

APPENDIX: FISSION-TRACK LENGTH C-AXIS PROJECTION SOURCE CODE

FORTRAN source code is provided below that implements the fission-track length *c*-axis projection model presented in this paper. Appendix Table 1 lists the contents of an input data file named *testl.dat* and the corresponding output data file named *testlc.dat*.

The model presented in this paper permits any fission-track length l_i oriented at θ_i degrees to the crystallographic *c* axis in apatite to be converted to an equivalent track length parallel to the crystallographic *c* axis, $l_{c,i}$, based on the results of annealing experiments for six different apatites (five calcian fluorapatites and Durango apatite). An iterative process of calculation is required to obtain values of $l_{c,i}$ according to the following steps:

Step 1

Using Equation 1, the value of l_c is determined iteratively by varying l_c until the following expression holds within some pre-defined level of tolerance:

$$l_i = 1/\{\sqrt{[(\sin\theta_i/l_a)^2 + (\cos\theta_i/l_c)^2]}\} \quad (A1)$$

Step 2

(a) For $l_c \geq 12.96 \mu\text{m}$, the fission track is an elliptical-model fission track and the value of l_c obtained using Appendix Equation 1 is accepted as $l_{c,i}$. (b) However, for $l_c < 12.96 \mu\text{m}$, fission-track populations exhibit accelerated length reduction at relatively high angles to the crystallographic *c* axis. These populations are characterized by a progressive collapse of the fitted ellipses from relatively high angles to the *c* axis near $l_c \approx 12 \mu\text{m}$ to increasingly low angles to the *c* axis as l_c approaches $9.31 \mu\text{m}$, the approximate minimum l_c value to be expected before a fission-track population completely anneals and becomes unetchable. For collapsed ellipses, two equations define the mean fission-track length as a function of track angle to the crystallographic *c* axis; at relatively low angles to the *c* axis, fission-track lengths remain distributed about a partial ellipse; at relatively high angles to the *c* axis, fission-track lengths are shortened sufficiently so that they plot below the ellipse on average. A parameter defined as the accelerated-length-reduction angle, θ_{air} , is defined corresponding to the orientation relative to the *c* axis that divides these two fission-track length behaviors. The parameter θ_{air} is correlated with l_c according to Equation 2. For $\theta_i \leq \theta_{\text{air}}$, the fission track is an elliptical-model fission track and the value of l_c obtained iteratively using Appendix Equation 1 is accepted as $l_{c,i}$. (c) For $\theta_i > \theta_{\text{air}}$, most fission tracks are accelerated-length-reduction fission tracks and the mean fission-track length is modeled as a line between two specific points. The first point, $(c_1 = 0, a_1)$, is the *c*-axis perpendicular intercept of the line passing through the accelerated-length-reduction fission tracks. The parameter a_1 is correlated with θ_{air} according to Equation 3. The second point, (c_2, a_2) , corresponds to the point at the angle θ_{air} to the crystallographic *c* axis on the ellipse passing through the accompanying elliptical model fission tracks. Using Appendix Equation 1, the Cartesian coordinates of this point are given by

$$c_2 = \cos\theta_{\text{air}}/\{\sqrt{[(\sin\theta_{\text{air}}/l_a)^2 + (\cos\theta_{\text{air}}/l_c)^2]}\}$$

$$a_2 = \sin\theta_{\text{air}}/\{\sqrt{[(\sin\theta_{\text{air}}/l_a)^2 + (\cos\theta_{\text{air}}/l_c)^2]}\}$$

The value of l_c is determined iteratively by varying l_c until the following expression holds within some pre-defined level of tolerance.

$$l_i = \sqrt{\{(l_i \cos\theta_i)^2 + [l_i \cos\theta_i (a_2 - a_1)/(c_2 - c_1) + a_1]^2\}} \quad (A2)$$

For an accelerated-length-reduction fission track, the value of l_c obtained using Appendix Equation 2 is accepted as $l_{c,i}$.

Step 3

This step in program *lcproj* is used to assign the minimum possible l_c value to a fission-track length that is too short at its respective crystallographic orientation to fall within the valid model space. The value used for this parameter in *lcproj* is $7.31 \mu\text{m}$ representing $9.31 \mu\text{m}$ minus $2.00 \mu\text{m}$; the former is the minimum l_c observed for the six selected apatites and the latter is equal to ~ 2 units of $\sigma_{c,\text{mod}}$. The last fission track listed in both the input and output data files in Table A1 is an example of such a fission track.

Step 4

This step in program *lcproj* is used to test for convergence of the iterative solution being implemented and to update boundary values of l_c to facilitate convergence after each iteration for which convergence has yet to be achieved.

```

program lcproj
dimension
xl(2000),xthet(2000),xkin(2000),xlc(2000)
c reads 3 column length file (length,
angle to c-axis in degrees,
c kinetic parameter), projects the
lengths onto the c-axis, writes
c projected lengths to a new 3 column
length file
call readl(n,xl,xthet,xkin)
call projlc(n,xl,xthet,xlc)
call writelc(n,xlc,xkin)
stop
end
c
subroutine readl(n,xl,xthet,xkin)
dimension
xl(2000),xthet(2000),xkin(2000)
c reads 3 column length file and con-
verts angles from degrees to
c radians
pi=3.1415927
n=0
open(1,file=' ')
5 n=n+1
read(1,'(3f15.7)',end=10)xl(n),xthet(n),xkin(n)
xthet(n)=xthet(n)*(pi/180.)
goto 5
10 close(1)
n=n-1
return
end
c
subroutine projlc(n,xl,xthet,xlc)
dimension
xl(2000),xthet(2000),xlc(2000)
c projects lengths onto the c-axis with

```

```

a tolerance given by
c parameter tol
  pi=3.1415927
c parameters for Equation 1
  eq11=1.632
  eq12=-10.879
c parameters for Equation 2
  eq21=0.304
  eq22=0.439
c parameters for Equation 3
  eq31=0.1035
  eq32=-2.25
c lc value below which evidence of accel-
erated length reduction
c exists
  xlccut=12.96
c minimum lc value possible minus ap-
proximately 2 standard deviations
  xlc0=9.31-2.*1.
c tolerance level
  tol=0.001
  do 10 i=1,n
c step 1: calculate c-axis parallel
length lci value for measured
c values of li, thetai
  xlcmin=xl(i)
  xlcmax=20.
5   xlc(i)=(xlcmax+xlcmin)/2.
  xla=eq11*xlc(i)+eq12
  xltry=ell(xla,xlc(i),xthet(i))
c step 2a: test if on full ellipse or on
collapsed part of
c ellipse using cutoff given by parameter
xlccut
  if (xlc(i).lt.xlccut) then
c step 2b: if on collapsed ellipse then
calculate accelerated
c length reduction angle xthetalr for
current lci and convert to
c radians; test if xtheti is greater than
xthetalr for current lci
  xthetalr=eq21*exp(eq22*xlc(i))*(pi/
180.)
  if (xthet(i).gt.xthetalr) then
c step 2c: for xtheti greater than
xthetalr calculate parameters
c for linear collapsed part of ellipse
with lci
  c1=0.0
  a1=eq31*xthetalr*(180./pi)+eq32
  c2=
  ell(xla,xlc(i),xthetalr)*cos(xthetalr)
  a2=
  ell(xla,xlc(i),xthetalr)*sin(xthetalr)
  xseg=xl(i)*cos(xthet(i))
  yseg=((a2-a1)/(c2-c1))*xseg+a1
c equation (A.2)
  xltry=sqrt(xseg**2+yseg**2)
  endif
endif

```

```

c step 3: if a length is too short for
the model then the model
c projects this length to the minimum lci
value possible where la
c equals 0.
  if (xlcmax-tol.le.xlc0) then
    xlc(i)=xlc0
    goto 10
c step 4: test for convergence and adjust
minimum or maximum value
  else if (xltry.gt.(xl(i)+tol)) then
    xlcmax=xlc(i)
    goto 5
  else if (xltry.lt.(xl(i)-tol)) then
    xlcmin=xlc(i)
    goto 5
  endif
10 continue
return
end
end
c
function ell(xla,xlc,xthet)
c equation (A.1)
  ell=(sin(xthet)/xla)**2+(cos(xthet)/
xlc)**2
  ell=1./(sqrt(ell))
return
end
c
subroutine writelc(n,xlc,xkin)
dimension xlc(2000),xkin(2000)
c writes 3 column length file and with
the orientation angles set
c to zero
  xthet=0.
  open(2,file=' ')
  do 10, i=1,n
  write
    (2,' (3f15.7)')xlc(i),xthet,xkin(i)
10 continue
close(2)
return
end

```

Appendix Table 1. Example input and output data files for program *lcproj*

<i>testl.dat</i> (input)			<i>testlc.dat</i> (output)		
xl(i) (μm)	xthet (degrees)	xkin(i) (μm)	xlc(i) (μm)	xthet (degrees)	xkin(i) (μm)
16.00	87	1.50	16.47	0	1.50
16.00	3	1.51	16.00	0	1.51
13.00	87	1.52	14.63	0	1.52
13.00	3	1.53	13.01	0	1.53
12.80	87	1.54	14.51	0	1.54
12.80	45	1.55	13.95	0	1.55
12.80	10	1.56	12.92	0	1.56
8.00	87	1.57	12.82	0	1.57
8.00	45	1.58	11.30	0	1.58
8.00	10	1.59	8.69	0	1.59
3.00	87	1.60	11.61	0	1.60
3.00	45	1.61	10.63	0	1.61
9.80	10	1.62	10.12	0	1.62
1.00	10	1.63	7.31	0	1.63