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Comments on: Higgins: "Closure in crystal size distributions (CSD), verification of CSD calculations, and the significance of CSD fans"

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ABSTRACT

Higgins found that the slope and intercept of crystal size distribution (CSD) are automatically correlated with each other when the crystal content is constant, but he failed to make the necessary conclusion that even if the crystal content is not constant, the slope and intercept are still automatically correlated, as shown by his example of Mt. Taranaki. CSD analysis in its present form is fundamentally flawed not only because of its automatically correlated slope and intercept but also because crystal content, together with crystal shape factor, is the only hidden information we can recover from CSD plots.

Higgins (2002, p. 172) concluded that at any fixed crystal content, "any process that changes the slope of the CSD [plot of logarithmic population density vs. size] must also change the intercept." Crystal size distribution (CSD) analysis based on the automatically correlated slope and intercept is not meaningful because "there is essentially only one degree of freedom" (Higgins 2002, p. 172). Higgins made the above findings but failed to step back further to make the necessary conclusion—that the CSD analysis is not meaningful at all, no matter whether the crystal content is fixed or not.

Rearranging Equation 1 from Higgins (2002) by moving $n_i(L)$ to the left side and volumetric proportion V_i to the right side, we have:

$$n_i(L) = v_i/(\sigma L^3 \Delta L) \quad (1)$$

Notice the original integral disappears because here v_i represents the volumetric proportion within size window ΔL (or dL), not the accumulated volumetric proportion V_i for all size windows. The terms $n_i(L)$, σ, and L are crystal population density, shape factor, and size within size window ΔL, respectively.

Equation 1 is only another representation of Higgins's Equation 1 and therefore "applies to all CSDs" (Higgins 2002, p. 172)[1]. The window size, ΔL, is not a true variable because it is just a scale of mathematical operation and bears no relationship with crystal nucleation and growth. The shape factor σ is a true variable but has a rather distant relationship with crystal residence time and nucleation and growth rates. The following discussion could include σ as a variable, but for simplicity, σ is regarded as a constant (Randolph and Larson 1988). Consequently there are three variables in Equation 1: crystal population density = $n_i(L)$; volumetric proportion or crystal content = v_i; and crystal size = L. If we know any two of the three variables, the remaining one would be determined (two degrees of freedom). When crystal content is fixed at any value from 0 to 100%, there is only one degree of freedom between $n_i(L)$ and L: as L changes, $n_i(L)$ must change correspondingly according to Equation 1. The closure effect presented by Higgins (2002) is nothing more than this simple relationship between $n_i(L)$ and L required by Equation 1 when the crystal content, v_i, is fixed. For example, in Figure 2d of Higgins (2002), a new closure limit curve with a crystal content of 15 vol% (not 100 vol% as shown), which is within the range of the actual crystal contents of the samples (~10–30 vol%), would go right through most of the samples. Any sample with straight CSD must fall right on its closure limit curve in the plot of characteristic length (or slope) vs. intercept, if its crystal content is regarded as its closure crystal content.

Closure, however, might not be appropriate to describe the relationship among crystal size, content, and population density required by Equation 1. It is true that the summation of all sub-crystal contents from all size bins of a sample must equal the (total) crystal content, but this is the requirement of addition rather than closure. The (total) crystal content can vary freely in all its space between 0 and 100% because it is the only variable of its kind in our correlation: it does not need to compete for closed space against other similar variables. A true case of closure would be the study of the correlation between the contents of crystals and groundmass because they must add up exactly to 100%. In our case, however, the sub-crystal contents from all size bins could add up to anything, from 0 to 100%. By ascribing closure to the crystal content that in fact

[1] Notice that "Equation 1 applies to all CSDs" (Higgins 2002, p. 172) is a universal truth and can be quoted independent of its context. Whether or not equations applicable only to straight CSDs are given in the same context has nothing to do with this quotation.

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changes freely in all its space, we are producing a rather awkward (and serious!) closure problem that occurs at every possible crystal content from 0 to 100% (Higgins 2002). Crystal size, content, and population density are governed by physical law (Eq. 1), not by closure. Likewise, that the slope and intercept of the plot of travel speed vs. time are automatically correlated with each other when travel distance is fixed says nothing about closure. More importantly, it is only natural (nothing wrong whatsoever) to have automatically correlated slope and intercept in the CSD plot, or any other plot like travel speed vs. time, if we fix crystal content, or travel distance, because the original two degrees-of-freedom problem is reduced artificially to a one degree-of-freedom problem.

Crystal content (or combined crystal content and shape factor, v_i/σ) is the only variable that determines the shape and position (line or curve) of the CSD plot of $\ln[n_i(L)]$ vs. L according to Equation 1 for similar crystal sizes[2]. For similar L, a plot with higher $\ln[n_i(L)]$ indicates higher crystal content and a plot with steeper slope indicates a higher volumetric proportion for small crystals than for large crystals. Inversely, what the CSD plot reflects is exactly the characteristic of crystal content (or v_i/σ), nothing else. Whether the crystal content is fixed or not makes no difference. Higgins (2002) "verification of CSD calculations" could only verify the crystal content as there are no other variables to be verified in the CSD plot. Crystal content can be measured more accurately, compared to both crystal size and number, by measuring areas in cross-sections using image-processing software and is more sensitive to magmatic processes (Higgins 2002, Fig. 2e). Because crystal content (or v_i/σ) is the only variable we can recover from CSDs, how additional information on crystal nucleation and growth rates and on residence time can be recovered from the slope and intercept, which are additionally subject to the inherited correlation of Pan (2001), needs to be explained.

The closure effect of Higgins (2002) and the inherited correlation of Pan (2001) are both based on Equation 1 and are related to each other, contrary to the statement that they are unrelated (Higgins 2002). Pan (2001) stated that for the correct two degrees of freedom, the CSD technique is flawed (more so for one degree of freedom) whereas Higgins (2002) stated that the technique is flawed for one degree of freedom when the crystal content is fixed. When the crystal content is not fixed, the inherited correlation contributes about 95% of the correlation observed in the CSD plot (Pan 2001, 2002a). The unexpected and unexplained significant correlation between the slope and intercept of the CSD plot (correlation coefficient R = 0.95) in Figure 2d of Higgins (2002), despite "the relatively low volumetric phase proportion of plagioclase, and its variability" (Higgins 2002, p. 173), is exactly the result of the inherited correlation. Higgins's closure effect would produce the same CSDs of all materials for similar crystal contents because crystal size, the only variable, determines everything, while Pan's inherited correlation allows different CSDs due to its two degrees of freedom. Differences in CSDs for similar crystal sizes are only due to differences in crystal content (or v_i/σ), nothing else.

Why Higgins (2002, p. 171) stated that the study of Pan (2001) "was flawed by inappropriate use of CSD equations" needs to be explained, because Equation 1, which applies to all CSDs in Higgins (2002), is the only equation used by Pan (2001), albeit in the form of weight proportion instead of volumetric proportion (Pan 2002b; Marsh and Higgins 2002)[3].

The CSD technique has been used extensively. In the light of its problems (Higgins 2002; Pan 2001), it is only reasonable for us to fully understand the technique and provide satisfactory answers to its major problems like those raised by Pan (2001, 2002a) before embarking on new endeavors in its present form (Randolph and Larson 1988; Marsh 1988, 1998). Whether CSD has been popular or not should not make any difference when we judge its scientific soundness.

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[2]Literally, of course, it is the crystal population density and size that determine the shape and position of the crystal population density vs. size plot. But crystal content is the hidden variable that determines the outlook of the plot for similar crystal sizes.

[3] An important correction to Higgins (2002) is due here: note that the writer was not a co-author in the mistaken reference to Marsh, B.D., Higgins, M.D., and Pan, Y. (2002). That reference should be to Marsh and Higgins (2002) only.