Supplementary Material

Appendix to tetrahedral plot diagram

Geometry of tetrahedron

Our equations 2a, 2b, and 2c are solved by geometry of a tetrahedron. The geometrical features are shown in Figure A1.

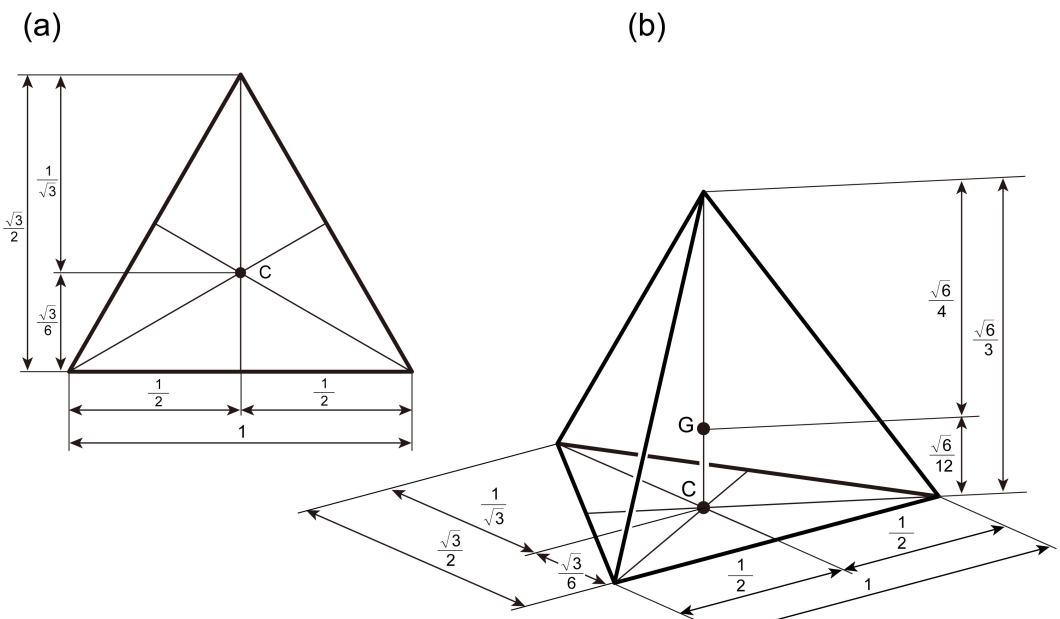


Figure A1. (a) Geometrical features of an equilateral triangle. (b) Geometrical features of a regular tetrahedron. C and G are the centers of mass of an equilateral triangle and a regular tetrahedron, respectively.

Rotation in 3D space

In the main article, the origin of the XYZ-orthogonal coordinate system was (0, 0, 0). For the purpose of rotation in three-dimensional space, the center of rotation is fixed to the geometric center of the tetrahedron, $\left(0.5, \frac{\sqrt{3}}{6}, \frac{\sqrt{6}}{12}\right)$, then the new coordinates for rotation (x_0, y_0, z_0) are,

$$x_0 = x - 0.5 \quad (3a)$$

$$y_0 = y - \frac{\sqrt{3}}{6} \quad (3b)$$

$$z_0 = z - \frac{\sqrt{6}}{12} \quad (3c)$$

Here, the rotation angles around the X-, Y-, and Z-axes are referred to as α, β, and γ, respectively (Fig. A2). For the purpose of observing the tetrahedral diagram, it is sufficient to change only two angles. One is the rotation angle $\gamma\,(0 \leq \gamma \leq 360^{\circ})$ and the other is the dip angle of the observer, d $(0 \leq d \leq 90^{\circ})$ (Fig. A2). Rotation of the tetrahedron can be calculated using the Euler angles equation. After the γ-rotation, the coordinates (x_γ, y_γ, z_γ) are,

$x_\gamma = x_0 \cos \gamma - y_0 \sin \gamma$ (4a)

$y_\gamma = x_0 \sin \gamma + y_0 \cos \gamma$ (4b)

$z_\gamma = z_0$ (4c)

The coordinates after changing the observer dip angle (x_d, y_d, z_d) are,

$x_d = x_\gamma$ (5a)

$y_d = y_\gamma \cos d - z_\gamma \sin d$ (5b)

$z_d = y_\gamma \sin d + z_\gamma \cos d$ (5c)

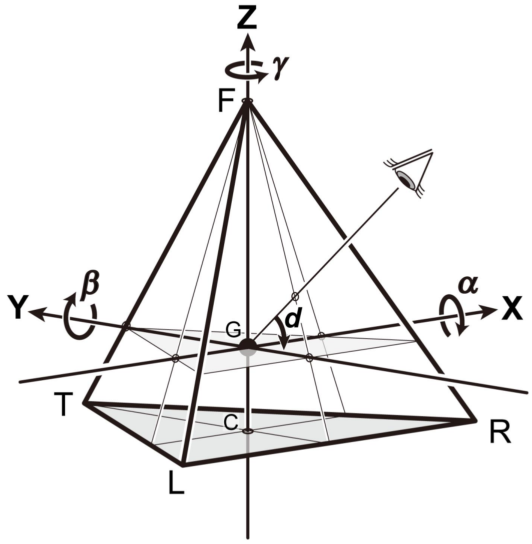


Figure A2. Tetrahedron in the right-handed orthogonal coordinate system. "*d* " is observer dip angle relative to the horizontal surface (X-Y plane). C and G are the centers of mass of an equilateral triangle and regular tetrahedron, respectively.

Projection to a 2D plane

The simplest projection from 3D to 2D is by a parallel projection. If the horizontal axis and longitudinal axis of a computer screen are defined as x_p and z_p, then

$(x_p, z_p) = (x_d, z_d)$ (6)

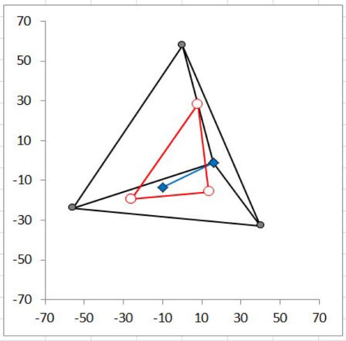
Normally, the parallel projection method is sufficient to observe the tetrahedral diagram.

The tetrahedral plot diagram can be easily constructed using the algorithms of this paper. It can be made using a spread sheet application (e.g. Microsoft Excel) as shown in Figure A3. If a scatter diagram is made by (x_p, z_p), at any given d, with changing γ, the tetrahedral diagram will rotate about the longitudinal axis.

If a perspective projection is required, the y_d value is used. Since y_d specifies depth from the computer screen, let (x_p, z_p) become smaller on a pro-rata basis with y_d. For hidden line processing, the mark is drawn on the order of the y_d value (Fig. A4). If two diagrams with slightly different γ (about 2-5 degrees) are constructed, these can be viewed as a 3D stereoscopic figure.

(a)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1									Frame								Plot data					
2									R	T	L	R	F	T	F	L	Sil	Bt	Grt	Crd	Kfs	Grt
3							Al_2O_3	T%	0	100	0	0	0	100	0	0	100.0	12.5	25.0	50.0	50.0	25.0
4							FeO	L%	0	0	100	0	0	0	0	100	0.0	43.8	62.5	15.0	0.0	62.5
5							MgO	R%	100	0	0	100	0	0	0	0	0.0	31.3	12.5	35.0	0.0	12.5
6							K_2O	F%	0	0	0	0	100	0	100	0	0.0	12.5	0.0	0.0	50.0	0.0
7								X	50.0	0.0	-50.0	50.0	0.0	0.0	0.0	-50.0	0.0	-6.3	-25.0	10.0	0.0	-25.0
8								Y	-28.9	57.7	-28.9	-28.9	0.0	57.7	0.0	-28.9	57.7	-14.4	-7.2	14.4	28.9	-7.2
9								Z	-20.4	-20.4	-20.4	-20.4	61.2	-20.4	61.2	-20.4	-20.4	-10.2	-20.4	-20.4	20.4	-20.4
10			degree	coarse	fine	radian		Xg	40.1	15.9	-56.0	40.1	0.0	15.9	0.0	-56.0	15.9	-10.0	-26.0	13.6	8.0	-26.0
11		gamma	344			6.00		Yg	-41.5	55.5	-14.0	-41.5	0.0	55.5	0.0	-14.0	55.5	-12.2	0.0	11.1	27.7	0.0
12								Zg	-20.4	-20.4	-20.4	-20.4	61.2	-20.4	61.2	-20.4	-20.4	-10.2	-20.4	-20.4	20.4	-20.4
13			degree	coarse	fine	radian		Xd	40.1	15.9	-56.0	40.1	0.0	15.9	0.0	-56.0	15.9	-10.0	-26.0	13.6	8.0	-26.0
14		dip	19			0.33		Yd	-32.6	59.1	-6.6	-32.6	-19.9	59.1	-19.9	-6.6	59.1	-8.2	6.6	17.2	19.6	6.6
15								Zd	-32.8	-1.2	-23.8	-32.8	57.9	-1.2	57.9	-23.8	-1.2	-13.6	-19.3	-15.7	28.3	-19.3
16								Xp	40.1	15.9	-56.0	40.1	0.0	15.9	0.0	-56.0	15.9	-10.0	-26.0	13.6	8.0	-26.0
17								Zp	-32.8	-1.2	-23.8	-32.8	57.9	-1.2	57.9	-23.8						
18																	-1.2	-13.6				
19																			-19.3	-15.7	28.3	-19.3



(b)

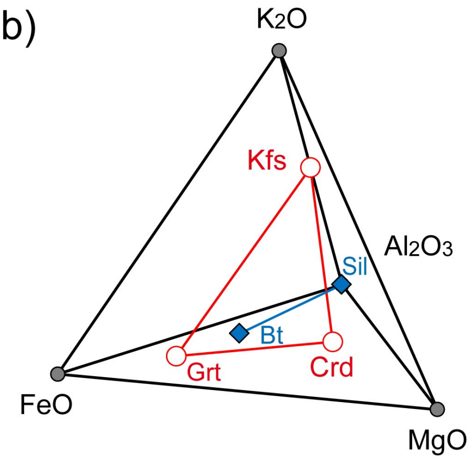


Figure A3. (a) An example of a quaternary diagram made by a spread sheet application (Microsoft Excel). (b) The tetrahedron depicts the Al_2O_3-FeO-MgO-K_2O system for pelitic metamorphic rocks. Filled blue diamonds show sillimanite + biotite, open red circles show garnet + cordierite + K-feldspar. T, L, R, and F are endmember names as described in the main article.

In this table, eight columns (from I to P in the spread sheet) describe the edges of the tetrahedron. Two garnet columns are required, duplicating a start point and an end point in order to draw the tie-line. X, Y, and Z are the orthogonal axis coordinates which are calculated by equations 2a - 3c. Xg, Yg, and Zg are calculated by equations 4a - 4c. Xd, Yd, and Zd are calculated by equations 5a - 5c.

In this example, Xp and Zp are the same as Xd and Zd, respectively (parallel projection, equation 6). If the spin buttons on the spread sheet are clicked, the tetrahedral diagram will rotate. This Excel file can be downloaded as a supplementary electronic file.

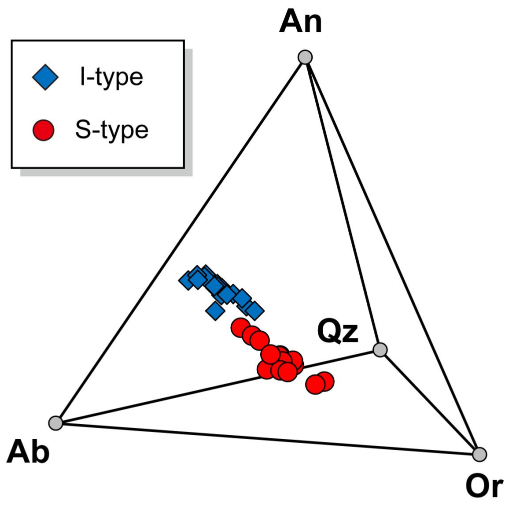


Figure A4. An example of normative An-Ab-Or-Qz tetrahedral diagram for granitic rocks. The plot data are 37 Ma (Kemp et al., 2007) I- and S-type tonalites of the Hidaka Metamorphic Belt, northern Japan (Shimura, 1999; Shimura et al., 1992, 2004). For hidden line processing, the marks are drawn on the order of the Yd value, at a condition of $\gamma = 223^{\circ}$ and $d = 14^{\circ}$. This example is also included in the supplementary Excel file.

REFERENCES

Kemp, A.I.S., Shimura, T., Hawkesworth, C.J. and EIMF (2007) Linking granulites, silicic magmatism and crustal growth in arcs: Ion microprobe (zircon) U-Pb ages from the Hidaka Metamorphic Belt, Japan. Geology, 35, 807-810.

Shimura, T. (1999) Genesis of the pyroxene-bearing I-type tonalite and melting degree of the source rock, in the Hidaka Metamorphic Belt, northern Japan. Journal the Geological Society of Japan, 105, 536-551.

Shimura, T., Komatsu, M. and Iiyama, J.T. (1992) Genesis of the lower crustal Grt-Opx tonalite (S-type) in the Hidaka Metamorphic Belt, northern Japan. Transactions of the Royal Society of Edinburgh: Earth Science, 83, 259-268.

Shimura, T., Owada, M., Osanai, Y., Komatsu, M. and Kagami, H. (2004) Variety and genesis of the pyroxene-bearing S- and I-type granitoids from the Hidaka Metamorphic Belt, Hokkaido, northern Japan. Transactions of the Royal Society of Edinburgh: Earth Science, 95, 161-179.