

AM 9331

Revision 1

Word Count: 9413

## Compositional effects on etching of fossil confined fission tracks in apatite

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### ABSTRACT

1 Fission-track analysis is a thermochronologic method for dating rocks and reconstructing their  
2 low-temperature thermal histories. We investigate the influence of the apatite composition on the  
3 etching of fossil confined fission tracks, and its consequences for the fission-track method. We con-  
4 ducted step-etch experiments with 5.5 M HNO<sub>3</sub> at 21 °C on samples with etch pit diameters (*Dpar*)  
5 spanning most of the range for natural apatites (Panasqueira: 1.60 μm, Slyudyanka: 2.44 μm, Brazil:  
6 3.92 μm, and Bamble: 4.60 μm) to determine their apatite etch rates  $v_R$  (the rate at which each lat-  
7 tice plane is displaced parallel to itself) as a function of crystallographic orientation ( $\phi'$ ). Our meas-  
8 urements revealed significant differences between the four samples. We fitted three-parameter  
9 functions,  $v_R = a(Dpar) \phi' e^{b(Dpar)\phi'} + c$ , describing  $v_R$  as a function of the angle to the apatite *c*-axis  
10 for our hexagonal samples (excluding Bamble) and Durango apatite. The parameters *a* and *b* both  
11 exhibit a linear correlation with *Dpar*, whereas the constant *c* is small ( $\sim 0.1 \mu\text{m}\cdot\text{min}^{-1}$ ) and its be-  
12 tween-sample variation negligible at the resolution of our measurements. Bamble exhibits a differ-  
13 ent, bimodal relationship between  $v_R$  and  $\phi'$ , which we fitted with a sum of two sine functions. In all  
14 cases, including Bamble, there is a striking correlation between the angular frequencies of horizon-  
15 tal confined tracks and the magnitude of the apatite etch rate  $v_R$  perpendicular to the track axes.  
16 This shows that the sample of confined tracks selected for measurement and modeling is to a much  
17 greater degree determined by the etching properties of the apatite sample than by geometric or  
18 subjective biases. The track etch rate  $v_T$  is constant along most of the track length but varies from  
19 track to track. The mean  $v_T$  correlates with *Dpar*, so that tracks etch to their full lengths in a shorter  
20 time in faster etching apatites. The mean rate of length increase between etch steps,  $v_L$ , also corre-  
21 lates with *Dpar*. The length increments of individual tracks are however irregular. This points to an  
22 intermittent structure at the ends of the tracks.

23 Keywords: apatite, fission-track, confined track, track revelation, apatite etch rate, track etch rate

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## 1. INTRODUCTION

24 Fission-track analysis is a thermochronologic method for determining the ages of rocks and for re-  
25 constructing their thermal histories. It is based on counting and measuring etched trails of lattice  
26 damage within suitable minerals, left by the fragments of fissioned uranium nuclei. There are unan-  
27 swered fundamental questions regarding how the tracks that are counted and measured after etch-  
28 ing are related to the original damage trails. Geometric biases affecting confined track-length  
29 measurements are understood (Laslett et al. 1982, 1984; Galbraith et al. 1990; Galbraith 2002,  
30 2005; Ketcham 2003, 2005), but biases related to the actual etching of confined tracks are less  
31 clear. The lengths of confined tracks in apatite are affected by the etching conditions, including  
32 etchant, concentration, duration, and temperature (Barbarand et al. 2003; Moreira et al. 2010; Rav-  
33 enhurst et al. 2003; Sobel and Seward 2010; Tello et al. 2006; Figure 1 and references in Jonck-  
34 heere et al. 2017). Ketcham (2003) found that geometric biases (Galbraith et al. 1990; Galbraith  
35 2002) alone cannot account for the angular distributions of confined tracks and proposed that "un-  
36 der-etching bias" was also a factor. The inclusion of under-etched tracks and subjective biases af-  
37 fecting the selection of confined tracks for measurement by a particular analyst have been inter-  
38 preted to account for the lack of inter-analyst agreement (Ketcham et al. 2015; Tamer et al. 2019;  
39 Tamer and Ketcham 2020; Ketcham and Tamer 2021).

40 The prevailing model describes track etching as the combined effect of the etch rate  $v_T$  of the  
41 damaged material along the track and the bulk etch rate  $v_B$  of the undamaged material in all other  
42 directions (Price and Walker 1962; Price and Fleischer 1971; Tagami and O'Sullivan 2005; Hur-  
43 ford 2019; Ketcham and Tamer 2021). Step-etch experiments showed that the lengths of individ-  
44 ual confined tracks in apatite increase  $\sim 0$  to  $>1$   $\mu\text{m}$  during 10-s etch-time increments following  
45 an initial 20 s etch (Jonckheere et al. 2017). This illustrates the importance of ascertaining the ef-  
46 fective etch times of individual tracks for modeling the track-length distribution (Aslanian et al.  
47 2021). Jonckheere et al. (2019, 2022) proposed a model explaining the geometries of etched  
48 tracks in which, instead of each individual point on a surface advancing at a rate  $v_B$  in all direc-  
49 tions, crystallographic planes move as units during etching. Their etch rate  $v_R$  is a vector perpen-  
50 dicular to the plane equal in magnitude to its rate of displacement. This model implies that the  
51 shapes of etched tracks depend on their crystallographic orientation. Aslanian et al. (2021)  
52 measured  $v_R$  as a function of orientation for Durango apatite (etched with 5.5 M  $\text{HNO}_3$  at 21 °C).  
53 The maximum width of a confined track ( $w$ ) and the apatite etch rate perpendicular to the track  
54 ( $v_R$ ) give an estimate of the true duration for which it was etched (effective etch time,  $t_E$ ):  $t_E = 1/2$   
55  $w/v_R$ . The effective etch time is shorter than the immersion time by the time  $t_A$ , needed for the  
56 etchant to travel down the host track ( $v_T$ ) and across to the confined track ( $v_R$ ). This "access time"  
57 is different for each confined track (Laslett et al. 1984).

58 Donelick (1993) and Burtner et al. (1994) found a correlation between the etch rate (etch-pit  
59 size,  $D_{par}$ ) and the anion composition (F, Cl, OH) of different apatites; apatites with higher of Cl-  
60 or OH-contents have a greater  $D_{par}$ . Thus, the apatite composition influences the effective etch  
61 times, widths, and lengths of the confined tracks selected for measurement and modeling. The  
62 following sections report step-etch experiments aimed at determining  $v_R$  for different apatites  
63 and relating them to a common reference. Our further aim is that this understanding should lead  
64 to improved etch protocols, tailored to individual apatites (Ravenhurst et al. 2003), as a step to-  
65 wards improved thermal histories.

## 2. MATERIALS AND METHODS

66 We report step-etch experiments on four apatites with different chemical compositions and etch  
67 rates, as reflected in the sizes of the etched track openings parallel to the *c*-axis (*Dpar*; Donelick  
68 1993): Panasqueira (PQ; *Dpar* = 1.60 μm), Slyudyanka (SY; *Dpar* = 2.44 μm), Brazil (BZ; *Dpar* = 3.92  
69 μm) and Bamble (BB; *Dpar* = 4.60). Prism sections of these apatites were mounted in resin, ground  
70 on SiC paper, polished with 6-, 3-, and 1-μm diamond suspensions, and fine-polished with 0.04-μm  
71 silica suspension until their surfaces appeared smooth under reflected light. We first etched the  
72 samples in a stirred 5.5 M HNO<sub>3</sub> solution at 21 °C (Carlson et al. 1999) for times in approximate in-  
73 verse proportion to their *Dpar*, so as to start off with more or less similar tracks and a minimum  
74 track overlap after a subsequent etch step: 35 s for PQ, 20 s for SY, and 10 s for BZ and BB (Figure  
75 1). Etching was stopped by consecutive immersion in two beakers with deionized water. We then  
76 rinsed the samples with ethanol and dried them in a curing cabinet at 35 °C. Following the first se-  
77 ries of measurements, we etched the samples for a second time in 5.5 M HNO<sub>3</sub> at 21 °C for 15 s for  
78 the slow-etching (PQ and SY) and a 10 s for the fast-etching apatites (BZ and BB). For these condi-  
79 tions, the initial widths and the width increments are comparable to those for Durango apatite (DR)  
80 in Aslanian et al. (2021). Before the track measurements, we measured the surface etch pits paral-  
81 lel to the *c*-axis (*Dpar*) in separate mounts of PQ, SY, BZ and BB etched in 5.5 M HNO<sub>3</sub> solution at 21  
82 °C for 20s (Table 2).

83 We built a database consisting of the locations and images of suitable confined tracks in each  
84 sample using a motorized Zeiss AxioImager Z2m microscope and Autoscan software. We scanned  
85 the samples in transmitted light at 250× optical magnification (100× dry objective and 2.5× post-  
86 magnification) recording image stacks of six frames with 0.25 μm offset (height 1.25 μm). Con-  
87 fined tracks (TinT) with both tips in sharp focus plunge <5° and can be considered as horizontal  
88 and measured with negligible error. Horizontal tracks are also sandwiched between two prism  
89 planes parallel to the surface; thus, the measured track width reflects the actual width of the  
90 blade of knife-blade shaped tracks, not an apparent width (Gleadow 1981). We measured the  
91 same tracks after the first and second immersion in the etchant, except for some that had come to  
92 intersect the surface or became obscured by neighboring tracks. Depending on the case, we ex-  
93 tracted the clearest image from a stack or compressed several into a single image, cut out a  
94 square frame centered on the confined track, and converted it to eight-bit greyscale. We import-  
95 ed these images into the CorelDraw graphics-suite software for measurement. We placed a circle  
96 tangent to facing sides of the track at its intersection with the host track to measure its width, and  
97 a second at some distance from the first to determine the track etch rate (Figure 1a-c). Tracks  
98 sub-orthogonal to the *c*-axis ( $\phi \gtrsim 80^\circ$ ) develop a diamond-shaped etch figure, bounded by the  
99 fastest-etching faces, at their intersection with the host track (Figure 1d; Jonckheere et al. 2022).  
100 In this case, we measured the distances ( $d_1$  and  $d_2$ ) between opposing sides of the diamond  
101 shapes for calculating their effective etch times.

102 The apatite etch rates ( $v_R$ ), track etch rates ( $v_T$ ), and effective etch times ( $t_E$ ) were calculated follow-  
103 ing Aslanian et al. (2021):

$$v_R \text{ (}\mu\text{m/min)} = \frac{1}{2} \frac{\Delta r_0 \text{ (}\mu\text{m)}}{\Delta t_l \text{ (min)}} \quad (1)$$

$$v_L (\mu\text{m}/\text{min}) = \frac{1}{2} \frac{\Delta l(\mu\text{m})}{\Delta t_l(\text{min})} \quad (2)$$

$$\theta = 2 \arcsin \left( \frac{(r_0 - r_1)(\mu\text{m})}{2s_1(\mu\text{m})} \right) \quad (3a)$$

$$v_T (\mu\text{m}/\text{min}) = \frac{1}{2} \frac{v_R(\mu\text{m})}{\sin(\theta/2)} \quad (3b)$$

$$t_E (s) = 30 \frac{r_0(\mu\text{m})}{v_R(\mu\text{m}/\text{min})} \quad (\phi \lesssim 80^\circ) \quad (4a)$$

$$t_E (s) = 15 \frac{(d_1 + d_2)(\mu\text{m})}{v_{R,MAX}(\mu\text{m}/\text{min})} \quad (\phi \gtrsim 80^\circ) \quad (4b)$$

104  $\Delta r_0$  is the track width increase at its intersection with the host track ( $r_0$ ) due to the second immer-  
105 sion;  $\Delta t_l$  the time increment from the first to the second etch;  $s_1$  is the distance between the centers  
106 of both circles ( $r_0$  and  $r_1$ ; Figures 1 and 2);  $\theta$  the angle between facing straight sides of the confined  
107 track;  $\phi$  its angle to the  $c$ -axis;  $d_1$ ,  $d_2$  the distances between opposite sides of the diamond shapes.  
108 (Figure 1d). Table 1 gives an overview of the symbols used.

### 3. RESULTS AND DISCUSSION

109 This section covers our length (3.1) and width measurements (3.2), including the calculation of  $v_R$   
110 and its correlation with  $Dpar$ , the calculation of the effective etch times  $t_E$  (3.3), the relationship be-  
111 tween  $v_R$  and angle to the  $c$ -axis ( $\phi' = 90 - \phi$ ) (3.4.), the calculation of the track etch rates  $v_T$  and the  
112 rates of length increase  $v_L$ , and their relationships with  $Dpar$  (3.5.). Tables 2 to 5 summarize the  
113 main statistics.

#### 3.1. Track Lengths

114 Table 2 summarizes the track-length data. Even considering the non-standard immersion times, PQ  
115 and SY, with mean lengths  $<14 \mu\text{m}$  and standard deviations  $>1.5 \mu\text{m}$ , plot squarely in the field of  
116 basement apatites (Gleadow et al. 1986; Figure 3). BB and DR (Durango data from Aslanian et al.  
117 2021), with mean lengths  $>14 \mu\text{m}$  and standard deviations of  $\sim 1 \mu\text{m}$ , are close to the volcanic apa-  
118 tites. BB plots at first between the basement and volcanic fields but in view of the short first etch  
119 (10 s), it belongs in the latter, wherein it plots after another 10 s. Although BZ contains fossil tracks,  
120 it plots at the edge of the induced-track field after 10 s immersion and in it after 20 s, with a greater  
121 mean length ( $>15 \mu\text{m}$ ) and a lower standard deviation ( $<1 \mu\text{m}$ ) than the average for induced tracks.  
122 Supplement Figure S1 shows the track lengths plotted against angle to the  $c$ -axis. Figure S2 plots  
123 the length increments  $\Delta l$  between the first and second etch against angle to the  $c$ -axis and the cor-  
124 responding distributions. The increments differ from track to track with no clear dependence on  
125 the track orientation.  $\Delta l$ -values range from  $\sim 0$  to  $\gtrsim 2 \mu\text{m}$ , which is well within the resolution of our  
126 measurements based on image pairs (Figure 1). The irregular increments suggest that latent tracks  
127 are discontinuous towards their ends (Paul and Fitzgerald 1992; Li et al. 2011; Jonckheere et al.

128 2017). The average increment is isotropic but differs from sample to sample with a minimum  $\Delta l_M <$   
129  $0.5 \mu\text{m}$  for PQ ( $\Delta t_i = 15 \text{ s}$ ) and maximum  $\Delta l_M > 1.0 \mu\text{m}$  for BZ ( $\Delta t_i = 10 \text{ s}$ ). The apparent isotropic  
130 length increase and the correlation with  $D_{par}$  support the notion that tracks become terminated by  
131 the slowest-etching apatite faces, i.e., the prism and basal face (Jonckheere et al. 2019). Figure 4  
132 plots the individual track lengths after the second etch against those after the first. Geometric mean  
133 regression lines fitted to the data have slopes  $S_L$  from 0.95 to 1.05 and correlation coefficients from  
134 0.87 to 0.99. However, constrained regression lines, with unit slope ( $S_L = 1$ ), also provide a good fit  
135 (Table 3). This suggests that the length increments are independent of the initial lengths, in con-  
136 trast with earlier findings for Durango apatite, indicating that the lengths of shorter fossil tracks in-  
137 creased less than those of longer ones for an etch time increment from 20 to 60 seconds (Jonck-  
138 heere et al. 2017).

139 Ellipses fitted to the lengths ( $l$ ) and orientations ( $\phi$ ) measured after the first immersion indicate  
140 that the fossil track lengths are less anisotropic than those of induced tracks annealed to the  
141 same mean  $c$ -axis value. The  $c$ -axis and  $a$ -axis intercepts plot between the isotropic line ( $1:1$ ;  $l_A =$   
142  $l_C$ ) and the trend for induced tracks annealed to different degrees under laboratory conditions  
143 (Figure 5; RD:  $l_A = 1.632 l_C - 10.879$ ). Donelick et al. (1999) report abundant data showing that  
144 this is a common observation for fossil fission tracks in geologic samples. They remark that its  
145 cause is uncertain, suspecting: (1) inaccurate identification of the  $c$ -axis, or (2) an unspecified ef-  
146 fect related to fitting ellipses to fossil track populations, comprising tracks with different thermal  
147 histories. In first instance, the  $c$ -axis and  $a$ -axis intercepts of an ellipse fitted to the sum of two or  
148 more populations on the RD line are the weighted means of those of its components, and also plot  
149 on the RD line. Possible causes for deviations are: (1) the component populations have different  
150 angular distributions, so that their relative weight varies with the angle to the  $c$ -axis; (2) length  
151 bias (Laslett et al. 1984; Galbraith et al. 1990) causes to overestimate the weight of populations  
152 consisting of longer tracks; this effect is somewhat greater for the shorter high-angle tracks than  
153 for the longer low-angle tracks.

154 In the case of our samples, neither cause could account for more than a small departure from the  
155 RD-trend. It is also improbable that the  $c$ -axis orientations have been misidentified in our prism  
156 sections. In addition to the usual criteria (habit, inclusions, track openings), the  $c$ -axis azimuth  
157 orientation can be inferred from the outline of each track (Figure 1; Aslanian et al. 2021; Jonck-  
158 heere et al. 2022). After the second etch, all samples, except PQ, plot near to the RD-trend. The  
159 mean length has increased by a different amount in different samples, but by the same amount in  
160 all directions in each sample, including along the  $a$ - and  $c$ -axes (Figure S2), i.e., parallel to the 1:1-  
161 line in Figure 5. We suggest that this is an indication that geologic annealing results in an aniso-  
162 tropic shortening of the tracks along the RD-line, like induced tracks annealed in the lab, but also  
163 in a lowering of the track etch rate  $v_T$ . The latter, we expect, depends on the composition of the  
164 apatite as well as on the thermal histories of individual samples or even those of individual tracks  
165 (section 3.5).

### 3.2. Apatite Etch Rates

166 Plots of the confined track widths ( $r_\rho$ ) against their  $c$ -axis angles ( $\phi$ ) for consecutive immersions  
167 reveal a distinct angular dependence (Figure S3). The tracks in PQ, SY, and BZ have maximum

168 widths of  $\sim 2.5$ ,  $\sim 2$ , and  $\sim 1$   $\mu\text{m}$  at  $60$ - $75^\circ$  to the  $c$ -axis after the first immersion. In contrast, the  
169 track widths in BB have a bimodal distribution with  $\sim 1$   $\mu\text{m}$  maxima at  $15$ - $30^\circ$  and  $60$ - $75^\circ$  to the  
170  $c$ -axis. We calculated the apatite etch rates  $v_R$  from the width increase  $\Delta r_0$  of individual tracks for  
171 our four samples (equation 1). PQ, SY, BZ, and BB all have  $v_R$ -maxima of  $\sim 3$ ,  $\sim 4$  and  $>5$   $\mu\text{m}\cdot\text{min}^{-1}$   
172 at  $15$ - $30^\circ$  to the  $c$ -axis, decreasing to  $<1$   $\mu\text{m}\cdot\text{min}^{-1}$  (BZ, BB) or  $<0.5$   $\mu\text{m}\cdot\text{min}^{-1}$  (PQ, SY) parallel and  
173 perpendicular to the  $c$ -axis. The Bamble apatite has a second local  $v_R$ -maximum of  $>4$   $\mu\text{m}\cdot\text{min}^{-1}$  at  
174  $40$ - $50^\circ$  to  $c$ , owing to its different crystal structure (Figure 6a and b). Although we have no crys-  
175 tallographic data for our samples, Bamble is known to have a patchy hexagonal and monoclinic  
176 structure (Taborszky, 1972), which may explain its broader maxima compared to the other apa-  
177 tites. We constructed polar plots of the radial etch rate  $v_R$  by mirroring the  $v_R$  interval between  $0$   
178 and  $90^\circ$  about the  $c$ -axis and about an axis perpendicular to it (Figure 6c).

179 We fitted empirical equations to the angular  $v_R$ -distributions of our four apatites and the pub-  
180 lished Durango data of Aslanian et al. (2021) (Table 4). For the hexagonal apatites, we used an  
181 equation of the form:

$$v_R = a\phi' e^{b\phi'} + c \quad (5)$$

182 The constant  $c$  is not correlated with  $Dpar$  and has almost no influence on the fit;  $c = v_R(0)$  is the  
183 etch rate parallel to the  $c$ -axis, where  $v_R$  has a cusp-shaped minimum and its angular variation is  
184 greatest, so that it is difficult to estimate  $c$ . As  $v_R$  cannot be  $0$ , we set  $c = 0.1$ .  $a$  and  $b$  exhibit a line-  
185 ar dependence on  $Dpar$ ; their common dependence on  $Dpar$  means that  $a$  and  $b$  are not inde-  
186 pendent of each other (Figure 7). Despite its small range, the variation of  $b$  in tandem with  $a$  is  
187 significant in the angular interval  $0^\circ \leq \phi' \leq 90^\circ$ .

$$a = 0.141(8) Dpar + 0.09(2) \quad r = 0.99 \quad (6)$$

$$b = -0.0017(5) Dpar - 0.047(1) \quad r = 0.93 \quad (7)$$

188 which gives:

$$v_R = (0.141 Dpar + 0.09)\phi' e^{(-0.0017 Dpar - 0.047)\phi'} + 0.1 \quad (8)$$

189 Equation (8) highlights that, for hexagonal apatites,  $v_R$  correlates with  $Dpar$  for all  $c$ -axis angles.  
190 We fitted a bimodal trigonometric equation to the  $v_R$  vs.  $\phi'$  data for the non-hexagonal Bamble  
191 apatite:

$$v_R = 0.88(11) \sin(5.56(12)\phi') + 3.69(34) \sin(2.08(3)\phi') + 0.76(28) \quad (9)$$

### 3.3. Effective Etch Times

192 Figure S4 plots the effective etch times of the confined tracks against their  $c$ -axis angles. The  
193 boomerang shapes reflect the fact that tracks sub-orthogonal to the  $v_R$ -minima parallel and per-  
194 pendicular to the  $c$ -axis require longer etching than those at intermediate angles to become wide  
195 enough to be selected for measurement (Gleadow 1981). Some  $t_E$ -values for thin tracks at low  
196 and at high angles to the  $c$ -axis exceed the immersion time. This could indicate that  $t_E$ -calculations  
197 have limited precision, although, since the difference is  $<2$  s, it could in part be due to a lingering

198 residue of the etchant in the thinnest tracks at the moment the samples are immersed in water to  
199 halt the etching process. The  $t_E$ -distributions are right-skewed, with geometric means just over  $\frac{1}{2}$   
200  $t_I$  and standard deviations just under  $\frac{1}{5} t_I$  (Figure S4). The mean track lengths and track densi-  
201 ties, defining the fundamental geometric relationships, e.g., the average separation, between host  
202 tracks and confined tracks appear to have no significant influence on the effective etch times of  
203 the confined track sample (Tables 2 and 5). For example, the surface track density of BZ is more  
204 than  $\sim 17$  times greater than that of BB but their mean effective etch times after their initial 10 s  
205 immersion in the etchant differ by less than  $\sim 10\%$ .

206 Jonckheere et al. (2017; their Figure 1) proposed "*etchant strength*" as the product of the etchant  
207 concentration and immersion time to measure their combined influence on the etched lengths of  
208 confined tracks in apatite. Although it is not evident that etchant strength alone determines the  
209 track length, it did harmonize the step-etch data obtained with different protocols. We define  
210 "*etch action*" ( $t'_I = Dpar \times t_I$ ) as the product of the apatite etch rate and immersion time to com-  
211 pare the results of one etchant (5.5 M HNO<sub>3</sub>) across different apatites. Although the apatite etch  
212 rate is anisotropic, the previous result implies that  $Dpar$  (Donelick 1993) characterizes the over-  
213 all apatite etch rate. Figure 8a shows a line of equal etch action:  $Dpar \times t_I = 50 \mu\text{m.s}$ , and the posi-  
214 tions of our samples relative to that line. Most samples have a reasonable, although not a perfect,  
215 fit. The  $t_I$  for BZ is short because a longer immersion would have caused excessive track overlap  
216 ( $\rho_S > 10^7 \text{ cm}^{-2}$ ). In the same manner, we define the "*effective etch action*" ( $t'_E = Dpar \times t_E$ ) of individ-  
217 ual tracks with calculated individual effective etch times. Figure 8a and 8b show the means and the  
218 standard deviations of the effective etch time distributions plotted against  $Dpar$ . The means plot  
219 close to the line  $Dpar \times t_{EM} = 25 \mu\text{m.s}$  and the standard deviations plot close to the line  $Dpar \times \sigma_{tE} =$   
220  $10 \mu\text{m.s}$ . Thus, the effective etch time distributions scale with the immersion times, with  $t_{EM} \approx \frac{1}{2} t_I$ .  
221 The fact that the standard deviation also scales with  $t_I$  implies that it reflects actual  $t_E$ -variation and  
222 not just random measurement errors.

### 3.4. Angular Distributions

223 Figure S3a-d plots the widths of the confined tracks against their  $c$ -axis angles. The dotted lines  
224 (1) represent the maximum track width for each orientation ( $v_R(\phi) \times t_I$ ;  $t_I = 35 \text{ s}$  (PQ),  $20 \text{ s}$  (SY),  
225  $10 \text{ s}$  (BZ), and  $10 \text{ s}$  (BB)). The solid line (2) is an estimate of the actual maximum widths, allowing  
226 for the access time  $t_A$  that the etchant needs to reach the confined track before it can begin to etch  
227 it, i.e., for travelling down the host track and across to the confined track (Laslett et al. 1984;  
228 Rebetz et al. 1988; Ketcham and Tamer 2021). We set  $t_A = 6 \text{ s}$  (PQ),  $3 \text{ s}$  (SY),  $2 \text{ s}$  (BZ), and  $2 \text{ s}$   
229 (BB) based on the effective etch time calculations (Figure S4). The long-dashed line (3) repre-  
230 sents a minimum width for tracks to be distinguishable and considered to be measurable under  
231 the microscope. We assumed a value of  $\sim 0.2 \mu\text{m}$  based on the minimum measured widths of the  
232 tracks in our samples. The short-dashed line (4) is the minimum width at the host track intersec-  
233 tion at the moment that the etchant reaches the track tips; the calculated value for confined  
234 tracks etched from the middle towards both ends is:

$$w_{MIN}(\phi)(\mu\text{m}) = \left( \frac{v_R(\mu\text{m}/\text{min})}{v_T(\mu\text{m}/\text{min})} \right) l(\mu\text{m}) \quad (10)$$

235 The difference  $\Delta w(\phi)$  between (2) and  $\max(3,4)$  reflects the range of track widths for a given ori-  
236 entation. Figure S3e-h shows  $\Delta w(\phi)$  superimposed on the distributions of the angular frequen-  
237 cies of the measured confined tracks  $F(\phi)$ . The good fit and the striking contrast between the  
238 hexagonal apatites (PQ, SY, BZ) and the distinctive distributions for the Bamble apatite (BB;  
239 Donelick et al. 1999) are convincing proof that the angular frequencies are controlled by the ani-  
240 sotropic rate of widening of the confined tracks rather than the host track cross-section (Gal-  
241 braith et al. 1990; Donelick et al. 1999; Galbraith 2002; Ketcham 2003). This also supports the  
242 notion that track width is the main criterion for confined track selection, although it is modified  
243 by factors depending on width and length. For instance, longer tracks attain a greater width be-  
244 fore being etched to their ends than shorter tracks with the same orientation (equation 12).  
245 However, their impact on the angular distributions of the confined tracks is almost negligible due  
246 to the limited annealing of our samples.

### 3.5. Track Etch Rates

247 Figure S5 shows that the distributions of  $v_T$  are right-skewed; this could in part be related to its cal-  
248 culation, as random errors on the small angle  $\theta$  in the denominator produce greater positive than  
249 negative deviations from the true  $v_T$ -value (equation 3). In this case, the geometric means provide  
250 more robust central estimates; these are 103 (PQ) and 95  $\mu\text{m}\cdot\text{min}^{-1}$  (SY) for the basement apa-  
251 tites and 197 (BZ) and 192  $\mu\text{m}\cdot\text{min}^{-1}$  (BB) for apatites with volcano-type length distributions  
252 (Gleadow et al. 1986). There is a significant positive correlation between the mean  $v_T$  and  $Dpar$  ( $r$   
253 = 0.93; Figure 9a). A geometric mean regression line has the equation:  $v_T$  ( $\mu\text{m}\cdot\text{min}^{-1}$ ) = 9.2  
254 ( $\mu\text{m}\cdot\text{min}^{-1}$ ) + 43.1 ( $\text{min}^{-1}$ )  $Dpar$ . However, a regression line anchored at the origin provides an  
255 equally good fit:  $v_T$  ( $\mu\text{m}\cdot\text{min}^{-1}$ ) = 46.3 ( $\text{min}^{-1}$ )  $Dpar$ . There is also a correlation between the stand-  
256 ard deviations of the  $v_T$  distributions and  $Dpar$  ( $r$  = 0.67; Figure 9a). This lends support to the in-  
257 terpretation that the within-sample ranges of  $v_T$ -values are not just an artefact of their measure-  
258 ment and calculation. Here too, an unconstrained geometric mean regression line,  $\sigma_{v_T}$  ( $\mu\text{m}\cdot\text{min}^{-1}$ )  
259 = 4.6 ( $\mu\text{m}\cdot\text{min}^{-1}$ ) + 17.6 ( $\text{min}^{-1}$ )  $Dpar$ , and a regression line through the origin,  $\sigma_{v_T}$  ( $\mu\text{m}\cdot\text{min}^{-1}$ ) =  
260 23.2 ( $\text{min}^{-1}$ )  $Dpar$ , provide an almost equally good fit.

261 A causal connection implies that  $v_T$ , while tens of times greater than  $v_R$ , is nevertheless under  
262 compositional control. The track etch-rate measurements in apatite therefore reveal a complicat-  
263 ed picture: (1) the straight edges of all confined tracks show that  $v_T$  is constant over most of the  
264 track length (Figures 1 and 2); (2) in contrast,  $v_T$  varies from track to track, with no clear depend-  
265 ence on orientation, giving rise to broad  $v_T$ -distributions with high standard deviations (Figure  
266 S5); (3) the differences between volcanic and basement apatites suggests an effect of time or  
267 temperature, such as seasoning (Bull and Durrani 1975), ageing (Gleadow et al. 1983), or thermal  
268 annealing (Fleischer et al. 1965). Price et al. (1973) concluded with respect to the track etch rates  
269 in silicate minerals that a "*gradual rearrangement of the damage at ambient temperature makes*  
270 *the properties of fresh tracks and of ancient tracks different*". All in all, this presents a quite differ-  
271 ent image of the track etch rate from the traditional concept, i.e., as a process controlled by chem-  
272 ical reaction rates rather than by physical factors related to track formation, such as along-track  
273 damage densities (Jonckheere 2003) or latent-track diameters (Li et al. 2012). The  $v_T$ -variation  
274 within a sample could then be a consequence of the individual histories of the tracks. On reflec-  
275 tion, it is indeed not at all evident that the geologic histories of individual fission tracks lead to



276 shortening of their etchable lengths without affecting the chemical properties of the remaining  
277 cores. If this result is confirmed, then the  $v_T$ -signatures of natural apatites could hold information  
278 about their geologic histories. For example, BB and BZ have similar  $v_T$ -distribution whereas those  
279 of SY and PQ, which are offset to lower values consistent with their lower  $Dpar$ , also present dif-  
280 ferent characteristic shapes.

281 The  $v_L$ -distributions are right skewed (Figure S6): a small fraction of the tracks increases in  
282 length at twice to several times the average rate while the remainder increases at lower and more  
283 uniform rates. The means and standard deviations correlate with  $Dpar$  (Figure 9b). A geometric  
284 mean regression line to the  $v_L$  data is given by:  $v_L$  ( $\mu\text{m}\cdot\text{min}^{-1}$ ) =  $-0.39$  ( $\mu\text{m}\cdot\text{min}^{-1}$ ) +  $0.78$  ( $\text{min}^{-1}$ )  $Dpar$   
285 ( $r = 0.99$ ); one anchored at the origin is also a good fit:  $v_L$  ( $\mu\text{m}\cdot\text{min}^{-1}$ ) =  $0.65$  ( $\text{min}^{-1}$ )  $Dpar$  ( $r =$   
286  $0.99$ ). Given that  $v_L$  is almost two orders of magnitude lower than  $v_T$ , this suggests that the length  
287 increase following the first immersion ( $t'_1 = t_1 \times Dpar \approx 50$ ) is due to a chemical process controlled  
288 by the apatite compositions, with little influence of the properties or histories of the latent tracks,  
289 except perhaps to explain the differences between individual tracks. Through its measured width  
290 and orientation, each individual confined track is characterized by its effective etch time  $t_E$  after  
291 the first immersion (section 3.3). This allows us to track the mean length increase and length dis-  
292 tribution between the first and second immersion as a function of the effective etch action (Fig-  
293 ure 10a). Length estimates at regular intervals are obtained through linear interpolation between  
294 the measurements after the first and second immersion. The slopes of regression lines fitted to the  
295 data fall within a narrow range ( $1.11$ - $1.13$   $\text{min}^{-1}$  for PQ and SY and  $1.34$ - $1.56$   $\text{min}^{-1}$  for BZ and BB;  
296 Table 3). Given the nature of the calculation, the difference needs not to be significant. In that case,  
297 past the limit of ca.  $20$   $\mu\text{m}\cdot\text{s}$ , the mean lengths of all our apatites have a common dependence on the  
298 effective etch action ( $1.29$   $\text{min}^{-1}$ ).

299 In one respect, this is a trivial result: if the apatite etch rate remains constant during the immersion  
300 of a sample (Sobel and Seward 2010), then it is interchangeable with the immersion time. It follows  
301 that samples on a line of equal etch action (Figure 8a) are etched to the same degree and the tracks  
302 present similar widths and shapes. A careful scientist, selecting confined tracks based on their  
303 etched appearance, can then expect to measure track lengths and widths that are comparable be-  
304 tween samples of different composition (sections 3.4 and 3.5). On the other hand, this means that  
305 apatites with different compositions are to different degrees under- or over-etched when using  
306 etching protocols with fixed immersion times. Figure 10b shows the data for step-etched fossil  
307 tracks in Durango apatite (Aslanian et al. 2021); the slope ( $1.64$   $\text{min}^{-1}$ ) of a geometric mean regres-  
308 sion line is somewhat steeper than the average for our samples but not inconsistent with it. Figure  
309 10b also plots the mean lengths of unannealed induced tracks in a large set of apatites against the  
310 mean effective etch action, calculated from their  $Dpar$  and an assumed mean effective etch time of  
311  $11.5$  s, which is our average for a  $20$  s immersion in  $5.5$  M  $\text{HNO}_3$  at  $21$  °C (data from Carlson et al.  
312 1999; Barbarand et al. 2003). The geometric mean regression line is given by:  $l_M$  ( $\mu\text{m}$ ) =  $15.64$   $\mu\text{m}$  +  
313  $1.38$  ( $\text{min}^{-1}$ )  $\times t'_E$  ( $\mu\text{m}\cdot\text{min}$ ), with correlation coefficient  $r = 0.89$ , in almost perfect agreement with  
314 equation (1) of Carlson et al. (1999). The slope of the regression line ( $1.38$   $\text{min}^{-1}$ ) is also within the  
315 range of those of our samples (Table 3). The most economical interpretation is that the mean  
316 lengths of unetched induced tracks in all investigated apatites is  $15.64$   $\mu\text{m}$ , regardless of their  
317 chemical composition, and that the measured differences among mean lengths are a consequence  
318 of bulk etching at different rates (Carlson et al. 1999). Our data show that the latter also applies to  
319 samples annealed in the geologic environment.

#### 4. IMPLICATIONS

320 A simple empirical equation fitted to step-etch data describes the apatite etch rate  $v_R$  as a func-  
321 tion of angle to the  $c$ -axis ( $\phi$ ):  $v_R = a \phi' e^{b\phi'} + c$ . This equation applies to hexagonal apatites etched  
322 in 5.5M HNO<sub>3</sub> at 21 °C. The fitted constants,  $a$  and  $b$ , depend on a parameter related to the com-  
323 position of the apatite ( $Dpar$ ). This makes it possible, for each confined track in each apatite, to  
324 calculate the true duration for which it has been etched from its width at its intersection with the  
325 host track and  $Dpar$ , eliminating the need to determine the etch rates of each different sample or  
326 grain. This is the first empirical criterion for distinguishing well-etched from over- or under-etched  
327 tracks, and thus for limiting one source of spurious variation in Tt-modelling (Trilsch et al., in re-  
328 view). Our equation does not extend to non-hexagonal apatites, which exhibit a different depend-  
329 ence of  $v_R$  on  $\phi$ .

330 The maximum attainable width of a confined track is proportional to the etch rate perpendicular  
331 to it, whereas its threshold (minimum) width is independent of its orientation. This accounts for  
332 the close correlation between the confined track widths and their angular frequencies. The impli-  
333 cation is that we must consider etching-related biases as well as geometrical biases. In principle,  
334 we can formulate a length- and-orientation-bias model for each apatite-etchant combination for  
335 which the etch rate  $v_R$  is known as a function of orientation. As long as  $v_R$  scales with  $Dpar$ , this  
336 model should fit different apatite compositions for a given etchant. All a priori bias models must  
337 however be approximate because they ignore other etching-related factors, such as effective etch  
338 time  $t_E$ , track etch rate  $v_T$ , and the rate of length increase  $v_L$ . Measuring these characteristics along  
339 with the track lengths and orientations after the confined tracks have been selected instead helps  
340 to define the selection bias for each given sample. It is evident that understanding the relation-  
341 ship between a confined track sample and the track population is a condition for meaningful Tt-  
342 modelling (Galbraith, 2005).

343 Our data provide the first indication of a correlation between the track etch rate  $v_T$  and the apa-  
344 tite composition. The full implications are not clear, although such a correlation appears to favour  
345 an amorphous track core over a depleted core, and a thermal-spike track formation mechanism  
346 over a pure ion-explosion-spike mechanism. It suggests that the track etch rate is under chemical  
347 control and thus less dependent on, or independent of, the variation of the calculated lattice  
348 damage along the tracks. The  $v_T$ -variation from track to track could be due to the different mass-  
349 es, charges and energies of the track forming particles, but the significant differences between  
350 samples, over and above those caused by their apatite etch rates, suggest that  $v_T$  might bear an  
351 imprint of their thermal histories.

352 In all our samples, the mean confined track lengths increase at a constant rate  $v_L$  between the  
353 first and second measurement;  $v_L$  is similar to the apatite etch rate, which implies that the tracks  
354 were etched to their ends after their first immersion for a time  $t_l \approx 50/Dpar$ , however short it  
355 was, even as little as 10 seconds for fast etching apatites. The mean length increase  $\Delta l$  between  
356 the first and second measurement is then due to "bulk etching". The lengths of shorter tracks in a  
357 sample increase as much on average as those of the longer tracks. In first approximation, this im-  
358 plies that after the first immersion all the selected tracks in all our samples have reached some-  
359 thing approaching their intrinsic length, which on continued etching increases in a predictable  
360 manner at an average rate proportional to  $Dpar$ . The  $\Delta l$ -distributions are nonetheless right-

361 skewed (average skewness = 1.7) due a small number of tracks with much longer than average  
362 length increments. The negative correlation between  $\Delta l$  and  $t_E$  (mean  $r = -0.23$ ) shows that these  
363 tracks began to etch last and were just short of bulk etching at the end of the first immersion.  
364 There are however too few in number to have an effect on the mean rate of length increase. The  
365 important implication is that, as of the end of the first immersion, the length distribution of the  
366 selected confined tracks in all our samples is a reflection of their formation and geological histo-  
367 ries, not of their etching histories.

### ACKNOWLEDGMENTS AND FUNDING

368 We are indebted to R. Donelick and R. Ketcham for reviewing our manuscript and for their help-  
369 ful comments, and to D. Harlov for efficient editorial handling. Research funded by the Innovation  
370 Team Project of Natural Science Foundation of Hubei Province (Grant Number: 2021CFA031),  
371 the National Natural Science Foundation of China (Grant Number: 42372181) and the German  
372 Research Council (DFG projects JO 358/4 and Ra 442/42).

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## Figure captions

498 **Figure 1.** Horizontal confined fossil tracks in prism faces of Bamble (BB), Slyudyanka (SY), Brazil  
499 (BZ), and Panasqueira (PQ) apatite. The left panel shows the track after the first immersion ( $t_{I1}$ )  
500 and the right panel after the second immersion ( $t_{I2}$ ) in 5.5 M HNO<sub>3</sub> at 21 °C. **(a)** BB:  $t_{I1} = 10$  s,  $t_{I2} = 20$   
501 s. **(b)** SY:  $t_{I1} = 20$  s,  $t_{I2} = 35$  s. **(c)** BZ:  $t_{I1} = 10$  s,  $t_{I2} = 20$  s. **(d)** PQ:  $t_{I1} = 35$  s,  $t_{I2} = 50$  s. Measurements:  
502 length ( $l$ ),  $c$ -axis angle ( $\phi$ ), maximum width ( $r_0$ ), width ( $r_1$ ) at distance ( $s_1$ ) from  $r_0$ , and perpendicu-  
503 lar distances ( $d_1$  and  $d_2$ ) between facing sides of the diamond shape, bounded by the fastest etching  
504 apatite faces at the intersection with the host track ( $\phi \gtrsim 80^\circ$ ).

505 **Figure 2.** **(a)** Transmitted-light microscope image of a horizontal confined track in apatite SY after  
506 20 s immersion. **(b)** Track contour showing the measured dimensions: length ( $l$ ),  $c$ -axis angle ( $\phi$ ),  
507 and maximum width ( $r_0$ ); the width ( $r_1$ ) at a distance ( $s_1$ ) from ( $r_0$ ), used for calculating the cone  
508 angle ( $\theta$ ) and the track etch rate ( $v_T$ ), and the apatite etch rate ( $v_R$ ) perpendicular to the track. **(c)**  
509 Principle for calculating  $\theta$  from  $r_0$ ,  $r_1$ , and  $s_1$ .

510 **Figure 3.** Plot of the standard deviations against the means of the confined track length distribu-  
511 tions. PQ: Panasqueira; SY: Slyudyanka; BZ Brazil; BB: Bamble; open symbols: lengths measured af-  
512 ter the first etch; filled symbols: after second etch. PQ and SY have basement and BZ and BB have  
513 volcano-type signatures (Gleadow et al. 1986).

514 **Figure 4.** Lengths of confined tracks measured after the second etch plotted against their lengths  
515 after the first etch. PQ: Panasqueira; SY: Slyudyanka; BZ Brazil; BB: Bamble; **(a)** PQ; **(b)** SY; **(c)** BZ;  
516 **(d)** BB. The dashed line is 1:1 (no length increase); the solid lines are geometric mean regression  
517 lines. Anchored (at origin) regression lines parallel to the 1:1 line are not shown but indistinguish-  
518 able.

519 **Figure 5.** Plot of the  $a$ -axis intercepts vs.  $c$ -axis intercepts of ellipses fitted to track-length and ori-  
520 entation data. PQ: Panasqueira; SY: Slyudyanka; BZ Brazil; BB: Bamble. Open symbols: data meas-  
521 ured after the first etch; filled symbols: after second etch. The RD line illustrates the relationship for  
522 induced fission tracks (Donelick et al. 1999).

523 **Figure 6.** **(a)** Apatite etch rates of PQ (white), SY (light grey), and BZ (dark grey) plotted against  
524 orientation ( $\phi' = 90^\circ - \phi$ ); the solid lines are empirical fits (equation 5). **(b)** Apatite etch rates of  
525 BB; the solid line is an empirical fit (equation 9). **(c)** Polar plots comparing the apatite etch rates  
526 of PQ (solid line), SY (long-dashed), BZ (medium-dashed), and BB (short-dashed) with the Duran-  
527 go data of Aslanian et al. (2021; DR: Durango apatite; red line). PQ: Panasqueira; SY: Slyudyanka;  
528 BZ Brazil; BB: Bamble; DR: Durango data of Aslanian et al. (2021).

529 **Figure 7.** Best-fit parameters  $a$  and  $b$  (equation 5) plotted against  $Dpar$ ; the solid lines are geomet-  
530 ric mean regression lines. PQ: Panasqueira; SY: Slyudyanka; BZ: Brazil; the fit further includes the  
531 Durango data of Aslanian et al. (2021; DR), which we refitted with equation (5) but not the Bamble  
532 data, which were fitted with equation (9).

533 **Figure 8.** **(a)** Immersion times ( $t_i$ ), geometric means ( $t_{EM}$ ) and standard deviations ( $\sigma_{TE}$ ) of the ef-  
534 fective etch-time distributions plotted against  $Dpar$ . PQ: Panasqueira; SY: Slyudyanka; BZ Brazil;  
535 BB: Bamble; DR: Durango data of Aslanian et al. (2021). Solid lines are first-order fits proportional  
536 to  $Dpar^{-1}$ . **(b)** Comparison of first-order fits (solid lines) to the means ( $25/Dpar$ ;  $t_{EM} \approx \frac{1}{2} t_i$ ) and



537 standard deviations ( $10/Dpar$ ;  $\sigma_{TE} \approx 1/5 t_i$ ), with best-fits of the form  $u + v/Dpar$  (dashed lines),  
538 showing good agreement.

539 **Figure 9. (a)** Geometric means (open) and standard deviations (filled) of the  $v_T$ -distributions (track  
540 etch rate). **(b)** Geometric means and standard deviations of the  $v_L$ -distributions (rate of length in-  
541 crease) plotted against  $Dpar$ . PQ: Panasqueira; SY: Slyudyanka; BZ: Brazil; BB: Bamble; DR: Duran-  
542 go data of Aslanian et al. (2021). Solid lines are geometric mean regression lines; dashed lines are  
543 anchored at the origin.

544 **Figure 10.** Mean track lengths plotted against effective etch action (effective etch time  $\times Dpar$ ). **(a)**  
545 Fossil tracks in Panasqueira (PQ), Slyudyanka (SY), Brazil (BZ), and Bamble (BB). **(b)** Fossil tracks  
546 in Durango (DR; Aslanian et al. 2021) and induced track data for different chemical compositions  
547 (Carlson et al. 1999; Barbarand et al. 2003). Solid lines are geometric mean regression lines; dotted  
548 lines are constrained to the same slope.

## Supplement figures

549 **Figure S1.** Lengths of fossil confined tracks in the studied apatites measured after the first ( $t_{I1}$ ) and  
550 second immersion ( $t_{I2}$ ) in 5.5 M HNO<sub>3</sub> at 21 °C, plotted against angle to the *c*-axis. PQ: Panasqueira  
551 ((**a**):  $t_{I1} = 35$  s, (**b**):  $t_{I2} = 50$  s); SY: Slyudyanka ((**c**):  $t_{I1} = 20$  s, (**d**):  $t_{I2} = 35$  s); BZ: Brazil ((**e**):  $t_{I1} = 10$  s,  
552 (**f**):  $t_{I2} = 20$  s); BB: Bamble ((**g**):  $t_{I1} = 10$  s, (**h**):  $t_{I2} = 20$  s).

553 **Figure S2.** Track length increments ( $\Delta l$ ) during the etch time increment ( $\Delta t_l$ ) from the first to the  
554 second immersion in 5.5 M HNO<sub>3</sub> at 21 °C, plotted against angle to the *c*-axis ( $\phi$ ), and correspond-  
555 ing frequency distributions of  $\Delta l$  in the studied samples. PQ: Panasqueira; SY: Slyudyanka; BZ: Bra-  
556 zil; BB: Bamble. PQ ((**a**) and (**e**);  $\Delta t_l = 15$  s); SY ((**b**) and (**f**);  $\Delta t_l = 15$  s); BZ ((**c**) and (**g**);  $\Delta t_l = 10$  s);  
557 BB ((**d**) and (**h**);  $\Delta t_l = 10$  s); the dashed lines in (e)-(h) represent polynomial fits.

558 **Figure S3.** Confined track widths ( $r_0$ ) after the first immersion in 5.5 M HNO<sub>3</sub> at 21 °C plotted  
559 against angle to the *c*-axis ( $\phi$ ), and corresponding angular distributions. PQ: Panasqueira; SY:  
560 Slyudyanka; BZ: Brazil; BB: Bamble. PQ ((**a**) and (**e**)); SY ((**b**) and (**f**)); BZ ((**c**) and (**g**)); BB ((**d**) and  
561 (**h**)). The lines (1)-(4) in (**a**)-(**d**) are inferred etching and selection biases; (1) theoretical maximum  
562 width:  $v_R(\phi) \times t_i$ ; (2) width assuming an average access time  $t_A$  to reach the confined track:  $v_R(\phi) \times$   
563 ( $t_i - t_A$ ); (3) threshold width of tracks judged suitable for measurement; (4) minimum width at the  
564 host track intersection of tracks etched to both ends (equation 4). The long-dashed lines superim-  
565 posed on the angular distributions (**e**)-(**h**) show the range of track widths  $r_0$  constrained by criteria  
566 (2)-(4) as a function of  $\phi$ .

567 **Figure S4.** Effective etch times ( $t_E$ ) of confined tracks plotted against their angles to the *c*-axis ( $\phi$ )  
568 and corresponding  $t_E$ -distributions. PQ: Panasqueira; SY: Slyudyanka; BZ: Brazil; BB: Bamble. PQ  
569 ((**a**) and (**e**)); SY ((**b**) and (**f**)); BZ ((**c**) and (**g**)); BB ((**d**) and (**h**)). Open symbols in (**a**)-(**d**): mea-  
570 sured using equation 4a (Figure 1a-c); filled symbols: measured using equation 4b (Figure 1d). The  
571 dashed lines in (**e**)-(**h**) represent polynomial fits.  $t_{EM}$  and  $\sigma_{tE}$ : geometric means and standard devia-  
572 tions of the effective etch-time distributions.

573 **Figure S5.** Etch rates ( $v_T$ ) of confined tracks plotted against their angles to the *c*-axis ( $\phi$ ), and cor-  
574 responding  $v_T$ -distributions. PQ: Panasqueira; SY: Slyudyanka; BZ: Brazil; BB: Bamble. PQ ((**a**) and  
575 (**e**)); SY ((**b**) and (**f**)); BZ ((**c**) and (**g**)); BB ((**d**) and (**h**)). The dashed lines in (**e**)-(**h**) represent poly-  
576 nomial fits.  $v_{TM}$  and  $\sigma_{vT}$ : geometric means and standard deviations of the track etch-rate distribu-  
577 tions.

578 **Figure S6.** Rates of length increase ( $v_L$ ) of confined tracks plotted against their angles to the *c*-axis  
579 ( $\phi$ ) and corresponding  $v_L$ -distributions. PQ: Panasqueira; SY: Slyudyanka; BZ: Brazil; BB: Bamble.  
580 PQ ((**a**) and (**e**)); SY ((**b**) and (**f**)); BZ ((**c**) and (**g**)); BB ((**d**) and (**h**)); the dashed lines in (**e**)-(**h**) rep-  
581 resent polynomial fits.  $v_{LM}$  and  $\sigma_{vL}$ : means and standard deviations of the distributions of the rate of  
582 length increase.

## Tables

**Table 1.** Symbols and their meaning

Symbols	Meaning
$r_0$	maximum width of confined track
$r_1$	non-maximum width of confined track
$s_1$	distance between $r_0$ and $r_1$
$d_1, d_2$	perpendicular distances between facing sides of the diamond shape in confined track
$\phi$	angle between the track axis and the apatite $c$ -axis
$\phi'$	angle between the apatite etch rate orientation and the apatite $c$ -axis, ( $\phi' = 90^\circ - \phi$ )
$\theta$	angle between facing straight sides of the confined track
$\Delta r_0$	confined track width increase at its intersection with the host track
$t_i$	immersion time
$t_E$	effective etch time
$t_A$	access time ( $t_A = t_i - t_E$ )
$t_{EM}$	geometric mean effective etch time
$\Delta t_i$	time increment from the first to the second etch
$t'_i$	etch action with apatite etch rate and immersion time
$t'_E$	etch action with apatite etch rate and effective etch time
$l$	measured length of confined track
$l_M$	mean track length of confined tracks
$l_{PM}$	mean $c$ -axis-projected length of confined tracks
$\Delta l$	length increment of confined track
$\Delta l_M$	mean length increment of confined track
$l_A$	short axis of unconstrained ellipses
$l_C$	long axis of unconstrained ellipses
$v_B$	apatite bulk etch rate
$v_R$	apatite radial etch rate
$v_T$	fission track etch rate
$v_L$	rate of length increase
$v_{TM}$	geometric mean of track etch rates
$v_{LM}$	geometric mean of rates of length increase
$\sigma_M$	standard deviation of mean track length
$\sigma_{PM}$	standard deviation of the $c$ -axis-projected lengths
$\sigma_{tE}$	standard deviation of the effective etch times
$\sigma_{vT}$	standard deviation of the track etch rates
$\sigma_{vL}$	standard deviation of the rates of length increase
$a, b, c$	constants of fitting equation for the hexagonal apatites
$I_L$	intercepts of regression lines in Figure 4
$I'_L$	intercepts of constrained regression lines in Figure 4
$I_T$	intercepts of regression lines in Figure 10
$S_L, S'_L$	slopes of regression lines and constrained regression lines in Figure 4
$S_T$	slopes of regression lines in Figure 10
$r_L$	correlation coefficients of regression lines in Figure 4
$r'_L$	correlation coefficients of constrained regression lines in Figure 4
$r_T$	correlation coefficients of regression lines in Figure 10

**Table 2.** Track length data for PQ, SY, BZ, BB and DR apatite (Aslanian et al. 2021).  $\rho_s$ : track densities of fossil tracks;  $N_{Track}$ : number of measured tracks;  $t_i$ : immersion time;  $l_M$ : mean track length;  $\sigma_M$ : standard deviation of  $l_M$ ;  $l_{PM}$ : mean  $c$ -axis-projected length;  $\sigma_{PM}$ : standard deviation of  $l_P$ ,  $l_C$  and  $l_A$ : long and short axes of unconstrained ellipses;  $\Delta l_M$ : length increase between the first and second measurement. Error are  $1\sigma$ .

Sample	$Dpar$ ( $\mu m$ )	$\rho_s$ ( $10^6 cm^{-2}$ )	$t_i$ (s)	$N_{Track}$	$l_M$ ( $\mu m$ )	$\sigma_M$ ( $\mu m$ )	$l_{PM}$ ( $\mu m$ )	$\sigma_{PM}$ ( $\mu m$ )	$l_C$ ( $\mu m$ )	$l_A$ (m)	$\Delta l_M$ ( $\mu m$ )
PQ	1.60 (1)	1.31 (5)	35	232	12.3 (1)	1.7	13.5 (1)	1.2	12.9 (3)	12.0 (2)	0.46 (2)
			50	221	12.8 (1)	1.7	13.8 (1)	1.3	13.3 (3)	12.5 (2)	
DR	1.85 (2)	0.18 (0)	30	205	14.2 (1)	1.0	14.9 (1)	0.7	15.0 (2)	13.8 (1)	0.67 (2)
			45	205	14.8(1)	1.0	15.4 (1)	0.7	15.7 (1)	14.4 (1)	
SY	2.44 (1)	4.15 (19)	20	226	13.1 (1)	1.7	14.0 (1)	1.3	14.1 (3)	12.8 (2)	0.80 (3)
			35	222	13.9 (1)	1.8	14.6 (1)	1.4	14.8 (3)	13.5 (2)	
BZ	3.92 (2)	11.1 (5)	10	209	15.6 (1)	0.8	16.0 (1)	0.6	15.7 (2)	15.6 (1)	0.97 (2)
			20	208	16.6 (1)	0.8	16.7 (1)	0.6	16.6 (2)	16.6 (1)	
BB	4.60 (4)	0.63 (3)	10	199	13.4 (1)	1.1	14.2 (1)	0.8	14.0 (2)	12.9 (2)	1.13 (4)
			20	199	14.5 (1)	1.0	15.1 (1)	0.8	15.3 (2)	13.9 (1)	

**Table 3.** Intercepts ( $I$ ), slopes ( $S$ ), and correlation coefficients ( $r$ ) of geometric mean regression lines to the track lengths measured after the second immersion plotted against the values measured after the first immersion (Figure 4;  $I_L$ ,  $S_L$ ,  $r_L$ ). Intercepts ( $I_L'$ ) and correlation coefficients ( $r_L'$ ) of regression lines to the same data, constrained to have slopes  $S_L'=1$ . Intercepts, slopes, and correlation coefficients of geometric mean regression lines to plots of the interpolated mean track lengths against the effective etch action  $t_E'$  (Figure 10;  $I_T$ ,  $S_T$ ,  $r_T$ ). DR data from Aslanian et al. (2021).

Sample	$Dpar$ ( $\mu m$ )	$I_L$ ( $\mu m$ )	$S_L$ (-)	$r_L$ (-)	$I_L'$ ( $\mu m$ )	$r_L'$ (-)	$I_T$ ( $\mu m$ )	$S_T$ ( $min^{-1}$ )	$r_T$ ( $min^{-1}$ )
PQ	1.60 (1)	0.28	1.01	0.985	0.458	0.985	11.88	1.13	0.989
SY	2.44 (1)	0.31	1.04	0.963	0.927	0.963	12.55	1.11	0.970
BZ	3.92 (2)	0.15	1.05	0.952	0.973	0.952	15.08	1.34	0.987
BB	4.60 (4)	1.79	0.95	0.865	1.135	0.844	12.75	1.56	0.976
DR	1.85 (2)	1.00	0.98	0.947	0.688	0.944	13.30	1.64	0.971

**Table 4.** Parameters of the  $v_R(\phi')$ -equation (4) fitted to the data for PQ, SY, and BZ, and DR of Aslanian et al. (2021) (Figure 6).

Sample	$Dpar$ ( $\mu m$ )	$a$ ( $\mu m/(min.^{\circ})$ )	$b$ ( $1/^{\circ}$ )	$c$ ( $\mu m.min^{-1}$ )
PQ	1.60 (1)	0.31 (1)	-0.049 (1)	0.06 (8)
DR	1.85 (2)	0.36 (1)	-0.051 (1)	0.26 (8)
SY	2.44 (1)	0.42 (1)	-0.051 (1)	0.10 (8)
BZ	3.92 (2)	0.64 (1)	-0.054 (1)	0.15 (8)

**Table 5.** Geometric means and standard deviations of the effective etch times, track etch rates, and rates of length increase of the studied samples, and their correlations with fitted power functions ( $t_{EM}$  and  $\sigma_{tE}$ ) and regression lines ( $v_{TM}$ ,  $\sigma_{VT}$ ,  $v_{LM}$ , and  $\sigma_{VL}$ ).

Sample	$Dpar$ ( $\mu m$ )	$t_i$ (s)	$t_{EM}$ (s)	$\sigma_{tE}$ (s)	$v_{TM}$ ( $\mu m.min^{-1}$ )	$\sigma_{VT}$ ( $\mu m.min^{-1}$ )	$v_{LM}$ ( $\mu m.min^{-1}$ )	$\sigma_{VL}$ ( $\mu m.min^{-1}$ )
PQ	1.60 (1)	35	15.8 (5)	7.2	103 (5)	66	0.76 (4)	0.51
SY	2.44 (1)	20	12.1 (2)	3.2	95 (3)	37	1.41 (6)	0.79
BZ	3.92 (2)	10	5.69 (1)	1.6	197 (5)	70	2.82 (6)	0.76
BB	4.60 (4)	10	5.24 (1)	1.9	192 (6)	78	3.08 (11)	1.52
DR	1.85 (2)	30	17.9 (7)	7.2	78 (2)	24	1.17 (6)	0.64
$r(Dpar)$		0.99	0.98	0.96	0.93	0.67	0.99	0.84

Figure 1

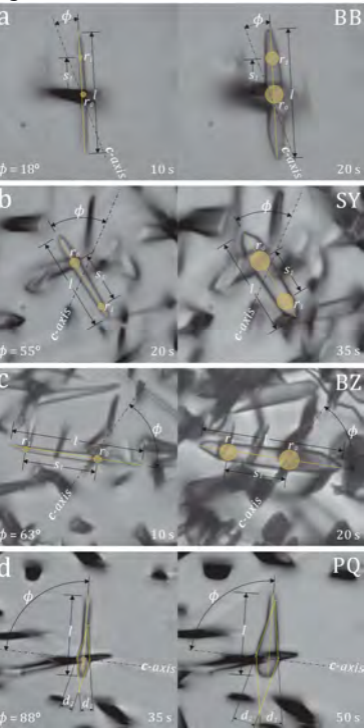
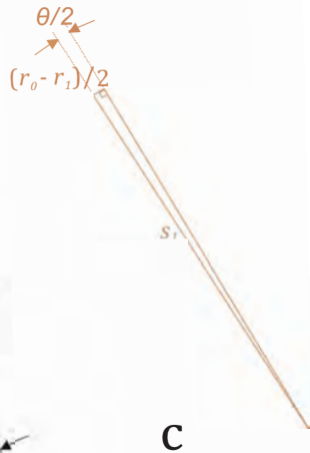
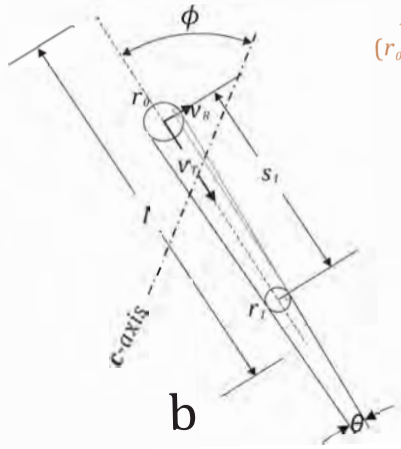
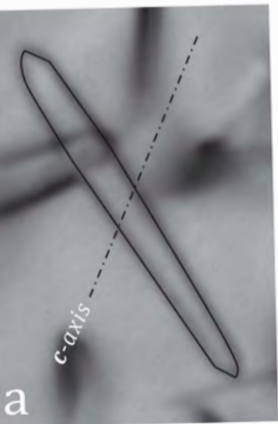


Figure 2



# Figure 3

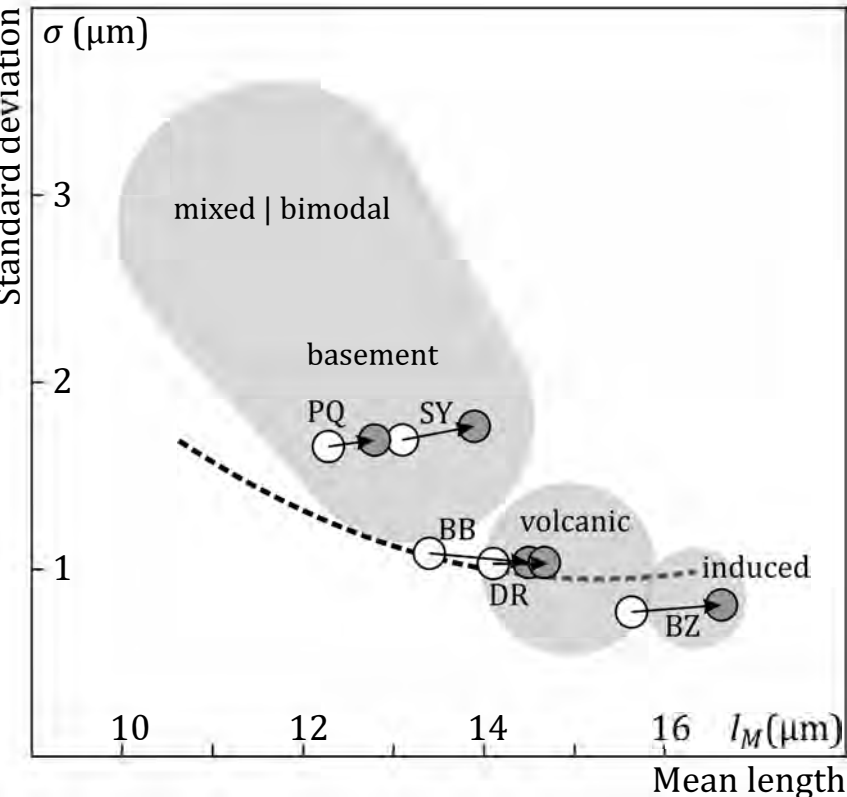
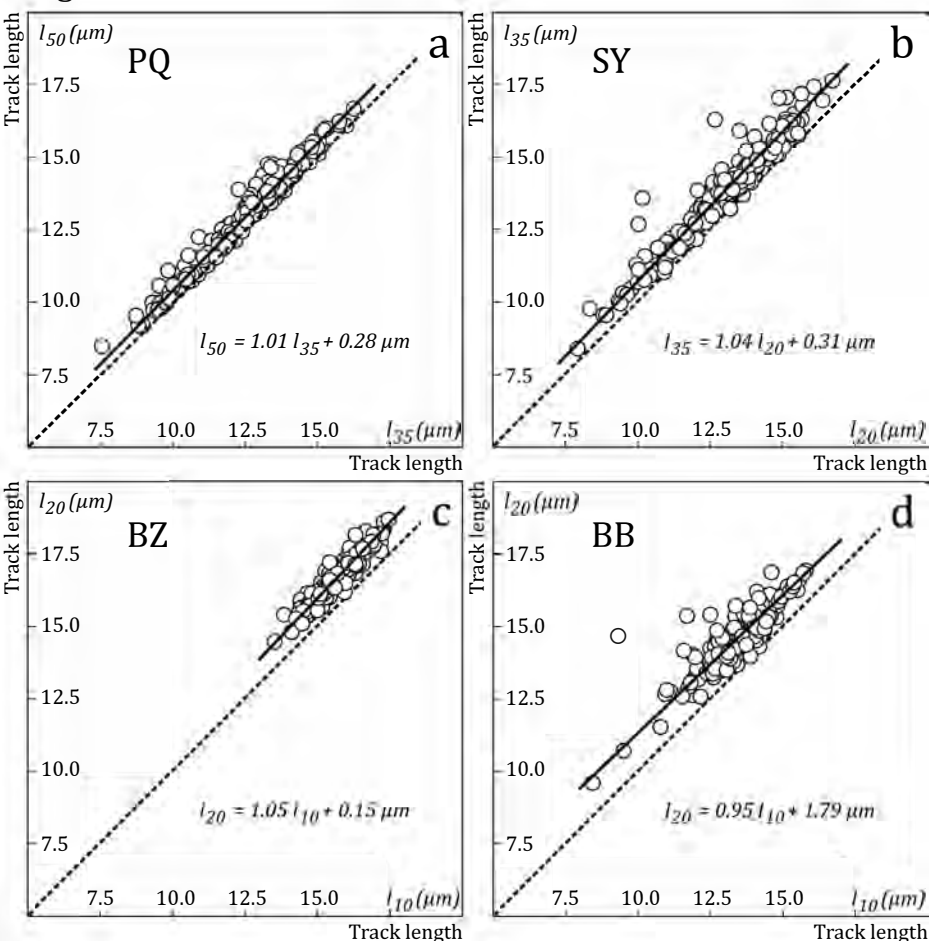


Figure 4





# Figure 5

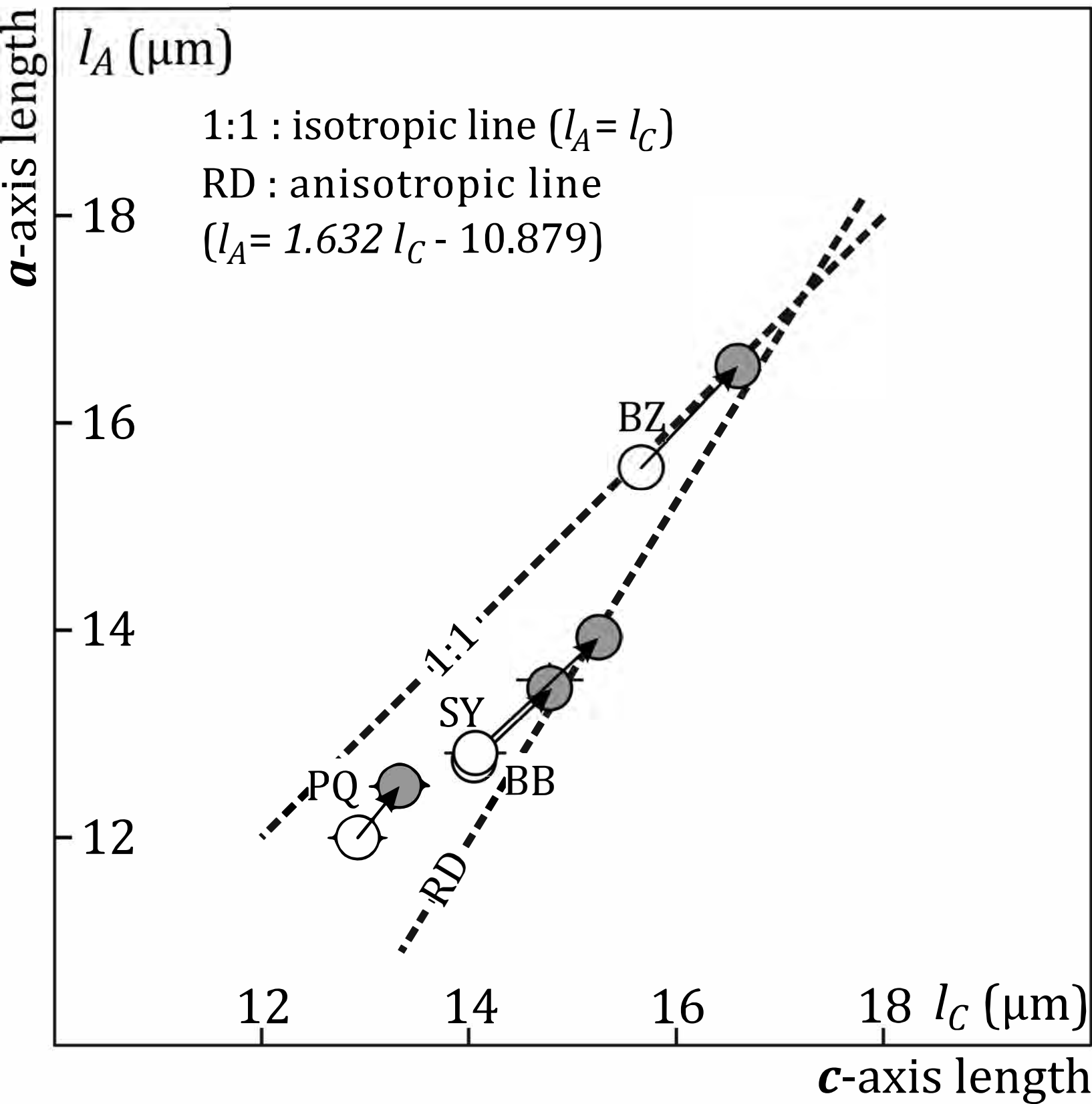
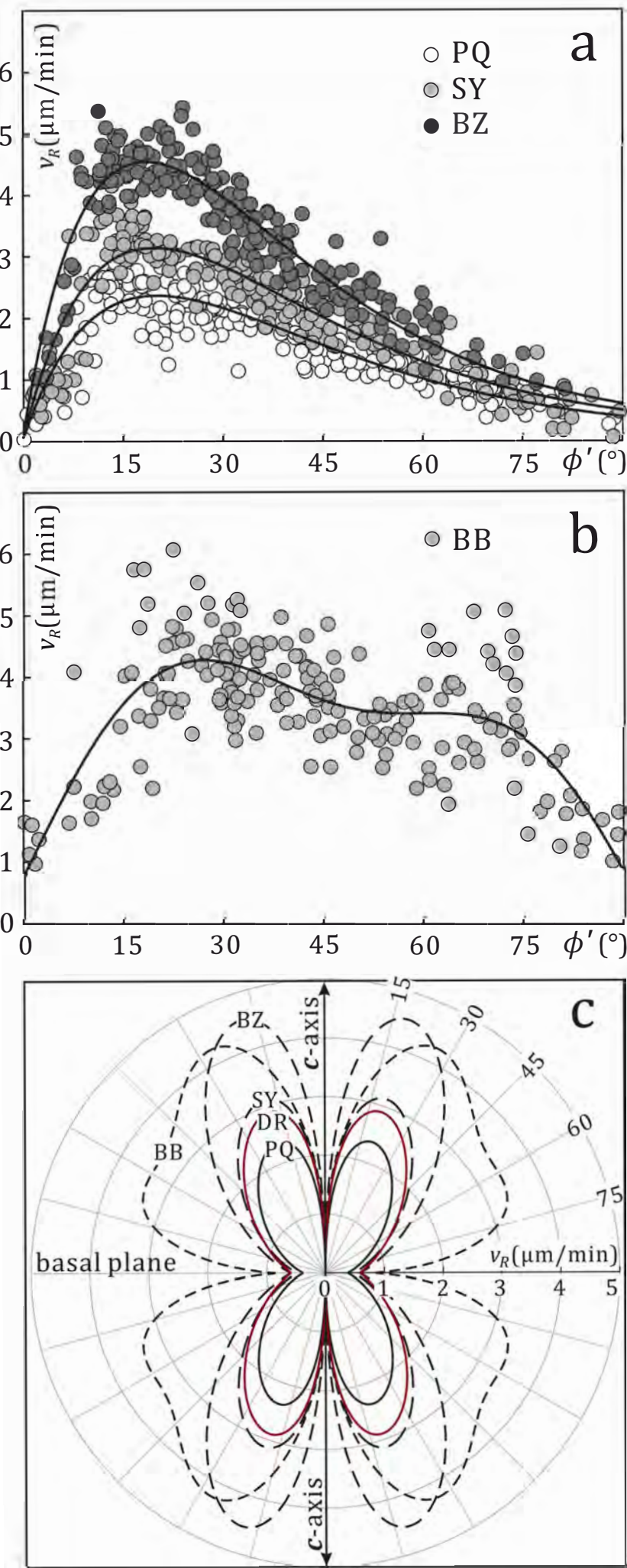


Figure 6



# Figure 7

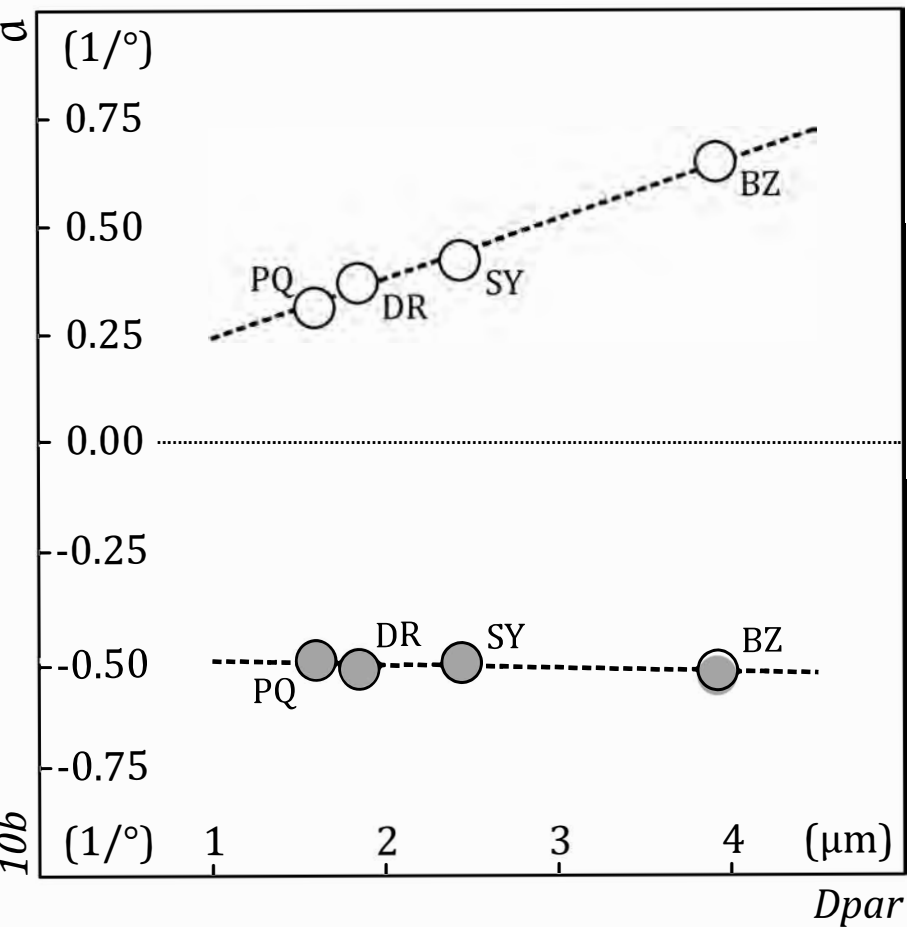


Figure 8

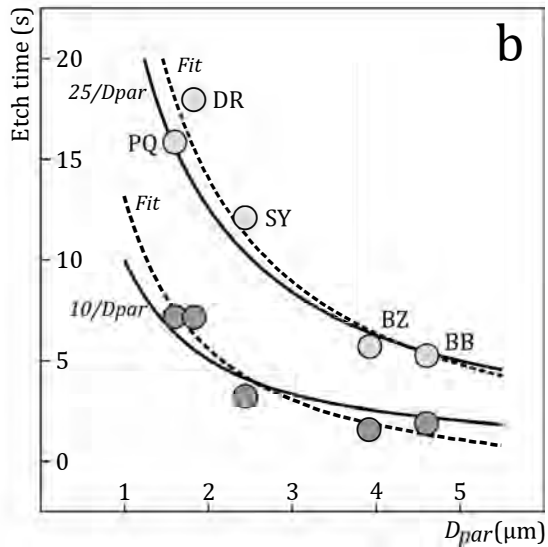
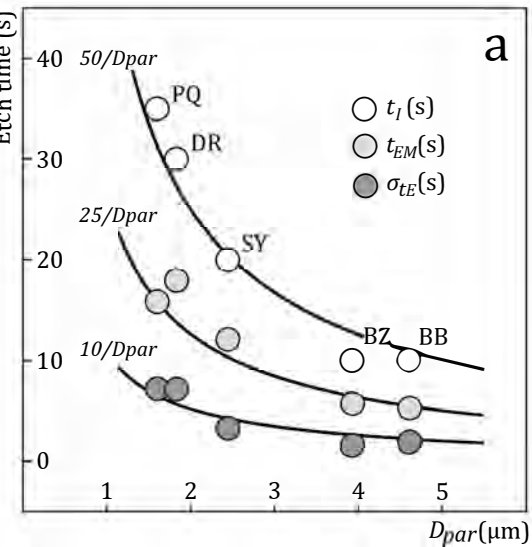


Figure 9

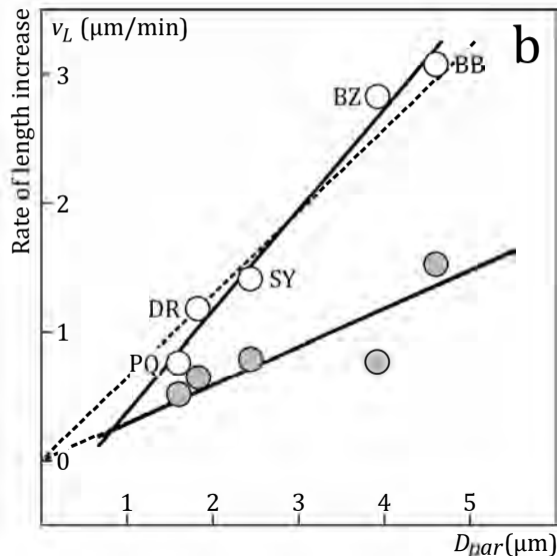
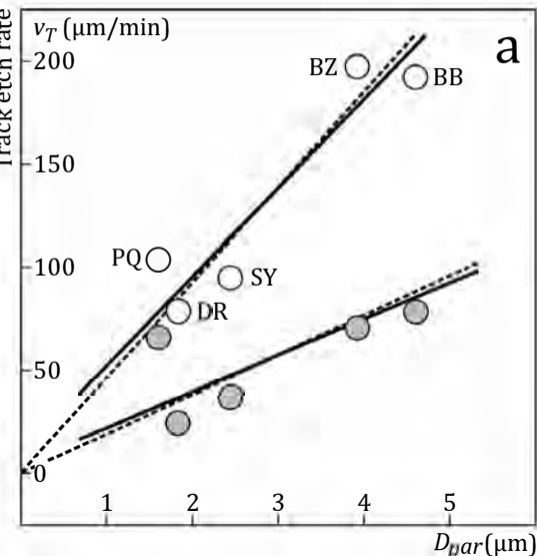


Figure 10

