Thermodynamics of nonconvergent cation ordering in minerals: III. Order parameter coupling in potassium feldspar

MICHAEL A. CARPENTER, EKWARD K. H. SALJE
Department of Earth Sciences, University of Cambridge, Downing Street, Cambridge CB2 3EQ, U.K.

ABSTRACT

The thermodynamic consequences of Al-Si ordering in potassium feldspar under equilibrium conditions have been described quantitatively using a Landau free energy expansion in two order parameters. The first order parameter, $Q_1$, relates to nonconvergent ordering between $T_1$ and $T_2$ sites and the second, $Q_2$, relates to the symmetry-breaking ordering between $T_{10}$ and $T_{1m}$ sites. Strong interactions occur between the two order parameters, as described by a term in $\lambda Q_1 Q_2^*$, and the form of the resulting expansion is

$$G = -hQ_1 + \frac{1}{2}a(T - T_0)Q_1^2 + \frac{1}{2}cQ_1^4$$

$$+ \frac{1}{2}a_{Q_2}(T - T_{Q_2})Q_2^2 + \frac{1}{2}c_{Q_2}Q_2^4 + \lambda Q_1 Q_2^*.$$ 

The coupling coefficient, $\lambda$, was assumed to be temperature dependent, as $\lambda = -20.132$, and a favored set of values for the coefficients is $h = 1650$ J/mol, $a = 22.28$ J/mol·K, $T_0 = 586$ K, $c_1 = 14700$ J/mol·K, $a_{Q_2} = 41.09$ J/mol·K, $T_{Q_2} = 239$ K, and $c_{Q_2} = 9818$ J/mol. The monoclinic = triclinic transition temperature was assumed to be 753 K.

Equilibrium variations of $Q_1$ and $Q_2$ calculated from the expansion give small changes in energy associated with $Q_1$ ordering in the stability field of monoclinic crystals. The monoclinic = triclinic transition is first order, with a jump in $Q_2$ from 0 to 0.94, a jump in $Q_1$ from 0.40 to 0.95, a latent heat of 7.2 kJ/mol, and a latent entropy of 9.5 J/mol·K. Below the transition, relatively large energy changes accompany $Q_1$ and $Q_2$ ordering. Coupling between $Q_1$ and $Q_2$ appears to outweigh coupling between $Q_2$ and the triclinic strain in crystals with the modulated orthoclase structure, with the result that both $Q_1$ and $Q_2$ may be close to being homogeneous in orthoclase and adularia.

INTRODUCTION

In the first and second papers of this series (Carpenter et al., 1994; Carpenter and Salje, 1994), the use of a Landau free energy expansion to describe nonconvergent cation ordering in minerals was introduced. To show that the approach is perfectly general, the analysis of a system with two order parameters, one convergent and the other nonconvergent, is now presented. In principle the treatment of order parameter coupling that involves a component of nonconvergent ordering should be the same as for any normal phase transition with coupled order parameters; only details of which terms in the free energy expansion are allowed by symmetry may differ. Potassium feldspar is also of interest as an important geological material, and the nature of the sanidine = microcline transition has been the subject of considerable speculation and debate. The two ordering processes involved are the nonconvergent partitioning of Al and Si between $T_1$ and $T_2$ sites and the convergent partitioning between $T_{10}$ and $T_{1m}$ associated with a symmetry change $C2/m = CI_T$.

The literature on alkali feldspars is extensive, and readers are referred to the reviews of Ribbe (1983a, 1983b, 1984), Kroll and Ribbe (1983), Yund and Tullis (1983), Smith and Brown (1988), and Brown and Parsons (1989) for a comprehensive survey. However, almost the only attempt to model the complete ordering behavior of potassium feldspar appears to be that of Thompson and coworkers (Thompson, 1969, 1970; Thompson et al., 1974), in their pioneering use of series expansions to describe thermodynamic changes in minerals. Part of the problem has been that there are few reliable data relating to equilibrium in a critical temperature range below ~800 K. In this range, the thermodynamic behavior has to be reconstructed indirectly from measurements at higher temperatures and observations of natural samples. On the experimental side, high-quality calorimetric data are available for natural and heat-treated samples, though the enthalpy variations appear to correlate only with the nonconvergent order and show no deviations that might obviously be ascribed to the phase transition (Waldbaum and Robie, 1971; Hovis, 1974, 1988). Direct ordering
and disordering experiments have been successful in bracketing the equilibrium degree of nonconvergent order at high temperatures but not so successful at low temperatures (summarized in Fig. 3 of Brown and Parsons, 1989).

Evidence from natural specimens for the evolution of equilibrium ordering states at low temperatures comes in two forms. On the scale of a metamorphic terrain, it has been possible to estimate the equilibrium transition temperature by characterizing the structural states of individual feldspar crystals as a function of paleotemperature. A transition temperature of approximately 450–500 °C seems to be accepted (Bernotat and Bambauer, 1982; Bambauer and Bernotat, 1982; Bambauer et al., 1984; Kroll, 1991; Kroll et al., 1991). On an electron optical scale, interpretations of possible equilibrium states below this temperature have been clouded by the development of complex microstructures. The monoclinic modulated orthoclase structure is believed to be metastable with respect to triclinic microcline, while intermediate (triclinic) microclines seem to be composed of orthoclase + microcline intergrowths (Eggleton and Buseck, 1980; Fitz Gerald and McLaren, 1982; McLaren, 1984; Krause et al., 1986; Kroll et al., 1991; and see discussion by Brown and Parsons, 1989; Kroll et al., 1991). On the other hand, there seems to be no doubt that microcline with well-developed transformation twinning tends to show a degree of order that is in excess of ~95% (Bambauer and Bernotat, 1982; Bernotat and Morteani, 1982; Kroll and Ribbe, 1983, 1987; Ribbe, 1984). With regard to the nature of the monoclinic = triclinic phase transition, it remains possible to choose between two hypotheses. For example, Kroll and Voll (in Ribbe, 1983a), Kroll (1991), and Kroll et al. (1991) favored a first-order transition with a significant jump in the equilibrium degree of order, whereas Brown and Parsons (1989) concluded that the assembled evidence is more consistent with a continuous transition.

Rather than interpreting the available data for potassium feldspar in isolation, an alternative strategy for predicting the possible behavior might be to compare it with the more fully documented behavior of albite. This is only partially informative, however, because the driving mechanism for the C2/m = C1 transition in albite is different, and triclinic states remain stable up to ~980 °C. The purely displacive part of the transition in albite has an equilibrium transition temperature that reduces steeply with increasing substitution of K for Na (Kroll et al., 1980). The pure order-disorder part has a lower transition temperature, but that appears to change only slightly with increasing K content. The transition lines must cross at some intermediate composition (e.g., see Fig. 6 of Brown and Parsons, 1989, or Fig. 9 of Carpenter, 1988). The result is, firstly, that the sequence of ordering and displacive driving forces is reversed between the endmembers, and, secondly, that the role of nonconvergent ordering in potassium feldspar assumes a significance over a much wider temperature interval than it does in sodium feldspar. Furthermore, although at least two order parameters are needed to describe the thermodynamic behavior of albite and potassium feldspar, the significant ones turn out to be different in each case. Symmetry-breaking (convergent) Al-Si ordering is common to both. In albite the second is the symmetry-breaking displacive order parameter. As will be argued here, for potassium feldspar the important second effect appears to relate to nonconvergent ordering between the T1 and T2 sites, and not, as was suggested by Carpenter (1988), to the displacive effects.

The paper is divided into five sections. The background to an appropriate Landau free energy expansion is first outlined briefly. An analysis of the spontaneous strains accompanying ordering then follows. In a sense this merely formalizes the well-known dependence of lattice parameters on structural states (cf. Kroll and Ribbe, 1983, 1987; Ribbe, 1984; Hovis, 1986), but it is needed to demonstrate both that the Landau approach can be valid and that a common-strain mechanism for the coupling is unlikely. Values for the Landau coefficients are then extracted from the available experimental data, and a number of different solutions are explored to establish the uniqueness, or otherwise, of each possible set of coefficients. Finally, the implications of the model solutions for both the equilibrium phase relations and the structural state of modulated crystals are discussed. Throughout, the emphasis is on an initial need to justify the Landau approach, followed by the more specific need to produce a thermodynamic model that accounts explicitly and quantitatively for the equilibrium states of potassium feldspar. The mechanisms of structural evolution under nonequilibrium conditions are not considered.

**Landau Theory**

Two independent order parameters are used here to describe the extent of Al-Si ordering in potassium feldspar. Q describes the nonconvergent ordering of Al and Si between T1 and T2 sites as

\[ Q = \frac{X_{Al}^{T1^o} - X_{Al}^{T1^m}}{X_{Al}^{T1^o} + X_{Al}^{T1^m}} \]  

where \( X_{Al}^{T1^o} \) is the fractional occupancy of the T1 site by Al, etc. Q corresponds exactly to the Z order parameter of Thompson (1969) and many subsequent workers. Given that \( X_{Al}^{T1^o} + X_{Al}^{T1^m} = 0.5 \), Q may be written in a simplified form (Harris et al., 1989; Kroll and Knitter, 1991; Salje and Kroll, 1991) as

\[ Q = 4X_{Al}^{T1^o} - 1. \]  

Q describes the symmetry-breaking ordering (C2/m = CT) between T1o and T1m sites as

\[ Q = \frac{X_{Al}^{T1^o} - X_{Al}^{T1^m}}{X_{Al}^{T1^o} + X_{Al}^{T1^m}}. \]  

This form was introduced by Salje (1985) and is scaled
Following the approach set out in the first and second papers of this series (Carpenter et al., 1994, and Carpenter and Salje, 1994), the nonconvergent ordering is described by a Landau free energy expansion of the form

\[ G = -hQ + \frac{1}{2}a(T - T_c)Q^2 + \frac{1}{4}bQ_1^2 + \frac{1}{6}cQ_2^3 + \frac{1}{6}dQ_3^3. \]  

(4)

The linear term accounts for an energetic difference due to locating Al on T1 sites and Al on T2 sites that remains at all temperatures, and the \( h \) coefficient is thus an effective field, ensuring that the fully disordered state (\( Q_x = 0 \)) does not have an equilibrium field of stability. Higher odd-order terms in \( Q \) are allowed by symmetry, since the ordering is nonconvergent, but these are assumed to be small. The remaining coefficients are typical of standard Landau free energy expansions, with the subscript 1 used to associate them with \( Q_1 \). As will be seen, there are insufficient experimental data to justify the inclusion of both fourth-order and sixth-order terms in the expansion. Trial values for the coefficients were therefore obtained with only \( Q_1^2 \) as the high-order term or with only \( Q_2^3 \) as the high-order term.

\( Q_2 \) describes the monoclinic = triclinic transition, and, by symmetry, odd-order terms in the free energy expansion must be strictly zero. Again, there are insufficient experimental data to warrant an expansion with more than one high-order term. Given the apparent tendency for order-disorder processes in minerals to have small fourth-order terms (e.g., CO orientational ordering in calcite: Redfern et al., 1989; cation ordering in omphacite: Carpenter et al., 1990), the expansion chosen to describe the contribution of \( Q_2 \) alone has the form for tricritical phase transitions:

\[ G = \frac{1}{2}a_{od}(T - T_{od})Q_2 + \frac{1}{6}c_{od}Q_2^3. \]  

(5)

The subscript \( od \) is used here to associate the coefficients with \( Q_{od} \).

Although \( Q_1 \) and \( Q_{od} \) are by definition independent order parameters in a geometric sense, they are expected to interact. In other words, a change in T1 and T2 ordering may induce a change in the degree of T10 and T1m ordering, and the energy changes due to such interactions may be accounted for by a coupling term in the free energy expansion. The lowest order term allowed by symmetry is linear in \( Q_1 \) and quadratic in \( Q_{od} \). For a complete description of the thermodynamics of ordering, the excess free energy (with respect to complete disorder, \( Q_x = Q_{od} = 0 \)) is then given by

\[ G = -hQ + \frac{1}{2}a(T - T_c)Q^2 + \frac{1}{4}bQ_1^2 + \frac{1}{6}cQ_2^3 + \frac{1}{6}dQ_3^3 + \frac{1}{4}eQ_1Q_{od} + \frac{1}{6}fQ_1Q_{od}^2 + \frac{1}{6}gQ_2Q_{od}^2 + \frac{1}{6}hQ_3Q_{od}^2. \]  

(6)

The coefficient \( \lambda \) defines the strength of the linear-quadratic coupling.

Conditions for equilibrium derived from Equation 6 are

\[ \frac{\partial G}{\partial Q} = 0 = -h + a(T - T_c)Q_1 \]

\[ + bQ_1^2 \text{ (or } cQ_1^3) + \lambda Q_2Q_{od} \]  

(7)

and

\[ \frac{\partial G}{\partial Q_{od}} = 0 = a_{od}(T - T_{od}) + c_{od}Q_2^3 + 2\lambda Q_1 \text{ (for } |Q_{od}| > 0). \]  

(8)

Excess enthalpies and entropies, again with respect to the fully disordered state, are given by

\[ H = -hQ + \frac{1}{2}a(T - T_c)Q^2 + \frac{1}{4}bQ_1^2 \text{ (or } \frac{1}{6}cQ_2^3) \]

\[ - \frac{1}{2}a_{od}(T - T_{od})Q_2^2 + \frac{1}{6}c_{od}Q_2^3 + \lambda Q_2Q_{od} \]  

(9)

and

\[ S = -\frac{1}{2}(aQ_1^2 + a_{od}Q_{od}^2). \]  

(10)

Many of the background formalities to order parameter coupling have been summarized by Salje and Devarajan (1986), Carpenter (1988, 1992), and Salje (1990), and a more complete free energy expansion, including strain and gradient effects, has been given by Harris et al. (1989).

In the present approach, the role of a displacive order parameter has been assumed to be small because the extrapolated transition temperature for a purely displacive transition in potassium feldspar would be well below 0 K (Thompson et al., 1974; Thompson and Hovis, 1979; Carpenter, 1988).

**Strain Analysis**

As discussed in the first paper (Carpenter et al., 1994), the development of a spontaneous strain accompanying cation ordering processes has two immediate implications. Firstly, the strain provides an indirect measure of the macroscopic order parameter and may be used to characterize the structural states of experimental samples. Secondly, strain-order parameter coupling that leads to strains in excess of ~0.001 acts to promote classical Landau-type thermodynamic behavior (Cowley, 1976; Folk et al., 1976, 1979; Als-Nielsen and Birgeneau, 1977; Salje et al., 1987; Carpenter and Salje, 1989; Salje, 1990, 1992; Carpenter, 1992). In the case of Al-Si ordering in potassium feldspar, a third implication of the spontaneous strain relates to the possibility that coupling between the two order parameters \( Q_1 \) and \( Q_{od} \) could occur by means of strains that are common to both.

For a monoclinic crystal, a suitable Cartesian coordinate system for describing the spontaneous strains has axes \( X, Y, \) and \( Z \) such that \( Y \) is parallel to the crystallographic \( b \) axis, \( X \) is parallel to the crystallographic \( a \) axis, and \( Z \) is perpendicular to both and is thus parallel to the \( c^* \) axis of the reciprocal lattice. The general equations of Schlenker et al. (1978) and Redfern and Salje (1987) may then be used to define the components \( e_1 - e_6 \) (in Voigt notation) of the strain tensor. The appropriate expres-
Definitions of components of the spontaneous strain tensor for nonconvergent (monoclinic → monoclinic) and convergent (monoclinic → triclinic) processes in potassium feldspar

<table>
<thead>
<tr>
<th>Monoclinic → monoclinic</th>
<th>Monoclinic → triclinic</th>
</tr>
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<tbody>
<tr>
<td>$e_1 = e_{11} - \frac{a}{a_0} - 1$</td>
<td>$e_1 = \frac{a}{a_0} \sin \gamma - 1$</td>
</tr>
<tr>
<td>$e_2 = \frac{a}{a_0} - \frac{b}{b_0} - 1$</td>
<td>$e_2 = \frac{b}{b_0} - 1$</td>
</tr>
<tr>
<td>$e_3 = \frac{c \sin \beta}{c_0 \sin \beta_0} - 1$</td>
<td>$e_3 = \frac{c \cos \alpha \sin \beta^<em>}{c_0 \sin \beta_0^</em>} - 1$</td>
</tr>
<tr>
<td>$e_4 = 2e_{13} - 0$</td>
<td>$e_4 = \frac{c \cos \alpha \sin \beta^<em>}{c_0 \sin \beta_0^</em>} + \frac{d \cos \beta^* \cos \gamma}{a_0 \sin \beta_0^*}$</td>
</tr>
<tr>
<td>$e_5 = 2e_{12} - 0$</td>
<td>$e_5 = \frac{a \cos \beta}{a_0 \sin \beta}$</td>
</tr>
</tbody>
</table>

Note: These equations are derived from the general equations of Schlenker et al. (1978); $a_0, b_0, c_0$, etc., are the lattice parameters of the reference state for the strain calculations.

* Reciprocal lattice dimensions.

For monoclinic strains, $e_1$ and $e_2$ are strictly zero. Both $b$ and $c$ vary linearly with $Q_0$ for $\beta \approx \beta_0$. For $\beta \approx \beta_0$, $e_1$ and $e_3$ are also linear functions of $Q_0$, as expected for the general strain-order parameter dependence in nonconvergent systems (Carpenter et al., 1994). The component $e_2$ is expected to be small but is zero if $a_0$ is set as the value of $a$ for each crystal to exclude strains of compositional origin. In monoclinc crystals, $e_1$ and $e_2$ are zero. In monoclinic and triclinic potassium feldspar crystals, the values of the lattice parameter $a$ and the lattice angle $\beta$ change very little with changing Al-Si order. The $a$ parameter is sensitive to compositional variations, however. Both $b$ and $c$ vary significantly with the degree of order, and it is well known that their variation is close to being linear with respect to $Q_0$ (Fig. 1, data from Kroll and Ribbe, 1983, and Kroll and Knitter, 1991; see also Fig. 1a of Kroll and Ribbe, 1987, and Fig. 11 of Hovis, 1986). Values of the reference state parameters, $b_0 = 13.043 \text{ Å}$ and $c_0 = 7.172 \text{ Å}$, have been obtained from a linear least-squares fit to the data for monoclinic crystals in Figure 1, and a value of $\beta_0 = 115.98^\circ$ has been determined by similar linear extrapolation of data from the same crystals (in Table 2 of Kroll and Ribbe, 1983, and Table 3 of Kroll and Knitter, 1991). Because of its sensitivity to composition and insensitivity to Al-Si order, the reference parameter $a_0$ is most conveniently taken as the value of $a$ for each crystal of interest; this reduces any contributions to the strain from compositional effects alone. Values of $a_0 = 8.543 \text{ Å}$ and $a_0 = 8.573 \text{ Å}$, respectively, were used in relation to strains determined from the data of Kroll and Knitter (1991) and Blasi et al. (1984), and $a_0 = a$ was used for calculations based on the data of Kroll and Ribbe (1983). Finally, since the reference state is monoclinic, $a_0$ and $\gamma_0$ are both $90^\circ$. Room-temperature strains have been calculated with respect to these reference parameters for monoclinic crystals (sandine, orthoclase, and adularia, in Kroll and Ribbe, 1983; Kroll and Knitter, 1991), and for a range of triclinic crystals produced by progressively disordering natural microcline (in Blasi et al., 1984).
Fig. 1. Variation of $b$ and $c$ lattice parameters as a function of $Q$, for monoclinic crystals (circles) and triclinic crystals (Xs). Data from Kroll and Ribbe (1983), open circles, and Kroll and Knitter (1991), solid circles. Extrapolation of the linear least-squares fit to the data for monoclinic crystals gives values of $b_0 = 13.043$ Å and $c_0 = 7.172$ Å for crystals with $Q = 0$ at room temperature. The slight divergence of $b$ for triclinic crystals from the monoclinic trend suggests that $Q_{\text{ad}}$ ordering may have a small influence on the $e_z$ strain component.

relations are, firstly, that $e_2, e_3,$ and $e_5$ arise almost exclusively from coupling with $Q$, secondly, that $e_1$ and $e_4$ are not affected significantly by coupling with $Q$, in triclinic crystals, and, thirdly, that when $Q_{\text{ad}}$ has nonzero values it varies approximately linearly with $Q$, in these samples. Note that $e_1, e_5,$ and $e_5$ do not extrapolate to the origin in Figure 2 because $Q$ does not go to zero at $Q_{\text{ad}} = 0$.

Since $e_2, e_3,$ and $e_5$ appear to couple predominantly with $Q$, and $e_1$ and $e_4$ couple with $Q_{\text{ad}},$ the principal strains associated with each order parameter can be analyzed separately. Strain matrices containing only $e_1, e_2, e_3,$ and $e_5$, or only $e_1$ and $e_4$, have been diagonalized to give the magnitudes and orientations of the principal strain components $e_1, e_2,$ and $e_3$ for microcline. From the $Q$ strain matrix, the principal strain values are $e_1, Q = 0.007, e_2, Q = -0.006, and e_3, Q = -0.001$; from the $Q_{\text{ad}}$ strain matrix the values are $e_1, Q_{\text{ad}} = 0.021, e_2, Q_{\text{ad}} = 0, and e_3, Q_{\text{ad}} = -0.021$ (with the microcline lattice parameters of Blasi et al., 1984). The relative orientations of the principal strains are shown in Figure 3. The largest $Q$ strains coincide with zero values of the $Q_{\text{ad}}$ strains, and, although there is a small common strain in a plane that nearly includes $e_1, Q$, $e_2, Q_{\text{ad}}$, and $e_3, Q$, the orientation relationship is such that the total overlap is minimized. Calculations of the strain ellipsoids for the intermediate structural states of Blasi et al. (1984) show that this orientation relationship does not change significantly as a function either of $Q$, or of $Q_{\text{ad}}$.

Relative magnitudes of the total strains accompanying the two types of Al-Si ordering can be given in terms of a scalar strain, $\epsilon$, which, following Redfern and Salje (1987), is given by

$$\epsilon = \sqrt{\sum_{i=1}^{3} \epsilon_i^2}. \quad (11)$$
Fig. 3. Orientation of the principal spontaneous strains in microcline at room temperature, with respect to crystallographic axes a, b, and c and reciprocal lattice axes a*, b*, and c* of sanidine, as calculated by diagonalizing matrices with only $e_1$ and $e_2$ and $e_3$=0 ($Q_o$ ordering, open triangles), and with only $e_1$, $e_2$, $e_3$, and $e_4$=0 ($Q_i$ ordering, Xs). Numbers by each point indicate the size of the principal strain in that direction. Note that the largest principal strain due to $Q_i$ ordering coincides with a point of zero strain due to $Q_o$ ordering. The next largest $Q_i$ principal strain also coincides with a zero value for the $Q_o$ strain, and there is almost no overlap between the $Q_i$ and $Q_o$ strains. X, Y, and Z denote Cartesian reference axes for the strain calculations.

This gives $e_{Q_0} = 0.010$ and $e_{Q_o} = 0.029$ for the Blasi et al. microcline. In each case the volume strains, defined as

$$V_i = \frac{V - V_0}{V_0} \approx \sum_{i=1}^{3} e_i$$  \hspace{1cm} (12)

are close to zero. In other words, the spontaneous strains are close to being purely shear deformations.

In summary, both $Q_o$ and $Q_o$ couple with macroscopic strains to a significant extent. This would be expected to promote Landau-like thermodynamic behavior, as discussed earlier. There is very limited overlap between these two strains, however, to the extent that they appear to be mutually exclusive in orientation. The strain components of both sets of strains conform closely to the relation $Q \propto e$, expected both for nonconvergent ordering and for convergent ordering when $e_4$ has the same symmetry as $Q$. The observation that the strain components associated with each order parameter vary linearly with each other also implies that $Q_o$ varies approximately linearly with $Q_i$ in triclinic crystals. Finally, the scalar strain, $e_{Q_o}$, may provide a useful parameter for calibrating the degree of order of crystals for which only lattice parameters are known. It includes changes in the parameters $b$, $c$, and $\beta$, and so depends to a lesser extent than in some calibrations on individual lattice parameter variations. Figure 4 shows a possible calibration using strains calculated from the data of Kroll and Ribbe (1983) and Kroll and Knitter (1991). A linear least-squares fit to the data for mono-

clinic crystals is

$$Q_i = 96.00 e_{Q_o}.$$  \hspace{1cm} (13)

**CALIBRATION OF THE LANDAU EXPANSION**

**Nonconvergent ordering: $Q_o$**

For crystals with $Q_o = 0$, the equilibrium condition represented by Equation 7 simplifies to

$$0 = -1 + \frac{a_i}{h}(T - T_o)Q_o + \frac{b_i}{h} Q_i \left( \text{or} \frac{c_i}{h} Q_i^2 \right).$$  \hspace{1cm} (14)

Values of the ratios $a_i/h$, $b_i/h$ (or $c_i/h$) and the critical temperature, $T_o$, can be determined if the equilibrium variation of $Q_o$ with temperature is known. The most recent data for homogeneous sanidine crystals equilibrated at high temperatures are those of Kroll and Knitter (1991) and Salje and Kroll (1991). Their four values, $Q_i = 0.240$ at 923 K, $0.180$ at 1023 K, $0.135$ at 1123 K, and 0.092 at 1323 K, together with the assumption that $Q_i$ would equal unity at 0 K in the absence of any Ti0/T1m ordering, have been used to constrain values for these coefficient ratios. A further constraint is provided by available data suggesting that $Q_i$ has a value between 0.30 and 0.44 at the equilibrium monoclinic = triclinic transition temperature. These limits correspond to the values of $Q_i$ for the most ordered natural crystals without ortho-
Values for the coefficient ratios were found by applying a linear least-squares fitting program (kindly provided by T.J.B. Holland) to a set of six linear equations, represented by Equation 14 with each of the four experimental values of $Q_i$ and $T$ inserted, plus $Q_i = 1$ at 0 K and $Q_i \approx 0.40$ at 753 K. This was repeated with either $Q_i^s$ or $Q_i^t$ as the high-order term, and, although somewhat different numerical results were obtained, the variations of $Q_i$ with temperature that they generate are not significantly different. Subsequent calculations using $Q_i^s$ and $Q_i^t$ as the high-order term in Equation 14 were continued in parallel. Again, however, although they lead to different values for the coefficient ratios, the overall thermodynamic picture does not differ in any major way. With more data it might be possible to distinguish between the two solutions, but for the present they would be equally valid. For the sake of consistency and with the assumption that the sixth-order term tends to be important in Landau free energy expansions for describing cation ordering reactions, only results for the tricritical type of equation ($Q_i^t$ in Equation 14) are given here. Attempts to fit values for the coefficients of both the $Q_i^s$ and $Q_i^t$ terms resulted in high correlations and are not justified by the available data.

The final fit coefficients were $a_i/h = 0.0135$, $T_{ct} = 586$ K, and $c_i/h = 8.91$. These give the variation of $Q_i$ shown in Figure 5, with $Q_i = 0.402$ at 753 K. This fit differs from that used by Salje and Kroll (1991), who were interested in kinetic aspects of the $Q_i$ ordering at high temperatures, primarily because of the constraint $Q_i = 1$ at 0 K introduced here.

**Coupling between $Q_i$ and $Q_{od}$**

There are insufficient experimental data to fit values for the coefficients of $Q_{od}$ in the same way as was achieved for $Q_i$. The nature of any general solution based on Equation 6 is quite clear, however. From the two criteria for equilibrium (Eqs. 7 and 8), two solutions are $Q_i = 0$, $Q_{od} = 0$ and $Q_i = 0.98$, $Q_{od} = 0.97$ at 700 K (broken line). Light lines indicate hypothetical variations of $Q_i$ and $Q_{od}$ in the absence of coupling.

Values for the coefficient ratios were found by applying...
A second constraint is provided by the observation that natural microcline has values of \( Q \) and \( Q_{\text{cd}} \) close to 1. Attempts to find values of the coefficients in Equation 15 that reproduce such high degrees of order at geologically reasonable temperatures with \( \lambda \) as a constant failed because they also produced the unphysical result \( Q > 1 \) and \( Q_{\text{cd}} > 1 \) at lower temperatures. It is well known that the order parameter for many phase transitions levels off more sharply at high degrees of order (characteristically well below room temperature) than would be predicted using a normal Landau series expansion, and so-called order parameter saturation (Salje, 1990; Salje et al., 1991) may be implicated here. The required leveling off can be reproduced adequately by adding terms that become significant as \( Q \to 1 \), but these terms are quantifiable only when rather good experimental data for the equilibrium temperature dependence of \( Q \) over a large temperature interval are available. Such data are unlikely to be generated for potassium feldspar because of the extreme sluggishness of Al-Si diffusion at low temperatures, and, in order to explore the implications of a constant coupling coefficient, it is necessary to resort to a gross simplification. Thus, arbitrarily, \( Q = Q_{\text{cd}} = 1 \) has been selected as an equilibrium state at 400 K. Although it is clearly unphysical, it has the merit of allowing some potentially reasonable approximations for the variations in excess free energy, enthalpy, and entropy at geologically relevant temperatures to be examined.

A value of the coupling parameter, \( \lambda/h \), can be obtained by inserting the chosen equilibrium values of \( Q \) and \( Q_{\text{cd}} \) at 400 K into Equation 7, rewritten as

\[
\frac{dG}{dT} = -1 + \frac{\alpha}{h} (T - T_c) Q + \frac{c}{h} Q_i + \frac{\lambda}{h} Q_{\text{cd}}. \tag{16}
\]

Using the values of \( \alpha/h \), \( c/h \), and \( T_c \) derived above gives \( \lambda/h = -5.40 \). The same condition inserted into Equation 15, along with this value of \( \lambda/h \), then provides the second equation constraining the possible values of \( \alpha_{\text{cd}}/h \), \( T_{\text{cd}} \), and \( c_{\text{cd}}/h \).

The third constraint needed to determine values for the three unknowns is provided by the fact that the excess free energies of the monoclinic and triclinic structures must be identical at the transition temperature, 753 K. Individual pairs of trial values of \( Q \) and \( Q_{\text{cd}} \) at 753 K, as permitted by Equation 16, were substituted into Equation 15 to yield an equation in the \( Q_{\text{cd}} \) coefficient ratios for this temperature. The three equations in these coefficient ratios were then solved in each case, and the values obtained used to calculate \( G/h \) at 753 K from Equation 6 (with \( Q_i \) terms in place of \( Q^i \) terms and divided by \( h \)) for the monoclinic (\( Q = 0.402 \)) and triclinic (\( Q = 0, Q_{\text{cd}} \neq 0 \)) structures. The procedure was repeated until a set of coefficients that gave an identical value of \( G/h \) for the two structures was found, namely, \( \alpha_{\text{cd}}/h = 0.027, T_{\text{cd}} = 547 K, \) and \( c_{\text{cd}}/h = 14.8 \) (for \( \lambda/h = -5.40 \)). By substitution of these coefficient ratios back into Equations 15 and 16, the equilibrium values of \( Q \) and \( Q_{\text{cd}} \) shown in Figure 5a were determined. The picture that emerges is reasonably consistent with limiting experimental data for the variation of \( Q \), below 450 °C, as extracted by Brown and Parsons (1989) from the data of Senderov and Yas'kin (1975, 1976). The calculated values of \( Q_{\text{cd}} \) and \( Q \) are also closely similar to each other, as is found in natural microcline crystals. On the basis of the tricritical description for \( Q_{\text{cd}} \), the monoclinic = triclinic transition temperature would be expected to be 547 K if there were no coupling to \( Q \).

If saturation effects are not important for Al-Si ordering in potassium feldspar at geological temperatures and Equation 6 provides an adequate representation of the real thermodynamic behavior, it follows that the coupling coefficient must be temperature dependent. For physically valid solutions of the equilibrium order parameters (\( Q_i \leq 1, Q_{\text{cd}} \leq 1 \) at 0 K), the value of \( \lambda \) must be zero at 0 K. As a first approximation, a linear temperature dependence may be anticipated, i.e.,

\[
\lambda = a_1 T. \tag{17}
\]

A value of \( a_1/h \) has been derived, as before, from Equation 16 for each of two assumed low temperature states, which may be regarded as being representative of a range of possible behaviors: \( Q = Q_{\text{cd}} = 0.98 \) at 450 K, or \( Q = 0.80 \) and \( Q_{\text{cd}} = 0.75 \) at 700 K. Iterating through permitted equilibrium values of \( Q \) and \( Q_{\text{cd}} \) at 753 K and the consequent free energy differences between monoclinic and triclinic crystals, to ensure \( \Delta G = 0 \) at the first-order transition as before, gave \( a_1/h = 0.0249, T_{\text{cd}} = 239 K, a_{\text{cd}}/h = 5.95, a_1/h = -0.0122 K^{-1} \) and \( a_{\text{cd}}/h = 0.0254, T_{\text{cd}} = 456 K, c_{\text{cd}}/h = 11.59, a_1/h = -0.00878 K^{-1} \), respectively. The predicted variations in \( Q_{\text{cd}} \) and \( Q \) derived from substituting these values and the earlier values for \( a_1/h, T_{\text{cd}}, \) and \( C_{\text{cd}}/h \) into Equations 15 and 16 are shown in Figure 5b. The phase transition remains first order in character, but only the first result is consistent with the high degrees of order found in natural microcline. The second result, however, is more consistent with the limited experimental data below 450 °C. If higher values of \( Q_i \) and \( Q_{\text{cd}} \) had been assumed for 450 K, the first order step at 753 K would have been larger.

**Absolute values for the coefficients from calorimetric data**

Trial values for the excess enthalpy due to ordering in crystals with \( Q_i = Q_{\text{cd}} = 1 \), i.e., \( H_{Q_i} \), were used to calculate absolute values of the coefficients in conjunction with the ratios derived from the conditions for equilibrium. For the case of \( \lambda = \text{constant} \), the expression for \( H \) has already been given (Eq. 9). If \( \lambda = a_1 T \), the excess enthalpy becomes

\[
H = -hQ_i - \frac{1}{2} a_1 T_{\text{cd}} Q_i^2 + \frac{1}{6} c_{\text{cd}} Q_i^6
- \frac{1}{2} a_{\text{cd}} T_{\text{cd}} Q_{\text{cd}}^2 + \frac{1}{6} c_{\text{cd}} Q_{\text{cd}}^6 \tag{18}
\]

(from \( H = G + TS, S = -dG/dT \)). Sets of coefficients

\[=\frac{c_{\text{cd}}}{c_{\text{cd}}},\text{ thereby reducing the number of independent variables by one.}\]
calculated for \( H_{Q^{-1}} = -8000 \) or \(-9000 \) J/mol are given in Table 2. Excess enthalpies for model equilibrium values of \( Q \) and \( Q_o \) are compared with the solution calorimetric data of Hovis (1988) in Figure 6.

When comparing calculated excess enthalpies, \( H \), with experimental heats of solution, \( \Delta H_{\text{soln}} \), it is necessary to select a value for \( \Delta H_{\text{soln}} \) that will correspond to \( H = 0 \). This can be done by choosing a common scale for which the greatest overlap between calculated and experimental values occurs. In Figure 6, the data are plotted as a function of \( Q \), and \( H = 0 \) has been selected at \( \Delta H_{\text{soln}} = 613.2 \) kJ/mol. There is a good general correlation between the calorimetric results and all the model solutions, but in detail the agreement is less convincing. At small degrees of order, within the sanidine stability field \((Q_o < 0.402)\), the constant \( \lambda \) solution (solution 1 in Fig. 6) gives enthalpy changes that are too small. Increasing the value of \( \Delta H_{\text{soln}} \) gives steeper variations in \( H \) but only slightly so. The steepest variation in \( H \) over this range of \( Q \) values is given by the \( \lambda = a_1 T \) solution with \( Q = Q_o \) at 450 K and \( H_{Q^{-1}} = -9000 \) J/mol (solution 4 in Table 2 and Fig. 6). The \( \Delta H_{\text{soln}} \) values for crystals that have \( Q \) in a range characteristic of modulated orthoclase and adularia (the four central points in Fig. 6) plot on or above an extrapolated line between the equilibrium triclinic and equilibrium monoclinic values in the case of solution 4. Thus, only for this solution would a homogeneous crystal with intermediate values of \( Q \) and \( Q_o \) have a lower enthalpy than a modulated crystal with the same average \( Q \) values. The other solutions (1, 2, and 3 in Table 2, Fig. 6) would predict homogeneous structures with higher (less negative) enthalpies of ordering than those of the observed modulated structures, which is counter to the expectation that the modulation represents a defect structure carrying some positive excess energy. The lowest dotted line in Figure 6 gives the excess enthalpy for homogeneous (unmodulated) crystals with metastable degrees of order, with the assumption of \( Q = Q_o \) in Eq. 18 with the coefficients of solution 4. It is interesting to note that \( \Delta H_{\text{soln}} \) for the sample with \( Q = 0.53 \) (the Madagascar orthoclase of Hovis, 1988) plots on this line and that the crystals did not appear to contain obvious modulations when examined by transmission electron microscopy (Hovis, 1988).

The most reliable solution calorimetric data are those obtained from crystals with different degrees of order but identical impurity contents, defect chemistries, etc. Hovis (1988) reported results for ion-exchanged Amelia albite and for the same ion-exchanged Amelia albite after disordering at \( \approx 1050 \) °C (Hovis, 1974, 1988). The measured differences in \( \Delta H_{\text{soln}} \) between these two samples, \( 8380 \pm 620 \) J/mol, can be compared with \( 8780 \) J/mol calculated from solution 4 using \( Q = 0.97 \rightarrow Q = 0.081 \) (the values of \( Q \), used by Hovis, 1988). The quoted uncertainty is \( \pm 1 \sigma \). Waldbaum and Robie (1971) measured an enthalpy difference of \( 8170 \pm 1340 \) J/mol between ion-exchanged Amelia albite and disordered crystals that were ion-exchanged after the disordering. Assuming \( Q = 0.97 \) for Amelia albite and taking \( Q = 0.14 \) for albite equilibrated at \( \approx 1060 \) °C, from the structure refinement of Winter et al. (1979), the equivalent enthalpy calculated from solution 4 is \( 8600 \) J/mol. The only natural sample of Hovis (1974, 1988) that appears to have equilibrated in the

### Table 2. Sets of coefficients for four solutions to the Landau free energy expansion

<table>
<thead>
<tr>
<th>Solution no.</th>
<th>Model</th>
<th>( \lambda = \text{constant} )</th>
<th>( \lambda = a_1 T )</th>
<th>( \lambda = a_2 T )</th>
<th>( \lambda = a_3 T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>( \lambda = \text{constant} )</td>
<td>( \lambda = a_1 T )</td>
<td>( \lambda = a_2 T )</td>
<td>( \lambda = a_3 T )</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>400 K</td>
<td>700 K</td>
<td>450 K</td>
<td>450 K</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>-8000</td>
<td>-8000</td>
<td>-9000</td>
<td>-11.75</td>
</tr>
<tr>
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<td></td>
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<td>1091</td>
<td>1467</td>
<td>1650</td>
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<tr>
<td></td>
<td></td>
<td>7.83</td>
<td>14.73</td>
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<td>22.28</td>
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<td>9721</td>
<td>13070</td>
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<tr>
<td></td>
<td></td>
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<td>27.77</td>
<td>36.53</td>
<td>41.09</td>
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<td></td>
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<td>456</td>
<td>239</td>
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<td></td>
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<td>12650</td>
<td>8735</td>
<td>9818</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-11.75</td>
<td>-11.64</td>
<td>-10.27</td>
<td>-11.56</td>
</tr>
</tbody>
</table>

Note: units for \( H, h, c, c_{\text{mol}} \), and \( \lambda \) are in J/mol; \( a_1, a_2, \) and \( a_3 \) in J/mol.

### Fig. 6. Calculated enthalpy variations (left scale) compared with measured \( \Delta H_{\text{soln}} \) values (right scale) from Hovis (1988), as a function of \( Q \). The calculated variations are for solutions 1–4. In each case there is a discontinuity corresponding to the first-order break in both \( Q \) and \( Q_o \). In the cases of solutions 3 and 4, both equilibrium (dot-dash lines) and metastable (dotted lines) variations are shown. Data represented by the dotted lines were calculated for metastable triclinic crystals with \( Q = Q_o \) and the coefficients from solutions 3 and 4; they pass approximately through the four central data points from modulated crystals with values of \( Q \) typical of modulated orthoclase and adularia. Modulations were not immediately apparent on a TEM scale in the crystals with \( Q = 0.53 \), however (Hovis, 1988).
monoclinic stability field is Eifel sanidine. The measured enthalpy difference between this sample ($Q = 0.248$) and its heat-treated equivalent ($Q = 0.106$) is $1300 \pm 680$ J/mol (Hovis, 1988). For comparison, the calculated value is 562 J/mol, which, if uncertainties of approximately $\pm 0.02$ in $Q$ were taken into account, would just be within 1σ of the measured value. Similar data for orthoclase and adularia have not been compared in the same way because of the possible influence of strain modulations. Perhaps an overall assessment of the success (or otherwise) of the Landau expansion in reproducing measured enthalpy variations, however, should be left as the comparison of predictions from solution 4 with all the calorimetric data of Hovis (1988) in Figure 6, since the individual data were not used explicitly to determine the absolute values of the Landau coefficients.

Excess entropies due to ordering, with $\lambda$ = constant, are given by Equation 10. If $\lambda$ is a linear function of $T$, the excess entropy becomes

$$S = -\frac{1}{2}a_i Q_i^2 - \frac{1}{2}a_{ad} Q_{ad}^2 - a_{Q} Q_{o}^2.$$  \hspace{1cm} (19)

The excess free energies, $G$, are given by Equation 6. Equilibrium variations in $H$, $S$, and $G$ have been calculated for solutions 1 and 4 and are shown in Figure 7. Calculated latent heats and latent entropies for the $C2/m \rightarrow CT$ transition are 3.02 kJ/mol, 4.01 J/mol·K (solution 1), and 7.17 kJ/mol, 9.53 J/mol·K (solution 4), respectively.

**DISCUSSION**

The most positive aspect of the approach set out here is that it reproduces the gross features of Al-Si ordering in potassium feldspar that are believed to represent equilibrium behavior. In particular, both the discrete monoclinic $\rightarrow$ triclinic phase transition and the nonconvergent ordering in monoclinic crystals are accounted for quantitatively. Many solutions for, and slight variations of, the basic free energy expansion have been explored, and these serve to show that, in spite of the nonuniqueness of each of the resulting sets of coefficients, the same general pattern emerges. With falling temperature, small free energy changes associated with $Q_o$ ordering are followed first by a first-order transition and then by relatively large energy changes accompanying $Q_i$ and $Q_{ad}$ ordering. The coupling term in $Q_i$ and $Q_{ad}$ causes the transition to become first order if it were continuous without coupling or would enhance the first-order character if the uncoupled transition were already discontinuous. The assumed tricritical behavior of pure $Q_{ad}$ ordering is probably not unreasonable in the light of closely similar behavior for pure ordering in albite (Salje et al., 1985). The solutions ascribe a substantial proportion of the excess energy associated with $Q_o$ ordering to the linear field term and a substantial proportion of the excess free energy at geological temperatures to the interactions between $Q_o$ and $Q_{ad}$.

One consequence of the strong coupling between $Q_i$ and $Q_{ad}$ is that crystals having $Q_{ad} \neq 0$ would tend to

![Fig. 7](image_url)
evolve with $Q_t \approx Q_{od}$ (values of $Q_{od}$ are restricted to $\leq Q_t$ by stoichiometry). This means that changes in physical properties arising from changes in $Q_{od}$ might show a close correlation with changes in $Q_t$, explaining, for example, why no discernible break occurs in the slope of the data of Hovis (1988) for $\Delta H_{od}$ against $Q_t$ for monoclinic and triclinic crystals. According to this argument, even in the modulated structures the local values of $Q_{ot}$ may be close to the values of $Q_t$ obtained from macroscopic measurements. Such a possibility is considered in more detail in the following section. On the other hand, particular strain variations appear to arise as a consequence of coupling only with $Q_t$ or only with $Q_{od}$. Making the connection between a macroscopic change of interest and the particular order parameters on which it depends, as set out formally here, should in the future allow clearer distinctions to be made as to the relative dependence of properties on the convergent and nonconvergent ordering processes.

Even with the limited data available, there are sound reasons for favoring solution 4 as the most valid quantitative description. Firstly, it is physically reasonable and does not require approximations that give unreal results, such as $Q_t, Q_{od} > 1$. It requires a temperature-dependent coupling coefficient that is also plausible from a physical point of view. The coupling mechanism does not appear to arise from overlapping macroscopic strains, since the separate strains due to $Q_t$ and $Q_{od}$ develop in an orthogonal, i.e., mutually exclusive, manner. A microscopic mechanism involving, for example, lattice vibrations is more likely. If local strains due to changes in $Q_t$ or $Q_{od}$ were influenced by, and themselves influenced the pattern of, phonons, the coupling would diminish with the phonon amplitudes. This would lead to a steady decrease in $\lambda_t$ with its possible disappearance at 0 K. The linear relation used in the solution may only be an approximation for some nonlinear variation, of course.

Solution 4 produces the best fit with the solution calorimetric measurements of Hovis (1988). It also gives the closest approximation to an equality of $Q_{ot}$ with $Q_t$ in triclinic crystals. This is consistent with the progressive disordering results for microcline of Blasi et al. (1984), who found $Q_{od} \approx Q_t$ between values of $-0.1$ and $1$. The lowest equilibrium degree of order in a triclinic crystal is predicted to be $Q_t \approx 0.95$ and $Q_{od} \approx 0.94$ (at 753 K). This limit can be adjusted to some extent by choosing values of $Q_t$ and $Q_{od}$ greater than $0.98$ at $450$ K, but it cannot be reduced by much if the high degrees of order found in natural microcline are to be reproduced. The equilibrium phase diagram in order parameter space is thus as shown in Figure 8. Modulated structures are regarded as being metastable with respect to microcline, and, in accordance with the TEM observations of Eggleton and Buseck (1980), Fitz Gerald and McLaren (1982), McLaren (1984), Krause et al. (1986), and Kroll et al. (1991), samples described as “intermediate” microcline are presumed to contain intergrowths of modulated regions and homogeneous triclinic regions. Favoring solution 4 in this way calls into question the experimental results of Senderov and Yas'kin (1975, 1976). Clearly, further order-disorder experiments below $\sim 750$ K and alternative solubility or phase equilibrium experiments would be of great value in providing an independent test of the model.

Up to this point no assumptions have been made concerning the origin of excess entropy accompanying the ordering. It must be predominantly configurational but can contain nonconfigurational contributions. The total expected excess configurational entropy is given in the usual way as $4R(0.25 \ln 0.25 + 0.75 \ln 0.75) = -18.7$ J/mol·K. This may be compared with calculated values for the excess entropy at $Q_t = Q_{od} = 1$ from Equation 19, which are in the range $-10$ to $-12$ J/mol·K (Table 2). The implicit assumption of $S \propto Q^2$ in standard Landau free energy expansions means that the calculated Landau excess entropy and the purely configurational entropy can be similar but are not necessarily identical (see Fig. 4 of Carpenter et al., 1994). In the present case, the comparison is also complicated by the proposed participation of phonons in the coupling mechanism, but the total calculated excess entropy is clearly small relative to the ideal configurational entropy. As a further refinement, the latter can be split artificially into two parts. For crystals with $Q_t = Q_{od} = 0$ evolving to $Q_t = 1$ and $Q_{od} = 0$, the ideal configurational entropy change would be $-7.17$ J/mol·K, and for crystals with $Q_t = 1, Q_{od} = 0$ evolving to $Q_t = 1, Q_{od} = 1$, it would be $-11.53$ J/mol·K. These values...
may be compared with -11.1 and -20.5 J/mol·K derived from the values of \( a \) and \( a_{\text{od}} \), respectively, when the coupling is ignored. At least the model solutions all have \( a_{\text{od}} > a \).

Finally, it is perhaps helpful to readers more familiar with the series expansions used by Thompson et al. (1974) to highlight those aspects of the present approach based on Landau theory that are most distinct. They relate to the treatment of all three of the primary excess properties, \( S \), \( H \), and \( G \). Here the excess entropy has been described entirely by quadratic terms in \( Q \) and \( Q^* \), with the additional contribution of the temperature-dependent coupling term. This greatly simplifies the problem of accounting for nonconfigurational or short-range ordering contributions, which are included automatically in the calibration of the coefficients \( a_i \), \( a_{\text{od}} \), and \( a \). Thompson et al. (1974) truncated their expansion for \( \Omega \) after second-order terms, whereas an additional high-order term for each order parameter is included in the Landau expansion. Experience gained from the study of many phase transitions and many materials is sufficient to indicate that such higher order terms are likely to be real. The most significant difference, however, is the inclusion in the expansion for free energy of a coupling term to describe the influence of \( \Omega \) on \( \Omega_{\text{od}} \), and vice versa. Such a coupling effect was not considered by Thompson et al. (1974) and is the principal cause of the discontinuity at the monoclinic = triclinic transition suggested here, in contrast with the thermodynamically continuous behavior shown in their Figure 13.

**Structural states of modulated crystals**

The modulations found in orthoclase and adularia crystals have attracted interest from both structural and energetic points of view since they were first described by McConnell (1965). McConnell’s interpretation of local triclinic distortions in a macroscopically monoclinic crystal appears to be generally accepted, as does his suggestion that they develop as metastable alternatives in place of the transition to microcline (Nissen, 1967; McConnell, 1971; McLaren, 1974, 1984; Eggleton and Buseck, 1980; Fitz Gerald and McLaren, 1982; McLaren and Fitz Gerald, 1987; Smith and Brown, 1988; Brown and Parsons, 1989). The energy changes associated with the relevant lattice distortions and with Al-Si ordering alone are known to be comparable in magnitude (Eggleton and Buseck, 1980), but there remains a question as to whether the Al-Si configuration varies on a local scale so as to follow the distortions exactly. Evidence from infrared spectroscopy excludes the possibility of large local variations in Al-Si order (Harris et al., 1989). As a consequence of the present analysis, there may now be sufficient quantitative information to suggest why relatively homogeneous states of Al-Si order should be favored.

The lattice distortions in orthoclase and adularia involve, primarily, variations in the \( \gamma \) angle and can therefore be characterized in terms of modulations in the strain component \( \epsilon_6 \). In macroscopically triclinic crystals large \( \epsilon_6 \) strains arise by coupling with \( \Omega_{\text{od}} \), and, if there are modulations in \( \epsilon_6 \), there may also be some energetic advantage due to the coupling that favors modulations in \( \Omega_{\text{od}} \). On the other hand, \( \Omega \), does not couple with a triclinic strain and would be expected to remain homogeneous in both monoclinic and triclinic crystals. It has been argued that \( \Omega \) and \( \Omega_{\text{od}} \) are strongly coupled, with the implication that if \( \Omega \) is homogeneous the coupling would tend to favor \( \Omega_{\text{od}} \) also being homogeneous. Thus, there is a competition between \( \Omega_{\text{od}}-\epsilon_6 \) coupling, favoring an inhomogeneous Al-Si distribution, and \( \Omega_{\text{od}}-\omega \) coupling, favoring a homogeneous distribution. Both these coupling energies can be estimated to find which is dominant.

The total excess free energy of the modulated structure may be written as

\[
G = L(\Omega_{\text{od}}) + L(\Omega) + a_{\text{od}} T \Omega \Omega_{\text{od}} + \lambda \epsilon_6 \Omega_{\text{od}} + \frac{1}{2} \epsilon_6 \epsilon_6 + \text{gradient terms} \tag{20}
\]

where \( L(\Omega_{\text{od}}) \) and \( L(\Omega) \) are the Landau free energy expansions in \( \Omega_{\text{od}} \) and \( \Omega \), much as before. Bilinear coupling between \( \epsilon_6 \) and \( \Omega_{\text{od}} \), with \( \lambda \) as the coupling coefficient, and the elastic energy, \( \frac{1}{2} C_{66} \epsilon_6 \epsilon_6 \), with \( C_{66} \) as the relevant elastic constant, are now included explicitly. The competing coupling term is \( a_{\text{od}} T \Omega \Omega_{\text{od}} \), as before, whereas energy contributions from gradients in \( \epsilon_6 \) or \( \Omega_{\text{od}} \) are not specified. From this equation, the condition for equilibrium with respect to an \( \epsilon_6 \) strain is

\[
\frac{\partial G}{\partial \epsilon_6} = 0 = \lambda \epsilon_6 \Omega_{\text{od}} + C_{66} \epsilon_6 \tag{21}
\]

and, hence

\[
\lambda = -\frac{C_{66} \epsilon_6}{\Omega_{\text{od}}} \tag{22}
\]

By extrapolation from the \( \gamma \) lattice angle of natural microcline, the value of \( \epsilon_6 \) in crystals with \( \Omega_{\text{od}} = 1 \) would be \( \sim 0.042 \). \( C_{66} \) for microcline has been measured as 37.4 GPa (Ryzhova and Aleksandrov, 1965), and substituting these values into Equation 22 gives \( \lambda = -1.71 \times 10^5 \) J/mol. If a crystal with \( \Omega_{\text{od}} = \Omega = 0.6 \) is considered, the coupling energy \( \lambda_0 \epsilon_6 \Omega_{\text{od}} \) would be approximately \( -2.6 \text{ kJ/mol} \) (again with the assumption of a linear relationship between \( \Omega_{\text{od}} \) and \( \epsilon_6 \), as expected for bilinear coupling). For the situation illustrated in Figure 9a, with modulations in both \( \epsilon_6 \) and \( \Omega_{\text{od}} \), the total energy from this coupling term would be substantially smaller because most of the crystal would have \( |\Omega_{\text{od}}| < 0.6 \). For a crystal with \( \Omega_{\text{od}} = \Omega = 0.6 \) at \( \sim 650 \) K, the coupling energy \( a_{\text{od}} T \Omega \Omega_{\text{od}} \) would be approximately \( -2.8 \text{ kJ/mol} \). For the situation envisaged in Figure 9b, with the modulation only in \( \epsilon_6 \), the whole of this coupling energy is available to stabilize the crystal. There would, in addition, still be some favorable \( \Omega_{\text{od}}-\epsilon_6 \) coupling, since regions of the crystal would be deformed in the correct sense with respect to \( \Omega_{\text{od}} \).

From these simple calculations it appears that the competing coupling effects would substantially favor the form
to a homogeneous triclinic state is also small, therefore.

Brown and Parsons, 1989; and references therein).

If the conclusion is correct that, ordering that are indistinguishable from, or only slightly monoclinic symmetry.

The amplitude of any data for orthoclase and adularia imply enthalpies of or-

tional approach also provides some insights into the factors controlling the stabilities of dif-

ferent structural states of potassium feldspar.

of variation shown in Figure 9b. In reality, local variations in Q_\text{od} must occur to some extent as a consequence of the coupling to the e_r modulations (Fig. 9c). The mod-

ulations in real crystals occur in two dimensions, giving a tweed texture, possible gradient energies have been ig-

oned, and the values of Q_r, Q_\text{od}, and T selected are slightly arbitrary, but the result is consistent with the infrared evidence of Harris et al. (1989). The amplitude of any modula-

tion in Q_\text{od} is probably small, but conventional structural refinements using diffraction data would not reveal either the modulation or the nonzero Q_\text{od} values, if the refinements included the initial assumption of monoclinic symmetry.

It has already been pointed out that the calorimetric data for orthoclase and adularia imply enthalpies of or-

dering that are indistinguishable from, or only slightly less than, the excess enthalpies of homogeneous crystals with Q_\text{od} \approx Q_\text{od} (Fig. 6). If the conclusion is correct that, be-

cause of the strong Q_r-Q_\text{od} coupling, the values of Q_\text{od} and Q_r are about the same in modulated crystals, the enthalpy change due to the formation of the strain mod-

ulations alone must be small. It follows that the driving force for a transition from the modulated monoclinic state to a homogeneous triclinic state is also small, therefore.

Such a lack of driving energy is often invoked as a con-

tributing factor to the metastable persistence of ortho-

clase and adularia in nature (e.g., Smith and Brown, 1988; Brown and Parsons, 1989; and references therein).

The overall importance of the triclinic strain in pro-

moting the monoclinic \text{\parallel} t triclinic transition can also be determined. T_\text{rect}, the equilibrium temperature for a hypo-

thetical C2/m = C1 transition with no coupling to Q_r, is renormalized by the linear coupling between Q_\text{od} and e_r. The unrenormalized value, T_\text{cod,unren}, for a transition in which e_r was also suppressed can easily be derived from Equation 20 (e.g., see Salje and Devarajan, 1986, or Car-

penter, 1992) as

\[ T_\text{cod,unren} = T_\text{cod} - \frac{\lambda^*_5}{a_\text{od}^5} \]  

For solution 4 (Table 2) and the value of \lambda^*_5 determined above, this gives T_\text{cod,unren} \approx 65 K. Such a small value indicates that the driving force for pure Q_\text{od} ordering is not very significant. Coupling with e_r raises the transition temperature, T_\text{cod}, to 239 K (solution 4). This latter effect and the coupling to Q_r evidently provide most of the energy that drives the phase transition.

\textbf{CONCLUSIONS}

A thermodynamic model has been developed in which the parameters can have a clear physical meaning. The reliability of values derived for these parameters is, of course, dependent on the quality of the experimental data used in the fitting procedures. Even so, the need simultaneously to generate a feasible equilibrium phase dia-

gram from the condition \delta G/\delta Q = 0 and to match an extensive set of enthalpy data provides a stringent test of self-consistency. The overall approach also provides some insights into the factors controlling the stabilities of dif-

ferent structural states of potassium feldspar.

A quantitative hypothesis against which further experi-

ments may be designed is encapsulated in the final pre-

ferred Landau expansion for the total excess free energy (in J/mol):

\[ G = -1650Q_r + 11.14(T - 586)Q_r^2 + 2450Q_r^3 + 20.55(T - 239)Q_\text{od} + 1636Q_\text{od}^2 - 20.13TQ_rQ_\text{od} \]

This describes a first-order C2/m = C1 transition at 753 K, with breaks in Q_r and Q_\text{od} of 0.40 \rightarrow 0.95 and 0 \rightarrow 0.94, respectively. Coupling between Q_r and Q_\text{od} and be-

 tween Q_\text{od} and the triclinic strain account for a substantial proportion of the energy stabilizing triclinic states.

It is hoped that this example, together with the inno-

vations suggested for single-order parameter systems and solid solutions in Carpenter et al. (1994) and Carpenter and Salje (1994), leads to practical formulations for the thermodynamic properties of minerals that undergo non-

convergent ordering in nature.

\textbf{ACKNOWLEDGMENTS}

The impetus for this study was provided by discussions with R. Powell during a period of study leave spent by M.A.C. in the Department of Geology, University of Melbourne. The former is thanked for his gener-

ous hospitality, as is the Royal Society of Great Britain for financial sup-

port from travel funds. The need for derivations of free energy functions to describe nonconvergent ordering has also been stimulated by a grant from the Natural Environment Research Council of Great Britain (GR3/8220 to M.A.C. and E.K.H.S.) to investigate the kinetics of cation order-

ing, which is gratefully acknowledged. We thank T.J.B. Holland for help with the linear least-squares fitting, A. Graeme-Barber for help with the strain calculations, and G.L. Hovis for valuable comments on the manu-

script. W.J. Carey is thanked, in particular, for a detailed and thoughtful review. Cambridge Earth Sciences contribution no. ES3579.
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MANUSCRIPT RECEIVED JUNE 10, 1993
MANUSCRIPT ACCEPTED JULY 19, 1994