APPENDICES

Appendix A1. The β-distribution, $N_T(\beta)\, d\beta$

We derive the β-distribution, $N_T(\beta)\, d\beta$, from the θ-distribution, $N_T(\theta)\, d\theta$ (equation (6)) using the Jacobian $|d\theta/d\beta|$, as follows:

$$N_T(\beta)\, d\beta = N_T(\theta) d\theta \left|\frac{d\theta}{d\beta}\right|\, d\beta \qquad \text{(A1)}$$

$$N_T(\beta)\, d\beta = A\, \sin\theta \left|\frac{d\theta}{d\beta}\right|\, d\beta \qquad \text{(A2)}$$

It follows from equations (6)-(8) and (10) that:

$$\tan\theta = \sin\alpha\, \cot\beta \qquad \text{(A3)}$$

Or:

$$\theta = \arctan(\sin\alpha\, \cot\beta) \qquad \text{(A4)}$$

Differentiating (A4) with respect to β gives:

$$\left|\frac{d\theta}{d\beta}\right| = \frac{\sin\alpha}{\sin^2\beta + \sin^2\alpha\, \cos^2\beta} \qquad \text{(A5)}$$

On the other hand, it follows from (A3) that:

$$\sin\theta = \sqrt{\frac{\sin^2\alpha\, \cos^2\beta}{\sin^2\beta + \sin^2\alpha\, \cos^2\beta}} \qquad \text{(A6)}$$

Substituting (A5) and (A6) in (A2) gives:

$$N_T(\beta)\, d\beta = \frac{A\, \sin^2\alpha\, \cos\beta}{[\sin^2\beta + \sin^2\alpha\, \cos^2\beta]^{3/2}}\, d\beta \qquad \text{(A7)}$$

<u>Appendix A2</u>. The (g/b)-distribution, $N_T(g/b)\ d(g/b)$

We derive the (g/b)-distribution, $N_T(g/b)\ d(g/b)$, from the θ-distribution, $N_T(\theta)\ d\theta$ (equation (6)), as follows:

$$N_T(g/b)\ d(g/b) = A \sin\theta \left|\frac{d\theta}{d(g/b)}\right|\ d(g/b) \tag{A8}$$

It follows from equation (16) that:

$$\sin\theta = \frac{\tan\alpha}{[(g/b)^2 - 1\,]^{1/2}} \tag{A9}$$

and:

$$\theta = arcsin\left[\frac{\tan\alpha}{[(g/b)^2 - 1\,]^{1/2}}\right] \tag{A10}$$

Differentiating (A10) with respect to (g/b) gives:

$$\left|\frac{d\theta}{d(g/b)}\right| = \frac{(g/b)\ \tan\alpha}{[((g/b)^2 - 1) - \tan^2\alpha\,]^{1/2}\ [(g/b)^2 - 1\,]} \tag{A11}$$

Substituting (A9) and (A11) in (A8) gives:

$$N_T(g/b)\ d(g/b) = \frac{A\ (g/b)\tan^2\alpha}{[((g/b)^2 - 1) - \tan^2\alpha]^{1/2}\ [(g/b)^2 - 1]^{3/2}}\ d(g/b) \tag{A12}$$

Appendix A3. The (h/a)-distribution, $N_T(h/a)\ d(h/a)$

The equations are more manageable if we first derive the (a/h)-distribution, $N_T(a/h)\ \mathrm{d}(a/h)$, from the θ-distribution $N_T(\theta)\ d\theta$:

$$N_T(a/h)\ d(a/h) = A \sin\theta \left|\frac{d\theta}{d(a/h)}\right|\ d(a/h) \qquad \text{(A13)}$$

It follows from equation (17) that:

$$\sin\theta = \frac{\left[(a/h)^2 - \sin^2\alpha\right]^{1/2}}{\cos\alpha} \qquad \text{(A14)}$$

and: $$\theta = \arcsin\left[\frac{\left[(a/h)^2 - \sin^2\alpha\right]^{1/2}}{\cos\alpha}\right] \qquad \text{(A15)}$$

Differentiating (A15) with respect to (a/h) gives:

$$\left|\frac{d\theta}{d(a/h)}\right| = \frac{(a/h)\ /\ \cos\alpha}{[1-(a/h)^2]^{1/2}\ [((a/h)^2 - \sin^2\alpha)/\cos^2\alpha]^{1/2}} \qquad \text{(A16)}$$

Substituting (A14) and (A16) in (A13) gives:

$$N_T(a/h)\ d(a/h) = \frac{A\ (a/h)/\cos\alpha}{[1-(a/h)^2]^{1/2}}\ \ d(a/h) \qquad \text{(A17)}$$

To obtain $N_T(h/a)\ d(h/a)$, we note that:

$$N_T(h/a)\ d(h/a) = N_T(a/h)\left|\frac{d(h/a)}{d(a/h)}\right|\ d(a/h) \qquad \text{(A18)}$$

with: $$(h/a) = \frac{1}{(a/h)} \qquad \text{(A19)}$$

and: $$\left|\frac{d(h/a)}{d(a/h)}\right| = \frac{1}{(h/a)^2} \qquad \text{(A20)}$$

Substituting (A17), (A19) and (A20) in (A18) gives:

$$N_T(h/a)\ d(h/a) = \frac{A/\cos\alpha}{(h/a)^2\ [(h/a)^2 - 1]^{1/2}}\ \ d(h/a) \qquad \text{(A21)}$$

Appendix A4. Exhumed tracks

For tracks exposed at the surface and etched due to surface etching (exhumed tracks), we have that (equation (7); main text):

$$N_E(\theta)\,d\theta = B\,d\theta \tag{A22}$$

And:

$$N_E(\beta)\,d\beta = N_E(\theta)d\theta\,\left|\frac{d\theta}{d\beta}\right|\,d\beta \tag{A23}$$

$$N_E(g/b)\,d(g/b) = N_E(\theta)d\theta\,\left|\frac{d\theta}{d(g/b)}\right|\,d(g/b) \tag{A24}$$

$$N_E(a/h)\,d(a/h) = N_E(\theta)d\theta\,\left|\frac{d\theta}{d(a/h)}\right|\,d(a/h) \tag{A25}$$

Substituting (A5) and (A22) in (A23) gives:

$$N_E(\beta)\,d\beta = \frac{B\,\sin\alpha}{\sin^2\beta + \sin^2\alpha\,\cos^2\beta}\,d\beta \tag{A26}$$

Substituting (A11) and (A22) in (A24) gives:

$$N_E(g/b)\,d(g/b) = \frac{B\,(g/b)\,\tan\alpha}{[((g/b)^2-1)-\tan^2\alpha\,]^{1/2}\,[(g/b)^2-1\,]}\,d(g/b) \tag{A27}$$

Substituting (A16), (A20) and (A22) in (A25) gives:

$$N_E(a/h)\,d(a/h) = \frac{B\,/\,(h/a)\cos^2\alpha}{[(h/a)^2-1]^{1/2}\,[(1-(h/a)^2\,\sin^2\alpha)]^{1/2}}\,d(a/h) \tag{A28}$$