Cation arrangement in iron-zinc-chromium spinel oxides

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Abstract

The distribution of ferric and ferrous ions within the quaternary Fe$^{2+}$-Fe$^{3+}$-Zn-Cr spinel solid solution series is documented using Mössbauer spectroscopy and X-ray diffraction. Along the binary joins FeCr$_2$O$_4$-Fe$_3$O$_4$, ZnCr$_2$O$_4$-Fe$_3$O$_4$ and ZnFe$_2$O$_4$-Fe$_3$O$_4$ the cation distribution changes from normal to inverse. From the different trends of iron distribution versus composition observed along these three joins and at other points within the Fe-Zn-Cr solid solution series, it was found that octahedral site ferric/ferrous charge hopping may be the most important phenomenon stabilizing the inverse ion distribution. For spinels whose compositions make charge hopping impossible or statistically unlikely, cation distribution is governed by the intrinsic spinel site preferences (tetrahedral site: Zn > Fe$^{2+}$ > Fe$^{3+}$, octahedral site: Cr > Fe$^{3+}$ > Fe$^{2+}$ > Zn), which result from the size and bonding characteristics of these atoms.

Near the ZnFe$_2$O$_4$-Fe$_3$O$_4$ edge of this spinel quadrilateral, the normal to inverse transition proceeds essentially to the maximum extent allowed by the Zn content, because the octahedral Fe$^{2+}$ produced by inversion can share electrons with the surrounding octahedral Fe$^{3+}$ ions. Inversion along the FeCr$_2$O$_4$-Fe$_3$O$_4$ edge of the quadrilateral, however, is inhibited until there is a sufficient octahedral Fe$^{3+}$ population to make electron sharing (with inversion produced octahedral Fe$^{2+}$) statistically likely. A rapid change of site preference of iron ions occurs in the central range of this composition subspace, offering a physical reason for the miscibility gap found along the chromite-magnetite join.

Introduction

Cation arrangement in spinel-type oxides has long been a topic of interest among mineralogists (Barth and Posnjak, 1932; Verwey and Heilmann, 1947; Robbins et al., 1971; Price et al., 1982), because the site distribution (particularly of iron) has a strong effect on magnetic, electrical and thermochemical properties. Studies of a variety of end member spinel compositions (Verwey and Heilmann, 1947, Navrotsky and Kleppa, 1967; Price et al., 1982; Ursusov, 1983; O’Neill and Navrotsky, 1983) and binary solid solutions (Olés 1970; Dobson et al. 1970; Levenstein et al. 1972; Grandjean and Gérard 1978, and Burns 1981, to mention a few) have brought into focus many problems concerning the compositional dependence of cation distribution in spinels. This paper will concentrate on changes in cation arrangement within a quaternary solid solution series, the Zn-Cr-Fe$^{2+}$-Fe$^{3+}$ spinel oxide system.

The spinel structure consists of an almost cubic close packed array of oxygen atoms with cations in interstices within the oxygen framework. Sixteen octahedral and eight tetrahedral sites within one unit cell of 32 oxygens are filled, in the system studied, with zinc in the tetrahedral sites, chromium in the octahedral sites and ferric and ferrous iron in either site. The octahedra form edge sharing chains along [110]. The tetrahedra are isolated from one another, sharing each of their corners with three octahedra.

Chromite (FeCr$_2$O$_4$), ZnCr$_2$O$_4$ and franklinite (ZnFe$_2$O$_4$) are all normal spinels, with divalent atoms on the tetrahedral sites. Magnetite (FeFe$_2$O$_4$) is an inverse spinel with all of its ferrous iron occupying the octahedral site. (The degree of inversion, which in these spinels is equal to the fraction of tetrahedral site occupied by trivalent ions, is commonly denoted by $I$ which can vary from 0 to 1.) Thus, this system exemplifies the transition from one end member ordering state to the other, across a quaternary solid solution series. By using Zn and Cr, which strongly prefer tetrahedral and octahedral sites respectively (Navrotsky and Kleppa, 1967), the only distributional variables are the position and valence of the iron atoms present. These constraints allow the cation distribution vs. composition of the experimentally produced phases to be found using X-ray powder diffraction (XRD) and Mössbauer spectroscopy.

Experimental procedures

Seventeen spinels were synthesized from mechanical mixtures of oxide powders within the Zn-Cr-Fe$^{2+}$-Fe$^{3+}$
spinel quadrilateral, except along the magnetite–chromite join, amply studied by Robbins et al., (1970); Levinstein et al., (1972); Ok et al., (1978); and others (compositions CO2) operated between 900°C and 1100°C.

All samples were quenched by dropping, under an argon atmosphere. The samples reached room temperature within minutes, although the exact quench rate was not measured. If wüstite or hematite was found in the X-ray pattern, or if the diffraction pattern showed broad peaks, the samples were reground and rerun at slightly increased oxygen fugacities until no other phases could be detected. Final conditions are included in Table 1. The compositions listed in Tables 1 and 2 are those assuming complete conversion of the starting mix to spinel.

Mössbauer and X-ray data were consistent with this assumption (to within 4% of the expected composition) except for sample 26. For that single sample, both the X-ray and Mössbauer measurements are more consistent with a composition of Zn 0.70, Cr 0.56, rather than the originally intended composition, Zn 0.75, Cr 0.50. The corrected composition is shown in Figures 2 and 3.

Mössbauer spectra were collected on a conventional constant acceleration spectrometer with a moving 57Co in Pd source and powdered sample absorbers. The mirror image spectra, accumulated on a 512 channel multichannel analyser, employed velocity increments of about 0.03 mm/sec/channel for paramagnetic spectra and about 0.08 mm/sec/channel for magnetic spectra. The samples were placed in 1-inch diameter polyethylene holders with inert adhesive. Velocity calibration utilized Fe metal and Nanotroisprusside.

Room temperature Mössbauer spectra were collected for each spinel synthesized, (see Table 2). In paramagnetic spectra, doublets were constrained to have equal areas and peak widths. In magnetic spectra, the areas for each sextet were constrained to a 3:2:1 ratio, with corresponding peaks constrained to have equal peak widths. (For a more detailed description of the experimental procedures, see Marshall, 1983.)

The calculated and measured cation arrangement of these spinels is assumed to correspond to the temperature of synthesis because the quench rates were rapid relative to ionic re-equilibration rates of similar spinels (Trestman-Mattes et al., 1983). The electronic configurations of the iron atoms, however, probably do not quench (O’Neill and Navrotsky, 1984; Trestman-Mattes et al., 1983) and could represent lower temperatures, although the cation arrangements predicted by Robbins et al., (1971) along the chrome-magnetite join correspond to the theoretical thermodynamic arrangement (O’Neill and Navrotsky, 1984) predicted for 1000°C, the synthesis temperature.

### Table 1. X-ray data for synthesized spinels

<table>
<thead>
<tr>
<th>No.</th>
<th>Zn, Cr (Zn, Cr)</th>
<th>Temp. (°C)</th>
<th>Dur. (hours)</th>
<th>f O2</th>
<th>f CO2</th>
<th>δ 2 (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>10.0, 20.0</td>
<td>1200.0</td>
<td>air</td>
<td></td>
<td></td>
<td>8.3275(6)</td>
</tr>
<tr>
<td>37</td>
<td>0.90, 2.0</td>
<td>960.26</td>
<td>16.5</td>
<td></td>
<td></td>
<td>8.33317(3)</td>
</tr>
<tr>
<td>38</td>
<td>0.80, 2.0</td>
<td>1000.24</td>
<td>16.5</td>
<td></td>
<td></td>
<td>8.33375(8)</td>
</tr>
<tr>
<td>39</td>
<td>0.85, 2.0</td>
<td>960.25</td>
<td>16.5</td>
<td></td>
<td></td>
<td>8.35030(11)</td>
</tr>
<tr>
<td>40</td>
<td>0.30, 2.0</td>
<td>940.25</td>
<td>16.5</td>
<td></td>
<td></td>
<td>8.36011(4)</td>
</tr>
<tr>
<td>41</td>
<td>1.0, 1.0</td>
<td>1060.4</td>
<td>13.6</td>
<td></td>
<td></td>
<td>8.3938(10)</td>
</tr>
<tr>
<td>42</td>
<td>1.0, 1.0</td>
<td>850.24</td>
<td>13.1</td>
<td></td>
<td></td>
<td>8.4252(11)</td>
</tr>
<tr>
<td>21</td>
<td>0.50, 1.0</td>
<td>1007.48</td>
<td></td>
<td></td>
<td></td>
<td>8.4252(8)</td>
</tr>
<tr>
<td>33</td>
<td>1.0, 0.55</td>
<td>1200.0</td>
<td></td>
<td></td>
<td></td>
<td>8.4099(24)</td>
</tr>
<tr>
<td>27</td>
<td>0.75, 0.50</td>
<td>965.3</td>
<td>10.0</td>
<td></td>
<td></td>
<td>8.3956(5)</td>
</tr>
<tr>
<td>28</td>
<td>0.50, 0.50</td>
<td>1010.11</td>
<td>13.0</td>
<td></td>
<td></td>
<td>8.3846(10)</td>
</tr>
<tr>
<td>29</td>
<td>0.05, 0.50</td>
<td>965.3</td>
<td>13.0</td>
<td></td>
<td></td>
<td>8.3846(4)</td>
</tr>
<tr>
<td>32</td>
<td>1.0, 0.0</td>
<td>1200.0</td>
<td></td>
<td></td>
<td></td>
<td>8.4415(12)</td>
</tr>
<tr>
<td>34</td>
<td>0.75, 0.0</td>
<td>912.10</td>
<td>12.1</td>
<td></td>
<td></td>
<td>8.4138(6)</td>
</tr>
<tr>
<td>35</td>
<td>0.50, 0.0</td>
<td>1053.4</td>
<td>12.0</td>
<td></td>
<td></td>
<td>8.4252(11)</td>
</tr>
</tbody>
</table>

δ assuming complete conversion of the starting mix
# Cu Kα 1 = 1.540598, Cu Kα 2 = 1.54443
+ synthesized with a Li-borate flux
* NBS silicon standard used
** Silver metal used as standard

### Table 2. Mössbauer data for synthesized spinels

<table>
<thead>
<tr>
<th>No.</th>
<th>Zn, Cr (Zn, Cr)</th>
<th>δIS (mm/ sec)</th>
<th>QS (mm/ sec)</th>
<th>PW (mm/ sec)</th>
<th>h/kβ</th>
<th>rel. area</th>
<th>δHoss.</th>
<th>UCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>10.0, 20.0</td>
<td>0.88(2)</td>
<td>0.17</td>
<td>0.30(1)</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>32</td>
<td>0.90, 2.0</td>
<td>0.30(1)</td>
<td>0.11</td>
<td>0.29(1)</td>
<td>0.11(1)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>33</td>
<td>0.85, 2.0</td>
<td>0.26</td>
<td>0.276(5)</td>
<td>0.35(8)</td>
<td>0.10</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>34</td>
<td>0.30, 0.50</td>
<td>0.40(15)</td>
<td>0.408</td>
<td>0.286(7)</td>
<td>0.67(5)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>35</td>
<td>0.25, 1.50</td>
<td>0.96(1)</td>
<td>0.33</td>
<td>0.435(9)</td>
<td>0.60(1)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>36</td>
<td>0.50, 0.0</td>
<td>0.38(1)</td>
<td>0.438(6)</td>
<td>0.395(1)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>37</td>
<td>0.75, 0.0</td>
<td>0.38(9)</td>
<td>0.385(9)</td>
<td>0.322(4)</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>38</td>
<td>0.50, 1.0</td>
<td>0.0</td>
<td></td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>39</td>
<td>0.75, 0.50</td>
<td>0.33(4)</td>
<td>0.343(5)</td>
<td>0.245(5)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>40</td>
<td>0.75, 0.0</td>
<td>0.20(8)</td>
<td>0.04</td>
<td>0.74(4)</td>
<td>0.11(1)</td>
<td>0.31</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>0.53(13)</td>
<td>0.02</td>
<td>2.36(4)</td>
<td>0.44(5)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

δ measured at room temperature, source was Co in Pd.
# assuming complete conversion of the starting mix to spinel
% rel to Fe. (errors are the same for δHoss.)
* Composition would be 0.70,0.50,0.56 (rather than 0.75,0.50,0.5) if the Mössbauer peaks area are correct.
** assuming excess ferrous iron on the octahedral site is charge hopping
? value was not obtained.

Experimental results

The parameters obtained from the Mössbauer spectra are isomer shift (IS), quadrupole splitting (QS) and peak
width at half height (PW). The IS is approximately constant for a particular iron valence state in a particular type of site. A literature search of oxide spinels gives the following average values relative to iron metal: tetrahedral Fe$^{2+}$ = 0.91 mm/sec (9 examples), octahedral Fe$^{2+}$ = 1.02 mm/sec (8 examples), tetrahedral Fe$^{3+}$ = 0.28 mm/sec (6 examples) and octahedral Fe$^{3+}$ = 0.37 mm/sec (14 examples). QS is a function of the electric field gradient and therefore measures the symmetry of the site. In oxide spinels, QS for either tetrahedral or octahedral Fe$^{3+}$ tends to be small (0.3-0.6 mm/sec) and relatively insensitive to cation disorder. On the other hand, Fe$^{2+}$ and Fe$^{3+}$ in octahedral and especially tetrahedral sites show small Qs values, depending upon disorder of near neighbor sites. The PW reflects the similarity of the sites within each crystal; for a spinel where all of the tetrahedral iron ions have an identical configuration of nearest neighbors, the peak width will be narrow (around 0.3 mm/sec.). Broader peaks result from the existence of dissimilar sites, differing, for example, by having different second or third nearest neighbors.

The room temperature Mössbauer spectra of all paramagnetic samples showed spinels with "normal" distribution ($\lambda = 0.0$), i.e., in these samples, all ferric iron is in octahedral coordination and all ferrous iron is in tetrahedral coordination.

Samples along the ZnCr$_2$O$_4$-ZnFe$_2$O$_4$ binary join showed IS, QS and PW consistent with octahedral ferric iron as was found also by Muthukumarasamy et al., (1980). Samples along the ZnCr$_2$O$_4$-FeCr$_2$O$_4$ binary join show typical tetrahedral ferrous iron values of IS, QS and PW. Along this join the QS remains almost constant and is very small (0.18 to 0.26 mm/sec.). This is expected because, along this join the local site symmetry is high with all of the nearest neighbor cations for this site being the same (Cr atoms). The small changes in QS are probably due to variations among the second nearest neighbor cations, i.e., the Fe and Zn on the tetrahedral site. The largest QS on this join occurs at an approximately half and half mixture of zinc and iron on the tetrahedral site, where the highest degree of disorder would occur.

Mössbauer spectra of samples 28 (Zn$_{0.75}$Fe$_{0.75}$Cr$_{1.50}$O$_4$) and 29 (Zn$_{0.25}$Fe$_{1.25}$Cr$_{1.50}$O$_4$) show contributions from both octahedral ferric and tetrahedral ferrous iron. The values of QS of the ferric iron are close to those obtained along the ZnCr$_2$O$_4$-ZnFe$_2$O$_4$ join. The QS for the ferrous iron, however, is higher than values obtained along the ZnCr$_2$O$_4$-FeCr$_2$O$_4$ join, as a result of decreased local site symmetry caused by mixing of different ion species on the nearest neighbor octahedral site. The width of both the ferric and ferrous Mössbauer peaks are broader than the average width measured along the simpler binary joins. According to the site distribution calculations utilizing unit cell edge, (next section) these two samples also contain minor amounts of tetrahedral ferric and octahedral Fe$^{3+}$, (i.e., they are partially inverse) which could also contribute to the peak broadening.

The magnetic spectra showed spinels which were either partially or fully inverse (0.3 < $\lambda$ < 1, see Table 2). These spectra are much more complicated and overlapped than the paramagnetic spectra and the errors in measurement are consequently larger.

In general the magnetic spectra looked similar to those of Robbins et al. (1971). In every magnetic spectrum, one sextet had an IS value corresponding to tetrahedral Fe$^{3+}$ (around 0.27 mm/sec). For these, QS is close to 0.0 as would be expected from the high symmetry of the site. The peak width averaged around 0.7 mm/sec, which is wider than ideal.

The octahedral site sextet in the magnetic spectra is more complicated. Whenever both ferric and ferrous iron are present on the octahedral site, an intermediate IS value of about 0.63 mm/sec was observed, corresponding to an intermediate valence state resulting from electron charge hopping between ferric and ferrous ions (Bauminger et al., 1961; Dobson et al. 1970; Robbins et al. 1971; Murray and Linnett, 1976; Ok and Evans, 1976; Lotgering and van Diepen, 1977 and Ok et al., 1978). The quadrupole interaction of this sextet varies between 0.06 to 0.25 mm/sec. The peaks are broad, around 1.4 mm/sec for samples with approximately equal numbers of ferric and ferrous ions, and up to 2.3 mm/sec when greatly different proportions of ferric and ferrous ions occupy the octahedral site. These extremely wide peaks are attributable to variable amounts of charge transfer between iron ions, depending on the ion distribution around each site.

Although a minor tetrahedral Fe$^{2+}$ component is present in some of the magnetic spectrum samples, its spectral contribution is too weak to show a third discernable sextet. Lack of resolution of this sextet no doubt contributes to the apparent broadened peakwidths of the two fitted sextets.

Several room temperature spectra in the central range of this composition space showed both paramagnetic and magnetic contributions of varying proportions. The paramagnetic component is due to the interaction of smaller magnetic domains within the samples, i.e. superparamagnetism (Greenwood and Gibb, 1971, p. 104).

The presence of both paramagnetic and magnetic components yield severely overlapped, low resolution spectra. Sample 34 (Zn$_{0.75}$Fe$_{2.25}$O$_4$) shows a largely paramagnetic spectrum with enough magnetic component to produce an undulating background, similar to that observed by Topsoe et al. (1974, Fig. 2-B). Sample 21 (Zn$_{0.50}$Fe$_{1.50}$Cr$_{1.0}$O$_4$) consists of about half paramagnetic and half magnetic components at room temperature, yielding a series of broad humps spanning the entire velocity range, similar to Figure 2, Grandjean and Gerard (1978). Although the paramagnetic component decreased when the temperature was lowered (also seen by Topsoe et al. 1974), individual peaks were still difficult to resolve.

**Site distribution calculations**

**Mössbauer parameters**

Using the chemical compositions of the starting mix and the relative peak areas of the Mössbauer spectra, site...
distributions for each of the synthetic spinels were calculated. The calculation is straightforward for paramagnetic spectra, where the ferric iron is confined to octahedral sites and the ferrous iron is in tetrahedral sites. The assumptions involved are: all Zn is in tetrahedral coordination, all Cr is in octahedral coordination and all the sites of the spinel structure are full.

For the magnetic spectra, only the tetrahedral ferric iron to total iron area ratio was used for the calculation. The Mössbauer peaks corresponding to the tetrahedral site are narrower and better resolved than those corresponding to the octahedral peaks.

From the calculated site occupancy, the inversion parameter \( \lambda \) is found (i.e., the amount of tetrahedral ferric iron, see Table 2, \( \lambda \) Möss. meas.). The estimated error in the inversion parameter is \( \pm 0.06 \) when errors in fit and relative areas on the Mössbauer spectra are taken into account.

**Cell dimensions**

Distinguishing between tetrahedral and octahedral ions using Mössbauer spectroscopy alone is somewhat imprecise, especially when magnetic or mixed magnetic and paramagnetic spectra are involved. Furthermore, changes in site occupancy that involve less than 5% of the total iron are not generally distinguishable by Mössbauer measurements. For these reasons, estimations of site occupancies were also made using unit cell edge measurements.

Interatomic distances (such as the tetrahedral atom-oxygen bond length and octahedral atom-oxygen bond length) can be calculated from a knowledge of the spinel cell dimension, \( a \), and the single variable positional parameter in the spinel structure, \( u \). (The oxygen atom lies on a 3-fold axis and has fractional coordinates \((u,u,u)\).) Conversely, the values of \( a \) and \( u \) can be calculated from a knowledge of the \( t-o \) (tetrahedral atom-oxygen) and \( m-o \) (octahedral atom-oxygen) bond lengths. The operative equations are:

\[
a_{\text{calc}} = \frac{2 d_{m-o} - d_{l-o}}{\sqrt{\frac{25}{48} d_{l-o}^2 + \frac{11}{16} (d_{m-o} - d_{l-o}) - \frac{5}{\sqrt{48}} (d_{l-o})}}
\]

\[
u_{\text{calc}} = \frac{d_{l-o}}{\sqrt{3} a_{\text{calc}}} + \frac{1}{8}
\]

where

- \( d_{m-o} \) = average octahedral atom-oxygen distances for that composition.
- \( d_{l-o} \) = average tetrahedral atom-oxygen distances for that composition.

\( a_{\text{calc}} = \) calculated unit cell edge.
\( u_{\text{calc}} = \) calculated oxygen atom fractional coordinate.

Related equations were derived to predict \( v \) and from this in turn, \( a \), for end member composition spinels by Hill et al., (1979). (These were also used by O’Neill and Navrotsky (1983) to calculate best fit cation radii for spinel oxides.)

Thus, given a spinel composition and an appropriate set of characteristic cation-oxygen bond lengths, the site occupancy can be varied until the calculated cell dimension matches the observed cell dimension. In considering the possible site distributions, it was assumed that where Zn is present, it is in tetrahedral coordination and where Cr is present, it is in octahedral coordination. Where both ferric and ferrous iron are present on the octahedral site, it is assumed that equal numbers of \( \text{Fe}^{3+} \) and \( \text{Fe}^{2+} \) are involved in charge hopping, i.e., the extra electron is shared between two octahedral sites creating a net charge of 2.5. The remaining iron, either \( \text{Fe}^{3+} \) or \( \text{Fe}^{2+} \), is not participating in charge hopping.

This latter assumption is probably not strictly correct as ratios other than 1:1 are possible, as shown by Ok and Evans, (1976), Evans and Ok, (1977), and Ok et al., (1978), but it was initially employed to simplify the calculations. Ignoring charge-hopping on the octahedral site altogether yield calculated unit cell edge dimensions that were much larger than measured values. Calculations made with octahedral Zn or tetrahedral Cr also yielded spurious results.

The values used for characteristic cation-oxygen distances must be chosen carefully. They should not be very different from those measured in other natural crystalline oxides (as given, for example, by Shannon 1976, or Brown and Wu, 1976). They must also be chosen so that the difference between calculated and measured unit cell dimensions of the end members are less than the error in measurement (0.001 Å).

For tetrahedral ferric and octahedral \( \text{Fe}^{2+} \) measured cation-oxygen distances from end member magnetite (Fleet, 1981) were used. The other values were chosen to fit the measured unit cell edge dimensions for the other end members of this spinel quadrilateral and, where known, their measured single variable positional parameters, \( u \). Unfortunately, this is an underconstrained system of equations (because the \( u \) parameters are not well known) so the values chosen here are not unique. That is, other values of cation-oxygen distances within 0.02 Å of those chosen will fit the end members just as well, as long as the difference between \( \text{Fe}^{2+} \) and Zn t-o distance is 0.035 Å and the difference between Cr and \( \text{Fe}^{3+} \) m-o distance is 0.041 Å. Calculated cation-oxygen distances for spinels by O’Neill and Navrotsky (1983) do not completely satisfy these constraints. The difference in their Cr and \( \text{Fe}^{3+} \) O distance is 0.030 Å and their tetrahedral \( \text{Fe}^{3+} \) oxygen distance is 1.865 Å rather than 1.888 Å as measured by Fleet (1981).

It is important that the cation-oxygen distances be fit to
the end members, because a change in 0.0007 Å in the
octahedral atom-oxygen distance or 0.004 Å in the octa-
mederal atom-oxygen distance changes the calculated unit
cell dimension by 0.001 Å, which is the typical error in
measurement. Also, because of the strong site prefer-
ences of chromium and zinc, it can be assumed for the
end members that a negligible amount of iron ion site
exchange has occurred. For tetrahedral sites the values
used are (in Ångstroms): Fe$^{2+}$ 2.015, Fe$^{3+}$ 1.988 and Zn
1.980. The octahedral values are: Fe$^{2+}$ 2.154, Fe$^{3+}$ 2.026,
Fe$^{2+}$ 2.059 and Cr 1.985.

From the calculated site occupancies, the inversion
parameter λ is found (i.e., the amount of tetrahedral ferric
iron), see Table 2 (λ UCE calc.) and Figure 3. The
estimated error for this calculation is ±0.02 when errors
in unit cell dimension and cation-oxygen distances are
both taken into account. For a complete tabulation of
these calculations, see Marshall, 1983.

**Discussion**

Before considering the general quaternary system, it is
useful to first review the binary joins. The ZnCr_2O_4-
FeCr_2O_4 join shows simple substitution of one ion for
another on the tetrahedral site, as can be verified by the
Mössbauer measurements. The simple substitution re-
ults in linear changes in unit cell edge with composition.
The ZnFe_2O_4-ZnCr_2O_4 join similarly shows simple sub-
stitution on the octahedral site. Measured unit cell di-
ensions of this study are in agreement with those of Oles' (1970). Measured Mössbauer parameters are consistent
with those of Muthukumarasamy et al. (1980). The calcu-
lated site occupancies using unit cell dimensions are also
consistent with simple substitution along both of these
binary joins.

The FeCr_2O_4-Fe_3O_4 join (chromite–magnetite), has
been well studied. Francombe (1957) and Levinstein et al.
(1972) found extraordinarily nonlinear changes in unit cell
dimension with composition. Robbins et al. (1971) and
Petric and Jacob (1982) presented predicted site distribu-
tion along this binary join in the basis of Mössbauer
spectra and magnetite activity measurements respectiv-
ely. Ok et al., (1978) predicted site occupancies for the
magnetite-rich end members.

Calculated unit cell dimensions (equation 1) utilizing
predicted site occupancies of Robbins et al. follow the
general trend for changes in unit cell with composition
(within 0.01 Å) as measured by Francombe (1957) and
Levinstein et al. (1972) (see Fig. 1). The predicted site
occupancies of Petric and Jacob (1982), however, show
trends in calculated unit cell edge very different than
those measured. This discrepancy could be due in part to
the difference in experimental conditions (1100°C for
Robbins et al. vs. 1400°C for Petric and Jacob), but the
fact that Petric and Jacob neglected to take into account
charge hopping on the octahedral site is also a factor.

In the present study, the “best fit” estimates of site
occupancies along the magnetite chromite join are only
slightly different from the predictions made by Robbins
et al. (1971). For the composition range Cr_0.8Fe_2.2O_4-
FeCr_2O_4 (see Fig. 1), some ferric iron substitution into
the tetrahedral site was necessary to produce a good fit
(±0.005 Å) between calculated and measured unit cell
dimension. For the composition range Fe_3O_4-Cr_0.8
Fe_2.2O_4, the calculated values using Robbins et al.’s
predictions and the initial assumptions in this study are
larger than the measured values by up to 0.007 Å, well
beyond measurement error.

The discrepancy is a result of assuming that charge
hopping only involves equal numbers of Fe$^{2+}$ and Fe$^{3+}$.
Whenever both Fe$^{2+}$ and Fe$^{3+}$ ions are present on the
octahedral site at appropriate hopping temperatures and
in sub-equal amounts, the “extra” electrons are shared
among all adjacent iron atoms, not only on a 1:1 basis. A
similar conclusion has been reached for magnetite-like
spinels by other authors (Adler, 1968; Goodenough, 1967;
Dobson 1970; Evans and Ok, 1977; Wu and Mason, 1981;
Trestman-Matts et al. 1984). To test this hypothesis, the
unit cell dimension was recalculated assuming that all the
octahedral iron atoms participate in charge hopping (i.e.,
the electrons form a conduction band on the octahedral
site), and that the cation-oxygen distance for the charge
hopping species is the same as measured in magnetite
(Fleet et al., 1981) where the ratio is 1:1. This assumption
produced a very good fit (±0.005 Å) throughout the
compositional join (see Fig. 1, dotted curve). The cation
distribution near the magnetite end member is essentially
deeply the predictions of Ok et al. (1978).

The ZnFe_2O_4-Fe_3O_4 join (franklinite–magnetite) has
also been well studied (Popov et al., 1963; Dobson et al.,
1970; Ok and Evans 1976; Srivastava et al., 1976; Evans
and Ok, 1977), though no one study included both XRD
and Mössbauer work across the entire join, and the

![Fig. 1. Unit cell edge as a function of composition along the
chromite–magnetite join. X's are measured values from
Levinstein et al. (1972). Solid line is calculated using equation (1)
assuming charge hopping only among equal numbers of Fe$^{2+}$ and
Fe$^{3+}$. Dashed line is calculated using the assumption that all the
octahedral iron atoms are charge hopping.

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*Table 2: Site Calculations for Various Compositions*

<table>
<thead>
<tr>
<th>Composition</th>
<th>Fe$^{2+}$</th>
<th>Fe$^{3+}$</th>
<th>Zn$^{2+}$</th>
<th>Cr$^{3+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FeCr_2O_4</td>
<td>2.015</td>
<td>1.988</td>
<td>1.980</td>
<td>2.059</td>
</tr>
<tr>
<td>CrFe_2O_4</td>
<td>1.980</td>
<td>2.059</td>
<td>2.026</td>
<td>1.985</td>
</tr>
<tr>
<td>Fe_2O_4</td>
<td>2.026</td>
<td>2.059</td>
<td>2.059</td>
<td>1.985</td>
</tr>
</tbody>
</table>

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*Figure 3: Unit Cell Dimension vs. Composition*

- Solid line: Calculated using the assumption that charge hopping among equal numbers of Fe$^{2+}$ and Fe$^{3+}$.
- Dashed line: Calculated using the assumption that all octahedral iron atoms are charge hopping.

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*Figure 1*: Unit cell edge as a function of composition along the chromite–magnetite join. X's are measured values from Levinstein et al. (1972). Solid line is calculated using equation (1) assuming charge hopping only among equal numbers of Fe$^{2+}$ and Fe$^{3+}$. Dashed line is calculated using the assumption that all the octahedral iron atoms are charge hopping.
Dobson and Srivastava studies are not completely consistent. For these reasons, three spinels were synthesized along this join, at 50%, 75%, and 100% franklinite mole proportion.

Unit cell edge measurements of these samples and those of Ok and Evans (1976) are consistent with those made by Popov et al. except for a constant of 0.004Å (Popov et al.'s data are consistently 0.004Å larger than this study's data and other published data on the end members). The unit cell edge vs. composition relation is not as sigmoidal as found on the chromite-magnetite join but does show a change in slope at 70 mole% franklinite, which has been correlated with a change in entropy by Popov et al. (1963).

The Mössbauer measurements of the present study along this join are consistent with those of Dobson et al. (1970). Franklinite shows a paramagnetic spectrum at room temperature. Between Zn$_{0.75}$Fe$_{2.25}$O$_4$ and Zn$_{0.60}$Fe$_{2.4}$O$_4$, spectra with paramagnetic and ferrimagnetic components are observed, with the paramagnetic component decreasing with decreasing Zn component. For samples less zinc rich than Zn$_{0.60}$Fe$_{2.4}$O$_4$, the spectra are entirely magnetic with all of the octahedral ions participating in charge transfer (as in Ok and Evans, 1976). Srivastava et al. (1976) found mixed spectra at 80 mole% franklinite and attributed the paramagnetic component to interactions of small magnetic domains in the crystals.

The tetrahedral site occupancy (calculated from cell dimensions and measured by Mössbauer spectroscopy) along this join between Fe$_3$O$_4$ and Zn$_{0.75}$Fe$_{2.25}$O$_4$ shows almost complete substitution of ferric iron for Zn. Between Zn$_{0.75}$Fe$_{2.25}$O$_4$ and ZnFe$_2$O$_4$, both ferric and ferrous ions substitute for Zn, with the larger proportion being ferrous (probably because of its more comparable size to Zn ions).

In Figure 2, contours of unit cell edge over the entire composition space studied are presented, incorporating data from this study and from Levinstein et al. (1972) and Popov et al. (1963) for the magnetite–chromite and magnetite–franklinite joins respectively. Although not all contours in Figure 2 are equally well constrained by the data, this particular configuration of contours is presented because it is most consistent with the site occupancy to be discussed below. The changes in unit cell edge in this composition space are due not only to changes in composition, but also reflect changes in site preference of the iron atoms, as the normal to inverse transition takes place.

It is also interesting to note that on each of the normal to inverse binary joins in this system, the onset of magnetic ordering at room temperature coincides with an inflection in the unit cell dimension and a sharp increase in the amount of ferric iron substituting on the tetrahedral site (i.e., a sharp increase in $\lambda$).

Figure 3 presents contours of the amount of tetrahedral ferric iron (that is, the inversion parameter, $\lambda$) throughout the composition space, as calculated using equation 1 and the measured and interpolated unit cell edge dimensions shown in Figure 2. The assumption used was that all
octahedral iron is charge hopping when there is a near 1:1 ratio of ferric to ferrous iron. The cation distribution of any spinel in this composition space (formed near 1000°C), can be determined using Figure 3.

To discuss the degree of inversion as a function of composition, it is useful to divide the composition space into three regions (as was done in Robbins et al., 1971, for the magnetite-chromite join). Although the boundaries between regions are not strictly constrained, Figure 4 shows the general form of the rate of change of inversion with composition, $d\ln I/dx$. The diagram was constructed by drawing compositional joins between points on the $ZnCr_2O_4-ZnFe_2O_4$ and $ZnCr_2O_4-FeCr_2O_4$ joins and magnetite (see dotted lines, Fig. 4). Along each of these compositional joins, the rate of inversion with composition was calculated. Boundaries separating regions with $d\ln I/dx < 1$, $d\ln I/dx > 1$ and $d\ln I/dx = 1$ were drawn. The error in placement of these boundaries is not more than 10 mole %.

In Region I, both $I$ and $d\ln I/dx$ are small, that is, very little ferric iron substitutes onto the tetrahedral site. This region can be described in terms of normal spinel site preferences arising from cation size and bonding considerations. In general, for the tetrahedral site, the preferences are: $Zn > Fe^{2+} > Fe^{3+} >> Cr$ and for the octahedral site: $Cr > Fe^{3+} > Fe^{2+} > Zn$. The difference in tetrahedral site preference between $Fe^{2+}$ and $Fe^{3+}$ is not very large, as is shown by the presence of small amounts of tetrahedral ferric iron.

In Region II, $d\ln I/dx$ is greater than one. This is a region of high rate of change of inversion with composition signifying a sudden transition from the normal spinel ordering scheme of Region I to the inverse spinel ordering scheme of Region III. Note that this region is not present near the franklinite-magnetite join because the presence of $Zn$ in the tetrahedral site limits the possible degree of inversion.

In Region III, $d\ln I/dx \approx 1$, that is, the rate of change of inversion is essentially constant with changes in composition. As the composition becomes more magnetite rich in this region, ferrous ions substitute directly onto the octahedral site (ferric ions onto the tetrahedral site) allowing charge hopping among octahedral iron atoms. Charge hopping stabilizes the inverse spinel by raising the entropy (de-localizing electrons) and lowering the strain in the structure (more similar-sized ions on the tetrahedral and octahedral sites). This region can be discussed in terms of a different site preference model than in Region I. That is: tetrahedral $Zn > Fe^{2+} > Fe^{3+} >> Cr$ and octahedral $Cr > Fe^{3+} > Fe^{2+} > Zn$. The energy saved by octahedral charge hopping ($Fe^{2+}$) more than compensates for the energy lost by replacing ferrous iron with ferric on the tetrahedral site, and so the inverse arrangement is preferred over the normal cation arrangement at these compositions.

Many factors influence the normal to inverse transition in spinel oxides. In this system, one important feature stabilizing the inverse spinel is the tendency of the iron atoms on the octahedral site to charge hop. This conclusion is similar to that of Robbins et al. (1971) who concluded from the Mössbauer spectra that along the magnetite-chromite join between $Fe_{3}O_4$ and $Fe_{1/2}Cr_{1/2}O_4$, the octahedral site maintained equal amounts of ferric and ferrous iron. The main difference between these conclusions is the phrase “equal amounts”. As was previously discussed (this study and Ok et al., 1978), a 1:1 correspondence of ferric and ferrous ions is not essential for charge transfer.

**Applications**

Octahedral site charge hopping is an important stabilizing phenomenon for iron rich inverse spinels, and must be taken into account in thermodynamic models of spinels (or any mineral system with ferric and ferrous ions in adjacent octahedral sites). Of geologic importance are the thermodynamic models using spinels as petrogenetic indicators (Sack, 1982).

In the spinel-olivine geothermometer (Irving, 1965), the partitioning of elements between olivine and spinel is assumed to be dependent only on the temperature of formation and mole fraction of ions on the octahedral site of the spinel. Roeder et al. (1979), however, showed that higher ferric iron spinels yield greater $Fe^{2+}/Mg$ ratios, and that the $Fe^{2+}/Mg$ distribution between ferric rich spinels and olivine is essentially independent of temperature and therefore not useful as a geothermometer. In view of the present study, this observation of Roeder et al. seems not only reasonable but expected, because a ferric rich spinel will incorporate as much ferrous iron in its octahedral sites as possible, to satisfy its charge.
hopping capacity. Consequently, the $\text{Fe}^{2+}/\text{Mg}$ ratio will be dependent more on the amount of ferric iron present in the spinel than the temperature of formation.

The change in site preference which results in the normal to inverse cation distribution is probably related to miscibility gaps in reported spinel systems, as shown theoretically by O’Neill and Navrotsky, (1984). One does not find nor would one expect a miscibility gap along the $\text{ZnCr}_2\text{O}_4 - \text{ZnFe}_2\text{O}_4$ or $\text{ZnCr}_2\text{O}_4 - \text{FeCr}_2\text{O}_4$ joins because the compositional changes result in simple substitution of one similar sized cation for another on the octahedral and tetrahedral sites respectively.

Along the franklinite–magnetite join, although a change in site preference does occur, the site substitution rate is still constant with composition. A miscibility gap is not expected until much lower temperatures are reached (O’Neill and Navrotsky, 1984). This join does not fit their model exactly, however, probably in part because they did not take into account conduction electrons on the octahedral site. Valentino and Scarl (1982) studied this join at temperatures as low as 500°C, and no miscibility gap was observed. Exsolution lamellae of magnetite in franklinite have been found by Burke and Kieft (1972), but no temperatures were estimated for this occurrence.

Along the chromite–magnetite join, a rapid exchange of ferric and ferrous ions occurs (Region II, Fig. 4). Compositions within this region of high rate of site substitution might be expected to exsolve at lower temperatures into a charge hopping species (inverse) and a non-charge hopping species (normal). Cremer (1969) studied the magnetite–chromite join and found a miscibility gap in this region at 900°C, although other studies have placed the miscibility gap as low as 500°C (Evans and Frost, 1975). The miscibility gap has also been calculated thermodynamically by O’Neill and Navrotsky (1984).

Increased pressure should also favor exsolution. As can be seen in Figure 1, the volume of normal and inverse spinels would be substantially smaller than that of the homogenous solid solution. Similarly, Figure 4 shows that the binary join connecting $\text{ZnCr}_2\text{O}_4$ with magnetite, crosses Region II and, by analogy with the magnetite–chromite join, a miscibility gap in this system is to be expected. No studies have yet been made at low temperatures or high pressures along this join.

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