Collapsing minerals: Crackling noise of sandstone and coal, and the predictability of mining accidents

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ABSTRACT

Mining accidents are sometimes preceded by high levels of crackling noise, which follow universal rules for the collapse of minerals. The archetypal test cases are sandstone and coal. Their collapse mechanism is almost identical to earthquakes: the crackling noise in large, porous samples follows a power law (Gutenberg-Richter) distribution $P \sim E^{-\epsilon}$ with energy exponents $\epsilon$ for near critical stresses of $\epsilon = 1.55$ for dry and wet sandstone, and $\epsilon = 1.32$ for coal. The exponents of early stages are slightly increased, 1.7 (sandstone) and 1.5 (coal), and appear to represent the collapse of isolated, uncorrelated cavities. A significant increase of the acoustic emission, AE, activity was observed close to the final failure event, which acts as “warning signal” for the impending major collapse. Waiting times between events also follow power law distributions with exponents $2 + \frac{\gamma}{2}$ between 2 and 2.4. Aftershocks occur with probabilities described by Omori coefficients $p$ between 0.84 (sandstone) and 1 (coal). The “Báth’s law” predicts that the ratio between the magnitude of the main event and the largest aftershock is 1.2. Our experimental findings confirm this conjecture. Our results imply that acoustic warning methods are often possible within the context of mining safety measures, although it is not only the increase of crackling noise that can be used as early warning signal but also the change of the energy distribution of the crackling events.

Keywords: Sandstone, coal, crackling noise, failure and collapse event, precursor effects

INTRODUCTION

Earthquakes and the collapse of porous materials are related phenomena deeply connected by the emission of crackling noise (Baró et al. 2013; Salje and Dahmen 2014; Sethna et al. 2001) where systems under slow perturbation respond through discrete events, so-called “jerks,” with a huge variety of sizes and energies. The signatures of seismic events in geophysics coincide with laboratory-scale experiments (“labquakes”) of compressed porous and fractured materials (Davidsen et al. 2007; Diodati et al. 1991; Hirata 1987; Kun et al. 2007, 2009, 2013; Lebyodkin et al. 2013; Nataf et al. 2014a; Niccolini et al. 2009, 2010, 2011; Petri et al. 1994; Salje et al. 2013; Weiss and Miguel 2004) and have been simulated by numerical discrete element calculations of porous materials (Kun et al. 2013, 2014). In laboratory experiments, external loading is applied to the samples and the system’s response is obtained by recording acoustic emission (AE). Other recordings are the stepwise change of the macroscopic strain or the emission of calorimetric heat jerk (Baró et al. 2014; Gallardo et al. 2010). All recordings mimic earthquakes since the main stresses underlying tectonic quakes are considered compressive (Baró et al. 2013). Previous experimental studies of labquakes have given some evidence that major collapse events are preceded by increased precursor crackling noise, which may allow the prediction of failure during earthquakes. Equally importantly, predicting failure is needed in the mining context. The main question is then: is it possible to predict a main collapse event from pre-shocks before the failure event actually occurs? First observations of large-scale foreshock sequences go back to 1988, where a full sequence was observed at the Chalfat earthquake by Smith and Priestley (1988) and large sequences of Californian earthquakes by Dodge et al. (1996). In each case, the statistical evidence was limited and related to technical issues of seismological observations. More complete data from laboratory experiments, such as from observations in porous goethite, FeO(OH), revealed two scenarios (Salje et al. 2013). Samples with low porosity showed no evidence for any precursor effects and no “early warning” signal could be extracted from the compression noise. Samples with high porosities (>80%), on the other hand, did show precursor noise and opened the possibility to use pico-seismic observations to predict the collapse of a goethite mine (Salje et al. 2013). Subsequent work found no indication of increased crackling noise in berlinite (Nataf et al. 2014b). Instead there was an indication that the exponent of power-law energy distribution reduces near the critical point, in agreement with trends in numerical simulations.

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of collapse mechanisms by Kun et al. (2013).

No significant increase of the AE activity was observed in the first such study of SiO$_2$-based Vycor (Salje et al. 2011), while an extensive study of other SiO$_2$-based materials found only a very weak increase of precursor activity in sandstone (Nataf et al. 2014a). No effect was found in charcoal (Ribeiro et al. 2015). These results contradict the simulation results (Kun et al. 2013), where both a big increase of AE activity and a change of the power law exponent were predicted. To test this scenario, we changed the experimental arrangement from very small load stresses and samples (Salje et al. 2011) to one where much larger samples can be compressed under enhanced forces. This novel experimental arrangement allows us to study systematically the major collapse mechanisms in the mineralogical context.

**EXPERIMENTAL METHODS**

The samples of sandstone and coal were collected from the Sichuan and Shanxi provinces of China. The samples were drilled with a high-speed rotary saw from larger blocks, their shapes were cylindrical with 50 mm diameter and 100 mm length. The sides of the specimen were optically smooth, and the ends of the specimens were smooth within ±0.02 mm (according to ISRM testing guidelines, Fairhurst and Hudson 1999). The density and porosity were determined by wax seal methods and mercury intrusion analysis, respectively. They are very similar to those of smaller samples measured previously (Table 1).

Dry sandstone samples were heated to 110 °C for 24 h and cooled to room temperature just before the compression experiment. Saturated sandstones were immersed in water for 48 h.

The compression experiment was performed using the loading equipment in Figure 1. The slowly increasing load is provided by oil pouring into a container at a constant flow rate, the weight of oil is then transferred to the lower tilting beam in Figure 1. The samples were placed between the lower tilting beam and a static support. The maximum load is 300 kN and the maximum vertical displacement is 5 mm. The stress rate was chosen to be \( \frac{d\sigma}{dt} = 8.5 \text{kPa/s} \) (1 kN/min) for all samples.

Acoustic emission signals were measured during compression by two or more piezoelectric sensors (NANO-30 Physical Acoustics Company) that were fixed on the sample’s round surface by rubber bands. The sensors were acoustically coupled to the sample by a thin layer of grease. The acoustic signal was pre-amplified (40 dB) and transferred to the AE analysis system (DISP from American Physical Acoustics Company). The threshold for detection was chosen as the signal of an empty experiment (45 dB).

**RESULTS**

The energy of the AE signal, the number of AE hits per second (AE activity), and the cumulative AE activity are shown as a function of the run time in for dry sandstone (Fig. 2a), wet sandstone (Fig. 2b), and coal (Fig. 2c). Note the logarithmic scale for the event energies that spans up to four decades. AE activities and activities are smaller in the early stages of the compression experiment compared with the late stages. The AE emission reaches a steady state in coal that is terminated before the final major failure event. Some early signals come from the friction between sample’s flat faces and the compressive equipment, and the closure of some original micro-cracks in the samples. The AE signals increase dramatically near the major failure event, most notably in sandstone samples, where virtually no steady-state emission occurs.

The density and porosity of our sandstone samples are close to those of Nataf et al. (2014a), see Table 1, although our failure stress is about five times higher than theirs (see Table 2). This may be due to our faster loading rates (Table 2), and smaller levels of stress concentrations in our cylindrical samples compared with the previously used prismatic samples. Our sandstone is also particularly uniform with few preexisting cracks, which leads to fewer AE events during the middle stage (Fig. 2). Coal contains more original micro-cracks than sandstone and more AE signals were recorded before failure.

**ANALYSIS**

The energy distribution (Gutenberg-Richter law)

The probability distribution function, PDF, of jerk energies (events) is \( P(E) \) and is derived from the raw data by appropriate linear binning of the event energies. The number of bins was chosen between 10$^4$ and 10$^6$ with little influence of the bin number on the functional form of the PDF. Logarithmic binning did
not change the results. Figure 3 shows the energy distributions $P(E)$ in log-log plots for dry and wet sandstone, and coal. The histogram corresponds to the accumulation of signals along the whole experiment. A good power law behavior is observed over more than three decades for the early stages and late stages of each collapse sequence: that the distribution of energies follows a power law (Gutenberg-Richter law) $P(E) \sim E^{-a}$ is a very good approximation but with different exponents for each data set. A finer sequencing with five intervals in the early stages and three intervals for the late stages lead to the same result, namely that the exponents show only two different numerical values with a larger value at the early stages and the smaller value near the major failure event. In each case, the distribution is described by

**Figure 2.** The energy of the AE signals, the time evolution of the AE activity (s$^{-1}$), and the accumulated number of events for (a) dry sandstone, (b) wet sandstone, and (c) coal samples. In all cases does the activity strongly increase prior to the main failure event. This signal can be used as early warning sign for impending collapse, e.g., in coal mines and buildings.

**Figure 3.** Distribution of avalanche energies for (a) dry sandstone, (b) wet sandstone, and (c) coal samples in the different time windows of experiments. The inset shows the ML-fitting exponent $e$ as a function of a lower threshold $E_{\text{min}}$. 
\[ P(E) dE = \frac{E^{-\varepsilon}}{E_{\text{min}}} dE; \quad E > E_{\text{min}} \]

where \( E_{\text{min}} \) is a lower cutoff used for normalization. To examine the distribution in more detail we apply the Maximum Likelihood method (Clauset et al. 2009). This method avoids the construction of histograms and the choice of the number of bins. The analytical formula is

\[ \varepsilon(x_{\text{min}}) = 1 + \frac{1}{n} \sum_{i=1}^{n} \ln \frac{x_i}{x_{\text{min}}} \]

where \( x_i, i = 1...n \) are the observed values of \( x \) such that \( x_i \geq x_{\text{min}} \)

The standard error is

\[ \sigma = \frac{\varepsilon(x_{\text{min}})-1}{\sqrt{n}} + O(\frac{1}{n}) \]

The results are as shown in the inserts of Figure 3, this analysis leads to a plateau, which defines the exponential parameter \( \varepsilon \). Note that we define \( \varepsilon \) as a positive quantity; the negative sign in the exponent then defines the negative slope of the power law curve. As the number of data points at the late stages (blue curves in the inserts) is much greater than in the early stages we find a much better defined plateau in this case with well-defined exponents \( \varepsilon \sim 1.55 \) (sandstone) and \( \varepsilon \sim 1.32 \) (coal) (Fig. 4). The agreement between the two methods, namely direct binning and the Maximum Likelihood, for the determination of \( \varepsilon \) is excellent.

**Distribution of waiting times**

Figure 5 shows the distribution of waiting times, defined as \( \delta_j = t_j - t_{j-1} \), with \( j \) labeling only the events with energy larger than a given threshold energy \( E_{\text{th}}^a \). \( D(\delta, E_{\text{th}}^a) \) is the waiting time distribution function and indicates the probability to observe a waiting time \( \delta \) for a threshold energy \( E_{\text{th}}^a \). When plotting \( D(\delta, E_{\text{th}}^a) \) as function of \( \delta \), we scale the axes as \( \langle r(E_{\text{th}}^a) \rangle = \delta \) and \( D(\delta, E_{\text{th}}^a) = \langle r(E_{\text{th}}^a) \rangle \), where \( \langle r(E_{\text{th}}^a) \rangle \) is the mean number of events per unit time with an energy \( E > E_{\text{th}}^a \). This approach leads to a collapse of all data in a single curve showing double power-law behavior with exponents \( 1 - \nu \) for small arguments, and \( 2 + \xi \) for large arguments (we follow again the convention to define the exponents as positive quantities and maintain the negative sign to indicate the negative slope of the power law function). We have insufficient data to determine \( 1 - \nu \) while the results for \( 2 + \xi \) are sufficiently well constrained for all data sets. No significant change of \( 2 + \xi \) was found between dry and water-saturated sandstone.

**Aftershocks and Báth’s law**

The number of aftershocks (AS) is described by the Omori’s law (Utsu et al. 1995) as often employed for the analysis of earthquakes. It states that the number of AS decays as a power-law after each mainshock (MS). We define MS as AE signals with energies \( E_{\text{MS}} \) between \( 10^k \) to \( 10^{k+1} \) aJ, with \( k = 1, 2, \) and \( 3 \). The sequence of AS is then continued until another MS is found, which terminates...
the AS sequence. We calculate the aftershock rates as function of the lapse time since the MS. The results of dry sandstone, wet sandstone, and coal are shown in Figures 6a–6c. The power law dependence is emphasized by the averaged slope with Omori exponents $p = 0.84$ (sandstone) and $p = 0.95$ (coal). These values are typical for Omori sequences in natural earthquakes.

The Báth’s law (Báth 1965; Console et al. 2003; Helmstetter and Sornette 2003) states that the ratio of the energy magnitudes of a MS and an AS is, on average, near to 1.2. This ratio is independent of the magnitude of the MS.

We plot in Figure 7 the normalized ratio between the magnitude of the mainshock MS and its largest aftershock AS*

$$\Delta M = \log \left( \frac{E_{MS}}{E_{AS}} \right)$$

as function of the magnitude of MS. The ratio $\Delta M$ shows indeed a trend toward the value of the Báth’s law of 1.2 for large MS. The dependency of $\Delta M$ on the magnitude of the main shock is well described by extended Debye model:

$$\Delta M = A_1 + \left( \frac{A_2 - A_1}{1 + \left( \frac{E_{MS}}{E_0} \right)^p} \right)$$

$A_1 = 0.13, A_2 = 1.16, E_0 = 413.65, p = 1.21, \text{ and R-square} = 0.97.$

**DISCUSSION**

Our investigations represent a significant advancement with respect to previous experimental studies of collapsing porous materials because we use large sample volumes and high applied stresses. These conditions are closer to those in the context of mining or the collapse of monuments than previous studies (Baró et al. 2013, 2014; Castillo-Villa et al. 2013; Gallardo et al. 2010; Nataf et al. 2014a, 2014b; Salje et al. 2008, 2009, 2011, 2013; Soto-Parra et al. 2015). The large sample volume leads to a new phenomenon that was not possible to observe in small sample, which is the evolution of the event centers during the experiment. The results indicate that all avalanche characteristics can be divided into two groups, namely those at the early stages and those closer to the main failure event. Using eight AE detectors and standard localization software, we determine the distribution of event centers where AE jerks originate in the sample. These distributions are shown in Figure 8.

The initial stages are characterized by randomly distributed event centers with large distances between them. The events are
hence approximately randomly distributed in time and space. We describe these events as “uncorrelated.” With an increasing number of defect centers we find an accumulation of event centers along the shear diagonal that characterizes the final failure of the sample. The event centers are now very densely located and spatially correlated. The energy exponents for the randomly distributed events are generally larger than those of the correlated events although the difference is small. The difference between the exponents given in Table 3 are not visible in most other experiments while our high-resolution investigations make it possible to distinguish between exponents within error margins of $\Delta \varepsilon = 0.1$.

The power law exponents near the major collapse event ($1.54$ and $1.32$) are similar to those reported by Nataf et al. (2014a) who found exponents $\varepsilon$ between $1.44$ and $1.55$ for three different kinds of sandstone. Furthermore, the exponents of stage 2 for coal ($\varepsilon_2 = 1.32$) are also compatible with the statistical results come from ethanol-dampened charcoal ($1.27–1.33$) (Ribeiro et al.)

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**Figure 8.** Evolution of AE centers. (a) A few AE centers appear in the top and bottom areas of sandstone sample from the friction between sample faces and the loading device, (b) some AE centers occur randomly, (c) AE centers form the final crack, and (d) photo of the cracked sample.

**Table 3.** Exponents for statistical laws used in this and previous studies

<table>
<thead>
<tr>
<th>Material</th>
<th>$\varepsilon_1/\varepsilon_2$ or $\langle \varepsilon \rangle$</th>
<th>$1 - \gamma$</th>
<th>$2 + \delta$</th>
<th>$\rho$</th>
<th>$\Delta M$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry sandstone</td>
<td>1.77/1.53</td>
<td>2.2</td>
<td>0.84</td>
<td>1.2</td>
<td>This study</td>
<td></td>
</tr>
<tr>
<td>Wet sandstone</td>
<td>1.71/1.56</td>
<td>2.4</td>
<td>0.85</td>
<td>1.2</td>
<td>This study</td>
<td></td>
</tr>
<tr>
<td>Coal</td>
<td>1.51/1.32</td>
<td>2.0</td>
<td>0.95</td>
<td>1.2</td>
<td>This study</td>
<td></td>
</tr>
<tr>
<td>LGsan</td>
<td>$&lt;1.48&gt;$</td>
<td>0.86</td>
<td>2.0</td>
<td>0.78</td>
<td>Nataf et al. (2014a)</td>
<td></td>
</tr>
<tr>
<td>Rian</td>
<td>$&lt;1.55&gt;$</td>
<td>1.12</td>
<td>2.3</td>
<td>0.74</td>
<td>Nataf et al. (2014a)</td>
<td></td>
</tr>
<tr>
<td>Yian</td>
<td>$&lt;1.49&gt;$</td>
<td>1.01</td>
<td>1.8</td>
<td>0.74</td>
<td>Nataf et al. (2014a)</td>
<td></td>
</tr>
<tr>
<td>Vycor</td>
<td>$&lt;1.39&gt;$</td>
<td>0.93</td>
<td>2.45</td>
<td>0.75</td>
<td>Baró et al. (2013)</td>
<td></td>
</tr>
<tr>
<td>Charcoal</td>
<td>$&lt;1.3&gt;$</td>
<td>0.87</td>
<td>1.2</td>
<td></td>
<td>Ribeiro et al. (2015)</td>
<td></td>
</tr>
<tr>
<td>Cu-Zn-Al alloys</td>
<td>$&lt;2.15&gt;$</td>
<td>0.9</td>
<td>2.2</td>
<td></td>
<td>Baró et al. (2014)</td>
<td></td>
</tr>
<tr>
<td>Bamboo</td>
<td>$&lt;1.45&gt;$</td>
<td>1.68</td>
<td>1.7</td>
<td></td>
<td>Tsai et al. (2016)</td>
<td></td>
</tr>
<tr>
<td>Spaghetti</td>
<td>$&lt;1.35&gt;$</td>
<td>3.53</td>
<td>0.8</td>
<td></td>
<td>Tsai et al. (2016)</td>
<td></td>
</tr>
<tr>
<td>Simulation</td>
<td>$&lt;2.02&gt;$</td>
<td>2.0</td>
<td></td>
<td></td>
<td>Kun et al. (2014)</td>
<td></td>
</tr>
</tbody>
</table>
These observations suggest that our results are consistent with previous research while the exponents for stage 1 are not only higher than the value of the correlated stage, but also higher than the results of previous research. A solution to this conundrum may stem from recent results of Kun et al. (2013, 2014) who investigated cracking noise generated by failure in porous materials by computer simulation. A most remarkable feature of their results is that the value of their exponent $\epsilon$ did not show fix-point behavior but smoothly decreased during compression. The smallest value of $\epsilon$ was found near the final collapse. This simulation result is similar to the evolution of the exponent in this study. Our results suggest that we do not have a smooth time dependence of $\epsilon$ but a stepwise behavior between two fix points, however. Only the second, “correlated” fix point is close to the mean field solution while the first fix point may relate to uncorrelated, isolated collapses with a different collapse mechanism (Salje and Dahmen 2014). In the simulations by Kun et al. (2013), spatial positions of bursts in their sample show randomness in the early stage of compression with no memory, while damage bands are formed at the later stage. This picture is confirmed by our result in Figure 8 where first few AE centers appear in the top and bottom areas of sandstone sample (Fig. 8a), then some AE centers occur randomly (Fig. 8b), and finally many AE centers form the final crack (Fig. 8c). This indicates that the lower energy exponent relates to “criticality” near the final failure event and agrees with the observation that previous measurements on small samples lead to the same energy exponent together with a much reduced failure strength. This result can be understood if the sandstone samples of Nataf et al. (2014a) were already weakened by internal cracks so that the physical events leading to the cracking noise were intrinsically linked to correlated local failure events rather than to localized collapse mechanisms as seen at the early stages during our studies.

A crossover behavior of the avalanches size was found in the study of fiber bundles and electrical fuse models (Pradhan et al. 2005). The avalanches size distributions near to and far away from the catastrophic failure follow both power law behavior with different exponents. Multiple fix-point behavior was also reported by Soto-Parra et al. (2015) for the compression of martensitic porous Ti-Ni. The first series of events is generated by de-twinnin ($\epsilon = 2$), while later stages relate to fracture ($\epsilon = 1.7$). This fracture exponent is close to the early stage, uncorrelated value in our study, which appears to indicate that collective shear plane failure does not occur in Ti-Ni alloys.

The waiting time distributions in Figure 5 are similar to those of previous investigations of less uniform and smaller samples. For dry and wet sandstone, the exponents $2 + \xi = 2.2$ and 2.4 are similar with the results for previous light-gray sandstone waiting time analysis ($2 + \xi = 2.0$) (Nataf et al. 2014a). Exponents of $2 + \xi = 2.45$ were reported for uniaxial compression of Vykor (Baró et al. 2013), a mesoporous silica ceramics. A reasonable agreement also exists with the power-law exponent $2 + \xi = 2.0$ from DEM simulation (Kun et al. 2013, 2014).

Our data show an excellent adherence of the collapse sequence of sandstone to the Omori’s law ($p = 0.84$ in Fig. 6). Baró et al. (2013) reported a stable Omori decay in small samples with $p = 0.75$ for Vykor, and $p = 0.78$ for gray sandstone, and $p = 0.74$ for red and yellow sandstone (Nataf et al. 2014a). Coal shows an Omori behavior with $p = 0.95$, similar to charcoal ($p = 0.87$) (Ribeiro et al. 2015). In contrast, recent reports (Tsai et al. 2016) gave much higher values for broken bamboo chopstick and a bundle of spaghetti with $p = 1.68$ and $p = 3.53$, respectively.

The same tendency is seen for the numerical values of $\Delta M$. $\Delta M$ of charcoal was reported to be 1.2, which is identical with our data. Breaking of bamboo chopsticks lead to $\Delta M = 1.7$ and that of a bundle of spaghetti gave $\Delta M = 0.8$ (Tsai et al. 2016). For earthquakes, large fluctuations of $\Delta M$ usually exist for different aftershock sequences (Hainzl et al. 2010).

**IMPLICATIONS**

The results of our research have two major implications. First, it shows that the prediction of collapse events is possible. The two indicators for an impending disaster, such as a collapse of a mining shaft or the collapse of a building, are the increase of acoustic emission of cracking noise and the change of its energy exponent. Both indicators can be measured easily by highly sensitive microphones attached to the material (sandstone sculptures, houses, coals seams, etc.). The signals need to be processed as described in this paper, which can be easily done using a simple computer device. The sensitivity of the methods depends largely on microstructural properties of the minerals: uniform but porous sandstone is much more sensitive to “early warning noise” than coal seams that contain a multitude of local cracks.

Second, we now understand cracking noise much better because we developed a much better measurement technique that allows us to investigate very large samples. The analogy of cracking noise of porous minerals with earthquakes is now firmly established with the surprising additional feature: the energy exponent (or $b$-value in seismology) depends on the collectiveness of the collapse event. Local cavity collapse displays higher energy exponents than collective movements such in crack propagation and the formation of micro-faults. Only the latter is close to the theoretical mean field value. This result will doubtlessly stimulate more theoretical research in this field.

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