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Fractal distribution of mineral species among the crystallographic point groups

Daniel R. Hummer[1,*]

[1]School of Earth Systems and Sustainability, Southern Illinois University, Carbondale, Illinois 62901, U.S.A.

Abstract

Crystallographic data from 5289 IMA-approved mineral species in the RRUFF database were used to examine the distribution of species among the 32 crystallographic point groups. It is found that within each crystal system, minerals strongly prefer point groups with higher group orders. Within a crystal system, the abundance of minerals belonging to each point group approximately obeys a power law with respect to group order, the same mathematical formalism that describes objects with fractal geometry. In this framework, each crystal system has its own fractal dimension; crystal systems possessing threefold (or sixfold) symmetry elements (i.e., trigonal, hexagonal, isometric) have significantly lower fractal dimension (<2), while those with only one-, two-, or fourfold symmetry elements (triclinic, monoclinic, orthorhombic, tetragonal) have higher fractal dimension (>2). While higher symmetry is preferred within a crystal system, the opposite trend is observed when comparing between crystal systems, with more species preferring crystals systems with lower order symmetry elements than those with higher order symmetry elements at constant group order. The combination of these two competing trends leads to a complex distribution of minerals among the crystal systems, and to the monoclinic group 2/*m*, the orthorhombic group 2/*m*2/*m*2/*m*, and the triclinic group $\bar{1}$ being the three most popular point groups, respectively. The fractal behavior of symmetry distribution among minerals points toward universal scaling patterns not just in physical, geometric objects but also in the way that symmetry is incorporated into natural periodic structures.

Keywords: Fractals, point groups, crystal systems, symmetry, crystallography, mineral species