Supplementary materials

This Supplement contains the followings: (i) additional descriptions regarding the estimation of the composition (X_{Si}) of the silicate melt before and after the core segregation in a magma ocean and the determination of Si content in the core, (ii) one figure, and (iii) one table for the manuscript by Moriguti et al., 2021, entitled 'Melting phase equilibrium relations in the $MgSiO_3$–SiO_2 system under high pressures'.

(1) Procedures of calculations to estimate the composition (X_{Si}) of the silicate melt after the core segregation in a magma ocean, and to determine the Si content in the core

To evaluate the feasibility of the enstatite chondrite (E-chondrite) model which proposed that E-chondrite would be the original material of the bulk Earth compositions, we estimated the range of Si content in the core on E-chondrite model and evaluated the range with the Si contents in the core proposed in previous works. To obtain the Si contents in the core on E-chondrite model, we also estimated the X_{si} (= SiO_2/(SiO_2+MgO) (in mol)) of the silicate melt in a magma ocean before and after the core segregation. In the estimations of X_{Si} in the silicate melts before and after the core segregation from a magma ocean, we assumed the following two melting conditions: (a) eutectic melting and (b) complete melting. In case (a), the temperatures of the magma ocean are the eutectic temperatures in the $MgSiO_3$–SiO_2 system, which change with the depth in the magma ocean and which are the lowest in the melting temperatures. In case (b), the temperatures are higher than liquidus temperatures. The X_{Si} of the silicate melts before the core segregation is equivalent to that of the source material, enstatite chondrite. In the calculations, we also assumed a single-stage process for the magma ocean model whose depth was 700 to 1200 km (e.g., Li and Agee, 1996; Righter et al., 1997).

Taking into account the mass balance of Si and Mg between the silicate melt and the metals during the core formation, the silicate melt compositions (X_{Si}) after the core segregation were estimated. The masses of Si and Mg in the silicate melt (M_{Si_melt} and M_{Mg_melt}) were calculated by the following equations:

$$M_{Si\_melt} = M_{Si\_melt\text{-}ini} - M_{Si\_c} \quad (1)$$

$$M_{Mg\_melt} = M_{Mg\_melt\text{-}ini} \quad (2)$$

where $M_{Si\_melt\text{-}ini}$ and $M_{Mg\_melt\text{-}ini}$ are the masses of Si and Mg in the initial silicate melt before the core segregation, respectively. M_{Si_c} is the mass of Si in the core. We assumed that magnesium was retained in the initial silicate melt without incorporation into the core as shown in Eq. (2) because Ito et al. (1995) found that Mg was not incorporated into the molten iron.

The data used in the following calculations are shown in Table S1: the mass, the radius, and the mean density of the Earth; the depth of core-mantle boundary; the proportion of the mass of the core to that of the Earth; the mean abundances of Si and Mg (wt.%) of two types of E-chondrites, EL and EH. As shown below, the densities of the magma ocean were estimated using the density of the present mantle from 40 to 2891 km depth in the PREM model (Dziewonski and Anderson, 1981). Furthermore, considering the pressure dependencies of the densities and the eutectic compositions in $MgSiO_3$ – SiO_2 system (see Fig. 9), the Earth's interior from the surface to 2891 km depth was divided into 44 layers according to the division in the PREM model in principle.

1. Estimation of densities of a magma ocean at the depth of each of the 44 layers

Considering that metals were contained in a magma ocean before core segregation, we assumed that the density of the magma ocean was higher than that of the present mantle. To estimate the density distribution of the magma ocean based on the PREM model, we determined the conversion factor (f_ρ) of the densities between the magma ocean and the present mantle as shown below. In the determination, we assumed that the mean density of the magma ocean (ρ_{mo}) was equivalent to that of the present

Earth (ρ_e) ($\rho_{mo} = \rho_e$). The f_ρ can be shown as the following:

$$f_\rho = \rho_{mo} / \rho_{mnt} \tag{3}$$

where ρ_{mnt} is the mean density of the present mantle. Given the volume and the mass of the mantle, ρ_{mnt} can be obtained. Based on the proportion of the mass of the core (M_c) to that of the Earth (M_e), M_c/M_e (= 0.35; Anderson and Kovach, 1967), we assumed that the proportion of the mass of the mantle to that of the Earth was 0.65 (= 1-0.35). Then, the obtained f_ρ was 1.2. Using this f_ρ (=1.2) and the densities from 40 to 2891 km in the PREM model, we estimated the densities of the magma ocean before the core segregation at each depth. As the density of each of the 44 layers, the mean of the densities at the top and the bottom at each layer was given to each layer.

2. Estimation of the X_{Si} of the silicate melts after the core segregation in a magma ocean and determination of the Si content in the core

2-1. Case (a): Eutectic melting model

The eutectic compositions at each depth of the magma ocean were obtained from the correlation curve shown in Fig. 9. The data point from the experiment undertaken at 41 GPa using the diamond anvil cell (DAC) (Ozawa et al., 2018) was excluded to give the correlation curve shown in Fig. 9 because the point was far from the correlation trend.

Large uncertainties in determining the chemical compositions due to the tiny size of the sample recovered from the DAC could be a plausible reason why the data point is out of the trend. The eutectic compositions in each of the 44 layers were determined by averaging the values of the top and the bottom of each layer.

Using the mass of each layer and the mean values of Si and Mg contents (wt.%) of the EL and EH of E-chondrite (Table S1), the masses of Si and Mg in each layer were estimated. Then, from the eutectic compositions in each layer and the mean X_{Si} *of* E-chondrite (0.553), the proportion of the melt to the solid was obtained for each layer, and subsequently the masses of Si and Mg in the silicate melt before the core segregation in each layer were estimated. The total masses of Si and Mg in the silicate melt in a magma ocean ($M_{\mathrm{Si\_melt\#1\_MO}}$ and $M_{\mathrm{Mg\_melt\#1\_MO}}$) can be obtained to be summation of those masses from the top of the layer to the layer which achieved to the bottom of the magma ocean as follows:

$$M_{\mathrm{Si\_melt\#1\_MO}} = \sum_{i=1}^{n} M_{\mathrm{Si\_melt\_lay-i}} \tag{4}$$

$$M_{\mathrm{Mg\_melt\#1\_MO}} = \sum_{i=1}^{n} M_{\mathrm{Mg\_melt\_lay-i}} \tag{5}$$

where $M_{\mathrm{Si\_melt\_lay-i}}$ and $M_{\mathrm{Mg\_melt\_lay-i}}$ are the masses of Si and Mg in the silicate melt in the *i*-th layer. Then, the mass of Si in the core was subtracted from $M_{\mathrm{Si\_melt\#1\_MO}}$ to obtain the mass of Si in the silicate melt after the core segregation, $M_{\mathrm{Si\_melt\#2}}$:

$$M_{\mathrm{Si\_melt\#2}} = M_{\mathrm{Si\_melt\#1\_sum}} - M_{\mathrm{c}} \times C_{\mathrm{Si\_c}} / 100 \quad (6)$$

where M_c is the mass of the core and $C_{\mathrm{Si\_c}}$ is the Si content (wt.%) in the core. As mentioned above, we assumed that magnesium was retained in the initial silicate melt without incorporation into the core. Then, the compositions (X_{Si}) of the silicate melt after the core segregation were calculated from the masses of Si ($M_{\mathrm{Si\_melt\#2}}$) and Mg ($M_{\mathrm{Mg\_melt\#1\_MO}}$).

In the eutectic melting model, the relation between the X_{Si} of the silicate melt after the core segregation and the depth of magma ocean are shown in Fig. S1a for the following four cases of Si contents in the core, 1.9, 2.7, 4.0 and 8.8 wt.%. Figure S1a indicates the followings: when Si content in the core is 2.7 wt.% concurrently that the depth of magma ocean is 700 km, the X_{Si} of the silicate melt after the core segregation shows 0.43 which is equivalent to that of the peridotitic mantle. Moreover, also when the content of Si in the core is 4.0 wt.% concurrently that the depth of magma ocean is 1200 km, the X_{Si} of the silicate melt after the core segregation shows 0.43 (Fig. S1a). Through those calculations, thus, Si content in the core was estimated to be 2.7 to 4.0 wt.% in the eutectic melting model if the source materials of the Earth are E-chondrites.

2-2. Case (b): Complete melting model

In the complete melting, the masses of Si and Mg in the initial melt ($M_{\mathrm{Si\_melt\#3}}$

and $M_{\mathrm{Mg\_melt\#3}}$, respectively) can be estimated using the contents of Si and Mg E-chondrite and the mass of MO which is summation of the masses of the layers from the top layer to the layer which located at the bottom of the magma ocean. Those values were obtained as follows:

$$M_{\mathrm{Si\_melt\#3}} = M_{\mathrm{MO}} \times C_{\mathrm{Si\_en\text{-}c}} /100 \tag{7}$$

$$M_{\mathrm{Mg\_melt\#3}} = M_{\mathrm{MO}} \times C_{\mathrm{Mg\_en\text{-}c}} /100 \tag{8}$$

where $C_{\mathrm{Si\_en\text{-}c}}$ and $C_{\mathrm{Mg\_en\text{-}c}}$ are the Si and Mg contents (wt.%) in the mean contents of the E-chondrite (Table S1), and M_{MO} is the mass of the magma ocean to the given depth, obtained as:

$$M_{\mathrm{MO}} = \sum_{i=1}^{n} M_{\mathrm{lay-i}} \tag{9}$$

where $M_{\mathrm{lay\text{-}i}}$ is the mass of the i-th layer. The mass of Si in the core was subtracted from $M_{\mathrm{Si\_melt\#2}}$ to obtain the mass of Si in the silicate melt after the core segregation, $M_{\mathrm{Si\_melt\#4}}$.

$$M_{\mathrm{Si\_melt\#4}} = M_{\mathrm{Si\_melt\#3}} - M_{\mathrm{c}} \times C_{\mathrm{Si\_c}} /100 \tag{10}$$

In also this complete melting model, we assumed that magnesium was retained in the initial silicate melt without incorporation into the core. Then, the compositions (X_{Si}) of the silicate melt after the core segregation were calculated from the masses of Si and Mg, $M_{\mathrm{Si\_melt\#4}}$ and $M_{\mathrm{Mg\_melt\#3}}$.

In Fig. S1b, we show the relation between the X_{Si} of the silicate melt after the core segregation and the depth of magma ocean in the complete melting model for the following four cases of Si contents in the core, 1.9, 5.0, 8.6 and 10.0 wt.%. Figure S1b indicates that Si content in the core estimated to be 5.0 to 10.0 wt.% in the complete melting model if the source materials of the Earth are E-chondrites. The obtained Si contents were higher than the those in the case of the eutectic melting model, which were 2.7 to 4.0 wt.%.

References

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