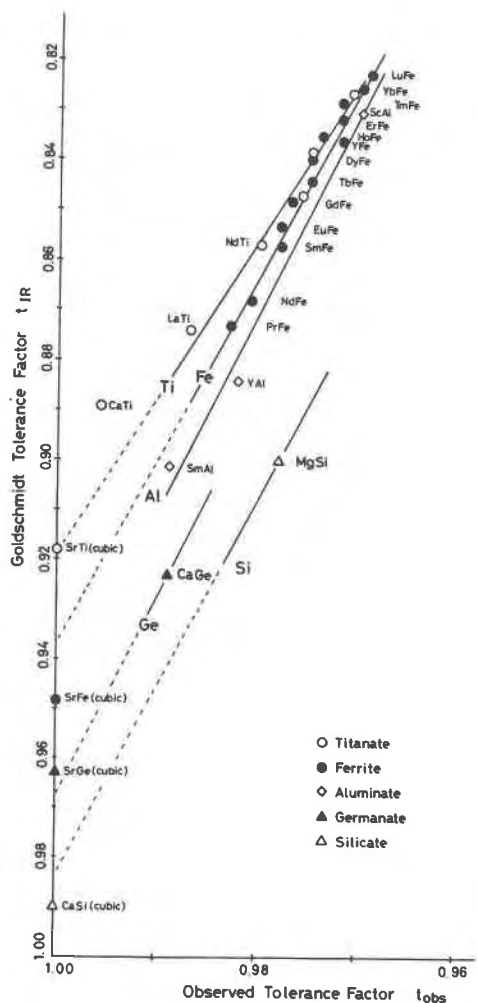


Fig. 8. The $t_{\text{obs}}-t_{\text{IR}}$ diagram of some GdFeO_3 -type perovskites such as orthoferrites, titanates, aluminates, germanate, and silicates.

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